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Cabibbo Suppressed Semileptonic Decay $D^0 \to \pi^- e^+ \nu_e$ *

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Abstract The transition rate for the Cabibbo suppressed decay $D^0 \to \pi^- e^+ \nu_e$ is calculated. This allows the extractions of the form factor $|f_+^\pi(0)|$ and the CKM matrix element $|V_{cd}|$ using the measured branching ratio.

Key words Semileptonic decay, decay rate, CKM matrix element $|V_{cd}|$, form factor $|f_+^\pi(0)|$

1 Introduction

The semileptonic decays of the charm mesons are theoretically simple to interpret because the effects of the weak and strong interactions can be well separated. The decay amplitude for the semileptonic mode $D^0 \to \pi^- e^+ \nu_e$ is propotional to the product of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cd}|$, which parametrizes the mixing between the quark mass eigenstates and the weak eigenstates, and the form factor $|f_+^\pi(0)|$ describing the strong interaction between the final state quarks. The Cabibbo-supressed decay $D^0 \to \pi^- e^+ \nu_e$ is illustrated by the Feynman diagram in Fig.1. Experimentally, the Cabibbo-favored decay $D^0 \to K^- e^+ \nu_e$ is well studied due to the relatively large number of reconstructed events. The Mark Ⅲ[1] finds the ratio of the CKM parameters $|V_{cd}/V_{cs}|^2$ and the form factor $|f_+^K(0)|$ based on measurements of the branching ratios for both $D^0 \to K^- e^+ \nu_e$ and $D^0 \to \pi^- e^+ \nu_e$. However, the extraction of the $|f_+^\pi(0)|$ and $|V_{cd}|$ in the Cabibbo-suppressed decay $D^0 \to \pi^- e^+ \nu_e$ has not been obtained from the direct measurement of the branching fraction yet. The magnitude of $|V_{cd}|$ is derived mainly from the neutrino and antineutrino production. With the better accuracy on $|V_{cd}|$, one is able to reproduce the experimental value for $|f_+^\pi(0)|$. Therefore $|V_{cd}|$ or $|f_+^\pi(0)|$ can be known with excellent precision, once the $D^0 \to \pi^- e^+ \nu_e$ distribution is accurately measured. In the near future, CLEO-c and BESⅢ detectors will provide high precision data in charm physics including data on D meson decays, which provide the possibility for a better understanding of the physics in charm sector.

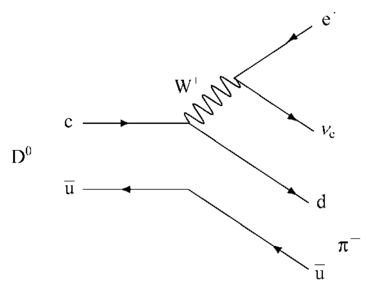

Fig.1. Semileptonic decay diagram.

In this paper, we derive the trantision rate for the Cabibbo suppressed decay of $D^0 \to \pi^- e^+ \nu_e$. This allows us to extract the form factor $|f_+^\pi(0)|$ using the measured branching fraction, the CKM matrix element $|V_{cd}|$ and the lifetime of the D^0 meson quoted from PDG[2]. We compare the extracted form factor with the theoretical results predicted recently by the lattice QCD calculation and the QCD sum rule. Conversely, we determine the CKM matrix element $|V_{cd}|$ using the form factor predicted by theoretical models and the measured

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branching fraction. We compare the extracted $|V_{cd}|$ with the value provided by PDG.

This paper is organized as follows. Section 2 gives the derivation of the decay width. The extractions of the form factor $|f_+^\pi(0)|$ and the CKM matrix element $|V_{cd}|$ are presented respectively in Section 3 and 4. The final section is reserved for summary.

2 Decay width

The decay rate for the semileptonic process $D^0 \to \pi^- e^+ \nu_e$ is described by

$$\Gamma = \frac{1}{2m_D}\int \frac{d^3\boldsymbol{p}_\pi}{(2\pi)^3}\frac{1}{2E_\pi}\int \frac{d^3\boldsymbol{p}_e}{(2\pi)^3}\frac{1}{2E_e}\int \frac{d^3\boldsymbol{p}_\nu}{(2\pi)^3}\frac{1}{2E_\nu}\times |\mathscr{A}|^2(2\pi)^4\delta^4(p_D - p_\pi - p_e - p_\nu). \quad (1)$$

The decay amplitude $\mathscr{A}$ is expressed as

$$\mathscr{A} = \frac{G_F}{\sqrt{2}}V_{cd}L^\mu H_\mu, \quad (2)$$

where G_F is the Fermi coupling constant and V_{cd} the CKM matrix element. The leptonic and hadronic currents are

$$L^\mu = \sum_{s,s'} u^s(p_\nu)\gamma^\mu(1-\gamma_5)v^{s'}(p_e), \quad (3)$$

$$H_\mu = (p_D + p_\pi - \frac{m_D^2 - m_\pi^2}{q^2}q)_\mu |f_+^\pi(q^2)| + \frac{m_D^2 - m_\pi^2}{q^2}q_\mu |f_-^\pi(q^2)|, \quad (4)$$

where s, s' are spin indices to be summed; $|f_+^\pi(q^2)|$ and $|f_-^\pi(q^2)|$ denote the transverse and longitudinal form factors at the square of the four-momentum transfer $q = p_D - p_\pi$.

Substituting the leptonic and hadronic currents into (2), we have

$$|\mathscr{A}|^2 = G_F^2 |V_{cd}|^2 Tr[\not{p}_\nu(\not{p}_D + \not{p}_\pi)\not{p}_e \times (\not{p}_D + \not{p}_\pi)(1-\gamma_5)] |f_+^\pi(q^2)|^2 \quad (5)$$

One can notice that only the part $(p_D + p_\pi)|f_+^\pi(q^2)|$ in the hadronic current contributes to the semileptonic process.

We integrate first the pion momentum integral in (1) by inserting the identity

$$\int dE_\pi \theta(E_\pi)[\delta(E_\pi - \sqrt{\boldsymbol{p}_\pi^2 + m_\pi^2}) + \delta(E_\pi + \sqrt{\boldsymbol{p}_\pi^2 + m_\pi^2})] = 1 \quad (6)$$

to obtain

$$\Gamma = \frac{1}{2m_D}\frac{1}{(2\pi)^2}\int \frac{d^3\boldsymbol{p}_e}{2E_e}\int \frac{d^3\boldsymbol{p}_\nu}{2E_\nu}\delta[(p_D - p_e - p_\nu)^2 - m_\pi^2] |A|^2_{p_\pi = p_D - p_e - p_\nu}. \quad (7)$$

In the rest frame of the D meson, one can express the three-momentum integrals of the electron and neutrino in terms of their energy integrals

$$\Gamma = \frac{G_F^2}{8\pi^3}|V_{cd}|^2 m_D^4 \int dE_e \int dE_\nu (4E_eE_\nu + m_D^2 - m_\pi^2 - 2m_DE_e - 2m_DE_\nu) |f_+^\pi(q^2)|^2. \quad (8)$$

Using the energy conservation, the four-momentum relations $q^2 = (p_D - p_\pi)^2$ and $t^2 = (p_D - p_\nu)^2$ can be written respectively as

$$E_e = \frac{(q^2 + t^2 - m_\pi)^2}{2m_D}, E_\nu = \frac{(m_D^2 - t^2)^2}{2m_D}. \quad (9)$$

We now change integral variables in (8) using (9) as

$$\Gamma = \frac{G_F^2}{32\pi^3 m_D}\int dq^2 \int dt^2 [-\frac{q^4}{m_D^2} + \left(1 - \frac{q^2}{m_D^2} + \frac{m_\pi^2}{m_D^2}\right)q^2 - m_\pi^2] |f_+^\pi(q^2)|^2. \quad (10)$$

By integrating t^2 integral, we obtain

$$\Gamma = \frac{G_F^2|V_{cd}|^2}{192\pi^3 m_D^3}\int dq^2 [(q^2 - m_D^2 - m_\pi^2)^2 - 4m_D^2m_\pi^2]^{3/2} |f_+^\pi(q^2)|^2, \quad (11)$$

where the integral limits used are

$$t^2 \geqslant \frac{m_D^2 - q^2 + m_\pi^2}{2} - \frac{\sqrt{(m_D^2 - q^2 + m_\pi^2)^2 - 4m_D^2m_\pi^2}}{2}, \quad (12)$$

$$t^2 \leqslant \frac{m_D^2 - q^2 + m_\pi^2}{2} + \frac{\sqrt{(m_D^2 - q^2 + m_\pi^2)^2 - 4m_D^2m_\pi^2}}{2}. \quad (13)$$

For the q^2 dependence of the form factor, we take the BSW model[3], that is the poledominated form of the form factor

$$|f_+^\pi(q^2)| = \frac{|f_+^\pi(0)|}{1 - q^2/m_*^2}, \quad (14)$$

where $m_* = 2.11\text{Gev}/c^2$ is the mass of the lowest-mass resonance, $f_+(0)$ is the form factor evaluated at the four momentum transfer q equal to zero. By performing q^2 integral, in which the integral limits are $0 \leqslant q^2 \leqslant (m_D^2 - m_\pi)^2$, one obtains the decay width expressed in term of the CKM matrix $|V_{cd}|$ and the form factor $|f_+^\pi(0)|$

$$\Gamma = 3.01 |V_{cd}|^2 |f_+^\pi(0)|^2 \times 10^{11} s^{-1}. \quad (15)$$

This allows to extract $|f_+^\pi(0)|$ or $|V_{cd}|$, using the measured branching fraction and the lifetime of the D^0 meson.

3 Form factor $|f_+^\pi(0)|$

The form factor $|f_+^\pi(0)|$ can be expressed in term of the branching ratio, the CKM matrix element, and the D^0 lifetime

$$|f_+^{\pi}(0)| = \sqrt{\frac{Br(D^0 \to \pi^- e^+ \nu_e)}{3.01 |V_{cd}|^2 \tau_D^0 \times 10^{11} s^{-1}}}. \quad (16)$$

If we take the PDG[2] values for $Br(D^0 \to \pi^- e^+ \nu_e) = (0.36 \pm 0.06)\%$, $|V_{cd}| = 0.224 \pm 0.016$ and $\tau_D^0 = (411.7 \pm 2.7) \times 10^{-15}$, we obtain the measurement of the form factor to be

$$|f_+^{\pi}(0)| = 0.76 \pm 0.08. \quad (17)$$

The error is obtained from the error propagation formula

$$\Delta f = \frac{1}{2} f \sqrt{(\frac{\Delta B}{B})^2 + 4(\frac{\Delta V}{V})^2 + (\frac{\Delta \tau}{\tau})^2}, \quad (18)$$

where $f \equiv f_+^{\pi}(0)$, $B \equiv Br(D^0 \to \pi^- e^+ \nu_e)$, $V \equiv V_{cd}$ and $\tau \equiv \tau_D^0$. The extracted form factor is compared with the predicted form factors calculated most recently by various theoretical models and enumerated in Table 1. The second column gives the form factor predicted by QCDSR, whereas the third and fourth columns lists the predictions of LQCD. The last column presents the result extracted from (15).

Table 1.　The form factors.

	QCD sum rule[4]	lattice QCD[5]	lattice QCD[6]	result
$\lvert f_+^{\pi}(0)\rvert$	0.65 ± 0.11	$0.64 \pm 0.05^{+0.00}_{-0.07}$	$0.57 \pm 0.06^{+0.01}_{-0.00}$	0.76 ± 0.08

4　CKM matrix element $|V_{cd}|$

Reversing the argument that presented in the previous section, the measured values of the CKM matrix element $|V_{cd}|$ can be obtained. Namely, by interchanging the form factor with CKM matrix element in (16) and (18), one can calculate $|V_{cd}|$. Table 2 presents the results of $|V_{cd}|$ obtained using the form factors predicted by various theoretical models, and compared with the PDG value. The second column gives $|V_{cd}|$ obtained by using the form factor predicted from the QCD sum rule, whereas the third and fourth columns list the result from the lattice QCD. The last column gives the PDG result.

Table 2.　The CKM matrix elements.

$\lvert f_+^{\pi}(0)\rvert$	QCD sum rule[4]	lattice QCD[5]	lattice QCD[6]	PDG
$\lvert V_{cd}\rvert$	0.262 ± 0.049	$0.266^{+0.030}_{-0.042}$	$0.299^{+0.041}_{-0.040}$	0.224 ± 0.016

5　Summary

In this paper, we derive the transition rate for the Cabibbo supressed decay $D^0 \to \pi^- e^+ \nu_e$. Using the measured branching ratio and the CKM matrix element $|V_{cd}|$ presented in PDG, we extracted the form factor $|f_+^{\pi}(0)|$. In addition, we determined the $|V_{cd}|$ using the form factors predicted by the QCD sum rule and lattice QCD calculations.

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Cabibbo压制半轻子衰变过程 $D^0 \to \pi^- e^+ \nu_e$ *

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摘要 计算了Cabibbo压制过程 $D^0 \to \pi^- e^+ \nu_e$ 衰变率.利用计算结果和测得分支比来研究形状因子 $|f_+^{\pi}(0)|$ 及CKM矩阵元 $|V_{cd}|$.

关键词 半轻子衰变 衰变率 矩阵元 $|V_{cd}|$ 形状因子 $|f_+^{\pi}(0)|$

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