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Rapidity Gap Analysis and Event Rapidity Correlation *

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Abstract The relation between event rapidity gap fluctuation and rapidity correlations is derived. It is shown that the new and interesting parts included in rapidity gap analysis are the generalized two-particle rapidity-correlation moments. The physical meanings of these moments for specific two particles and moment orders are clarified.

Key words rapidity, gap, fluctuation, correlation

Most of our present knowledge about the bulk properties of high energy collisions is based on the study of single-particle inclusive distributions. The newly developed measures of event-by-event fluctuations can provide additional valuable insight into the particle production mechanism and dynamical evolution of the system, which cannot be extracted from the conventional single-particle inclusive distributions. Most suggested measures among them are expected to provide us with information about new phenomena, in particular, the formation of quark gluon plasma (QGP)[1]. While the physical motivation of the erraticity of rapidity gap analysis[2] is still unclear, it has been analyzed by several experimental groups[3].

In accounting for the statistical instability of event factorial moments due to the finite number of final state particles[4] under current hadron-hadron collision energies, R. C. Hwa et al. introduced the mean rapidity gap of qth order per event defined as[2]:

$$G_q = \frac{1}{N+1}\sum_{i=0}^{N} x_i^q = \frac{1}{N+1}\sum_{i=0}^{N}(X_{i+1}-X_i)^q,\ i=0,1,\cdots,N, \quad (1)$$

where, N is the number of final state particles, q is the order of moment and X_i is the corresponding cumulant variable of the rapidity of ith particle,

$$X_i = \frac{\int_{y_{\min}}^{y_i}\rho(y)\mathrm{d}y}{\int_{y_{\min}}^{y_{\max}}\rho(y)\mathrm{d}y} \quad (2)$$

where, y_i is the rapidity of ith particle, which is ordered in each event. $X_0=0$ and $X_{N+1}=1$ are added for all events so that the number of gap in an event is $N+1$. The advantage of working in the cumulant variable X is to reduce the influence of energy conservation. If the final state particles are closer to each other, the mean rapidity gap of qth order per event for $q>1$ will be smaller, and vice versa. Therefore, the mean rapidity gap per event describes the characteristic of rapidity distribution of final state particles.

Then R. C. Hwa et al. argued that the anomalous behavior of rapidity gap in its moment order will imply the erraticity of final state particle rapidity distribution[2]. However, this is not as clear as the anomalous scaling of moment function in nonlinear physics, which indicates the self-similar structure of observed system. In this letter, we will demonstrate the relation between the rapidity gap analysis and the rapidity correlations, and pick out the new interesting measures included in the rapidity gap analysis, which are different from the conventional ones.

When multiplicity N is not high enough, the mean rapidity gap per event defined by Eq.(1) is unstable in statistics[5]. Moreover, the relation between such ratio-like event-by-event fluctuation and inclusive distribution is difficult to derive[6]. Therefore, we start from the simplest event rapidity gap defined by:

$$S_2 = \sum_{i=2}^{N}(X_i - X_{i-1})^2 . \quad (3)$$

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Its fluctuation is:

$$\Delta S_2^2 = \langle S_2^2\rangle - \langle S_2\rangle^2, \qquad (4)$$

The first and second terms of this fluctuation can be readily written as:

$$\langle S_2\rangle^2 = \left\langle 2\sum_{i=1}^{N} X_i^2 - X_1^2 - X_N^2 - 2\sum_{i=2}^{N} X_iX_{i-1}\right\rangle^2$$
$$= \left(2\langle \sum_{i=1}^{N} X_i^2\rangle - \langle X_1^2 + X_N^2\rangle - 2\langle \sum_{i=2}^{N} X_iX_{i-1}\rangle\right)^2$$
$$= 4\langle \sum_{i=1}^{N} X_i^2\rangle^2 - 4\langle \sum_{i=1}^{N} X_i^2\rangle\langle X_1^2 + X_N^2\rangle + \langle X_1^2 + X_N^2\rangle^2 +$$
$$\left[4\langle X_1^2 + X_N^2\rangle - 8\langle \sum_{i=1}^{N} X_i^2\rangle\right]\times$$
$$\langle \sum_{i=2}^{N} X_iX_{i-1}\rangle + 4\langle \sum_{i=2}^{N} X_iX_{i-1}\rangle^2, \qquad (5)$$

and

$$\langle S_2^2\rangle=\langle \sum_{i=2}^{N}\sum_{j=2}^{N} X_i^2X_j^2\rangle + \langle \sum_{i=2}^{N}\sum_{j=2}^{N} X_{i-1}^2X_{j-1}^2\rangle +$$
$$2\langle \sum_{i=2}^{N}\sum_{j=2}^{N} X_i^2X_{j-1}^2\rangle - 4\langle \sum_{i=2}^{N}\sum_{j=2}^{N} X_i^2X_jX_{j-1}\rangle +$$
$$4\langle \sum_{i=2}^{N}\sum_{j=2}^{N} X_iX_{i-1}X_jX_{j-1}\rangle - 4\langle \sum_{i=2}^{N}\sum_{j=2}^{N} X_{i-1}^2X_jX_{j-1}\rangle$$
$$= 4\langle \sum_{i=1}^{N} X_i^4\rangle - 4\langle X_1^2\sum_{i=1}^{N} X_i^2\rangle - 4\langle X_N^2\sum_{i=1}^{N} X_i^2\rangle -$$
$$8\langle \sum_{i=1}^{N}\sum_{j=2}^{N} X_i^2X_jX_{j-1}\rangle + 4\langle X_1^2\sum_{i=2}^{N} X_iX_{i-1}\rangle +$$
$$4\langle X_N^2\sum_{i=2}^{N} X_iX_{i-1}\rangle + 4\langle \sum_{i=2}^{N}\sum_{j=2}^{N} X_iX_{i-1}X_jX_{j-1}\rangle +$$
$$4\langle \sum_{i=1}^{N}\sum_{j\neq i}^{N} X_i^2X_j^2\rangle + \langle (X_1^2 + X_N^2)^2\rangle. \qquad (6)$$

They are complex combinations of various rapidity correlations.

Let us define a general event two-particle rapidity-correlation moment as,

$$B_2^{p,q} = \sum_{i=1}^{N}\sum_{j\neq i}^{N} X_i^pX_j^q, p = 0,1,2,\cdots, q = 0,1,2,\cdots, \qquad (7)$$

in particular, the two-neighboring-particle event rapidity-correlation moment,

$$B_{\rm nb}^{p,q} = \sum_{i=2}^{N} X_i^pX_{i-1}^q, p = 0,1,2,\cdots, q = 0,1,2,\cdots, \qquad (8)$$

and the event rapidity moment,

$$B_2^{p,0} = \sum_{i=1}^{N} X_i^p, \quad p = 0,1,2,\cdots . \qquad (9)$$

With these definitions the event-by-event rapidity gap fluctuation, Eq.(4), can be clearly written as:

$$\Delta S_2^2 = 4\langle B_2^{2,2}\rangle + 4[\langle (B_{\rm nb}^{1,1})^2\rangle - \langle B_{\rm nb}^{1,1}\rangle^2] + 4\langle X_1^2B_{\rm nb}^{1,1}\rangle +$$
$$4\langle X_N^2B_{\rm nb}^{1,1}\rangle - 8\langle B_2^{2,0}B_{\rm nb}^{1,1}\rangle - [4\langle X_1^2\rangle + 4\langle X_N^2\rangle -$$
$$8\langle N\rangle\langle\langle X^2\rangle\rangle]\langle B_{\rm nb}^{1,1}\rangle + 4\langle N\rangle\langle\langle X^4\rangle\rangle -$$
$$4\langle N\rangle^2\langle\langle X^2\rangle\rangle^2 - 4[\langle X_1^2B_2^{2,0}\rangle + \langle X_N^2B_2^{2,0}\rangle -$$
$$\langle N\rangle\langle\langle X^2\rangle\rangle\langle X_1^2\rangle - \langle N\rangle\langle\langle X^2\rangle\rangle\langle X_N^2\rangle] +$$
$$\langle (X_1^2 + X_N^2)^2\rangle - \langle X_1^2 + X_N^2\rangle^2, \qquad (10)$$

with

$$\langle B_2^{p,0}\rangle = \langle N\rangle\langle\langle X^p\rangle\rangle, \quad p = 0,1,2,\cdots, \qquad (11)$$
$$\langle B_2^{p,q}\rangle = \langle \sum_{i=1}^{N}\sum_{j\neq i}^{N} X_i^pX_j^q\rangle$$
$$= \int {\rm d}X_i{\rm d}X_j\,\rho(X_i, X_j)X_i^pX_j^q, \qquad (12)$$

and

$$\langle B_{\rm nb}^{p,q}\rangle = \langle \sum_{i=2}^{N} X_i^pX_{i-1}^q\rangle$$
$$= \int {\rm d}X_i{\rm d}X_{i-1}\rho(X_i, X_{i-1})X_i^pX_{i-1}^q, \qquad (13)$$

where $p = 0,1,2,\cdots, q = 0,1,2,\cdots$. The $\langle\cdots\rangle$ and $\langle\langle\cdots\rangle\rangle$ are respectively the event average for event variables such as N, $B_2^{p,q}$, and the inclusive average for single particle variable X.

So the basic measures included in the event-by-event rapidity gap fluctuation are the event two-particle rapidity-correlation moment $\langle B_2^{p,q}\rangle$, in particular, $\langle B_2^{2,2}\rangle$, the event-by-event fluctuation of two-neighboring-particle rapidity-correlation moment $\langle (B_{\rm nb}^{1,1})^2\rangle - \langle B_{\rm nb}^{1,1}\rangle^2$, the correlation between event rapidity moment $B_2^{2,0}$, and the two-neighboring-particle rapidity-correlation moment of an event $B_{\rm nb}^{1,1}$. Those event variables characterize the rapidity correlation of an event and open an new world to explore the underlying dynamics of high energy collisions.

Usually, the investigation of two-particle correlation is focused on the measure of joint density $\rho(X_i, X_j)$[7]. The event rapidity correlation, such as Eq.(12) or Eq.(13), is the average of two-particle rapidity with respect to joint density $\rho(X_i, X_j)$ in given phase space region. So it is the summation of various correlation lengthes and should have better statistical stability. For the lowest moment order $p = 0$ and $q = 0$, if we restrict the species of particle i to the positively charged particles and j to the negatively charged particles, Eq.(12) reduces to the integration of charge correlation, i.e., the dynamic part of the event-by-event charge fluctuations[8].

In this letter, the relation between event-by-event rapidity gap analysis and event rapidity gap correlation is derived. The general two-particle rapidity-correlation moment is suggested. It has a clear physical meaning and less statistical instability and therefore is recommended in future measurements.

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快度间隔分析与逐事件快度关联*

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摘要　本文推导了逐事件快度间隔的起伏与逐事件快度关联的关系. 分析表明:快度间隔分析中新的和有意义的部分是推广的两粒子快度关联矩.对于一些特殊的两粒子和矩阶数这些矩有明确的物理意义.

关键词　快度　间隔　起伏　关联

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