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ρ-ω Interference in J/ψ-Decays and $\rho\to\pi^+\pi^-\pi^0$ Decay*

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Abstract We study ρ-ω interference by analyzing $J/\psi\to\pi^+\pi^-\pi^0\pi^0$. PDG-2002 data on J/ψ decays into PP and PV (P denotes pseudoscalar mesons; V, vector mesons) are used to fit a generic model which describes the J/ψ decays. From the fits, we obtain anomalously large branching ratio $Br(\rho^0\to\pi^+\pi^-\pi^0)\approx10^{-3}$—$10^{-2}$. A theoretical analysis for it is also provided, and the prediction is in good agreement with the anomalously large $Br(\rho^0\to\pi^+\pi^-\pi^0)$. By the fit, we also get the η-η′ mixing angle $\theta=-19.68°\pm1.49°$ and the constituent quark mass ratio $m_u/m_s\approx0.6$ which are all reasonable.

Key words J/ψ-decays, ρ-ω interference, $SU(3)$-breaking effect, η-η′ mixing

1 Introduction

Recently it has been predicted in Ref.[1] that there is a large isospin symmetry breaking enhancement effect in the decay $\rho^0\to\pi^0\gamma$ comparing with $\rho^\pm\to\pi^\pm\gamma$ due to ρ-ω interference, which was called as hidden isospin-breaking effects in Ref.[1]. This prediction has been confirmed by the renewed data in PDG-2002[2]. Following the discussion of Ref.[1], it could be expected that a similar large hidden isospin-breaking effect should also exist in $\rho^0\to3\pi$ due to ρ-ω interference. In this paper we try to analyze $J/\psi\to(\rho^0,\omega)(\pi^0,\eta,\eta')\to(3\pi)(\pi^0,\eta,\eta')$, and to reveal ρ-ω interference effect and then finally to abstract out the branching ratio of $\rho^0\to3\pi$. Such an analysis should be necessary for further confirming the enhancement effects mentioned above, and be also interesting for the G-parity violating process studies.

It is very difficult to directly measure $Br(\rho^0\to3\pi)$ experimentally both because $\Gamma_\rho\gg(m_\omega-m_\rho)$ and because the G-parity conserving decay mode of $\omega\to3\pi$ is dominate. This is the reason why there is still no a reliable value for $Br(\rho\to3\pi)$ yet so far in the literature[2,3]. Fortunately, this quantity can be obtained by fitting the data of J/ψ→PP and PV (where P denotes the pseudoscalar meson nonet (π, K, η, η′), and V, the vector mesons (ρ, ω, K^*, ϕ))[4]. Actually, more than fifteen years ago, $Br(\rho^0\to3\pi)$ was estimated in Ref.[4] by using the MARK-Ⅲ data of (J/ψ→PP, PV)-decays with P = (π, K) and V = (ρ, ω, K^*). However, people including the authors of Ref. [4] does not think their result of $Br(\rho^0\to3\pi)$ is very reliable (see the discussion in Ref. [4] and Ref. [2]). The reasons are multi-ply, and some of them may be as follows: 1) the J/ψ-decay data quality at that time was not good enough; 2) the fitting is not complete because the processes of (J/ψ→ηV, η′V, Pϕ) were not considered; 3) lacking a theoretical understanding why their result is so significantly different from the result of Ref. [3] which is quoted by PDG[2]. Our motive of this paper is to try to solve those problems: 1) We shall use nowadays data of (J/ψ→PP, PV) in PDG-2002[2] to perform the fit; 2) (J/ψ→ηV, η′V, Pϕ) will be considered in our analysis; 3) And the rationality of the result will be argued, i.e., we will see that the result is just consistent with the theoretical analysis in Ref. [1].

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The contents of the paper are organized as follows. In section 2 we describe the ρ-ω interference in the process $J/\psi\to\pi^+\pi^-\pi^0\pi^0$, and give the the branching ratio formulas for ($J/\psi\to$PP, PV). In section 3, by using the PDG-2002 J/ψ decay data and the formulas given in the section 2 we perform the datum fits. The $Br(\rho^0\to\pi^+\pi^-\pi^0)$ is obtained. The section 4 is devoted to estimate $Br(\rho^0\to\pi^+\pi^-\pi^0)$ by theoretical analysis. Finally, we briefly discuss the results.

2 ρ-ω Interference and branching ratios of J/ψ decays into PV and PP

The elusive $\rho^0\to\pi^+\pi^-\pi^0$ decay could be observed in J/ψ decays into the $\pi^+\pi^-\pi^0\pi^0$ final state[4]. Indeed, this decay can proceed through the interfering channels $J/\psi\to\omega\pi^0\to\pi^+\pi^-\pi^0\pi^0$ and $J/\psi\to\rho\pi^0\to\pi^+\pi^-\pi^0\pi^0$. Because $J/\psi\to\rho\pi^0$ is caused both by strong interaction via 3 gluons and by electromagnetic (EM) interaction and $J/\psi\to\omega\pi^0$ is caused by EM interaction merely, the passibility of the decay $J/\psi\to\rho\pi^0$ is much larger than one of $J/\psi\to\omega\pi^0$, i.e., $\Gamma(J/\psi\to\rho\pi^0)\gg\Gamma(J/\psi\to\omega\pi^0)$ (by using the 2002-PDG data[2] we have $\Gamma(J/\psi\to\rho\pi^0)\approx 2.5\times 10^2\,\Gamma(J/\psi\to\omega\pi^0)$). Consequently, even though $\Gamma(\omega\to\pi^+\pi^-\pi^0)$ may be much larger than $\Gamma(\rho\to\pi^+\pi^-\pi^0)$, it is still hopeful to measure $\Gamma(\rho\to\pi^+\pi^-\pi^0)$ by studying the ρ-ω interference effects in the process of $J/\psi\to\pi^+\pi^-\pi^0\pi^0$.

To $J/\psi\to(\rho,\omega)\pi^0\to3\pi\pi^0$, the corresponding s-dependence Breit-Wigner is written as[4]

$$F(s)\equiv BW_\omega(s)+\epsilon e^{i\theta'}BW_\rho(s),\qquad(1)$$

where

$$BW_i=\sqrt{\frac{m_i\Gamma_i}{\pi}}\,\frac{1}{m_i^2-s-\mathrm{i}m_i\Gamma_i}$$

is the normalized Breit-Wigner curve for $i=\omega$, ρ resonance with the mass of m_i and total width of Γ_i. The factor $\epsilon(\theta')$ is the modulus (phase) of the amplitude proceeding through the ρ resonance relative to ω resonance, which can be written as

$$\epsilon=\frac{|A(J/\psi\to\rho^0\pi^0)|}{|A(J/\psi\to\omega\pi^0)|}\times\sqrt{\frac{Br(\rho^0\to3\pi)}{Br(\omega\to3\pi)}}\qquad(2)$$

and $\theta'=\theta_{(J/\psi\to i\pi^0)}+\theta_{(i\to3\pi)}$ with $i=\rho,\omega$.

As we have the relations $m_\rho\approx m_\omega=m$ and $\Gamma_\rho\neq\Gamma_{\omega'}$, the total effect is integrated over s,

$$\int ds\,|F(s)|^2=1+\epsilon^2+2\epsilon\cos\theta'\frac{2\sqrt{\Gamma_\omega\Gamma_\rho}}{\Gamma_\omega+\Gamma_\rho}.\qquad(3)$$

The third term of the expression (3), $2\epsilon\cos\theta'\times\left(\frac{2\sqrt{\Gamma_\omega\Gamma_\rho}}{\Gamma_\omega+\Gamma_\rho}\right)$, is the ρ-ω interference term. According to Ref. [4]1), the phase θ' is equal to zero, and the interference effect produces a magnificent factor as a whole (we assume that the ω resonance contribution is one), i.e.

$$\int ds\,|F(s)|^2=1+\epsilon^2+\epsilon,\qquad(4)$$

where $\Gamma_\rho\approx16\Gamma_\omega$[2] has been used. Then, the interference between ρ and ω in the $J/\psi\to4\pi$ provides a relation as follows,

$$Br(J/\psi\to\omega(\rho)\pi^0\to4\pi)=(1+\epsilon^2+\epsilon)Br(J/\psi\to\omega\pi^0\to4\pi).\qquad(5)$$

The $Br(J/\psi\to\omega(\rho)\pi^0\to4\pi)$ can be detected directly in experiments, but there is no a direct experiment way to measure $Br(J/\psi\to\omega\pi^0\to4\pi)$ in the right hand side of the above relation. Fortunately, it can be got by fitting the branch ratios of $J/\psi\to$PP and PV[4]. As both $Br(J/\psi\to\omega(\rho)\pi^0\to4\pi)$ and $Br(J/\psi\to\omega\pi^0\to4\pi)$ are known, we will have ϵ by Eq. (5) and then obtain desired quantity $Br(\rho^0\to3\pi)$ via Eq. (2).

Decays of J/ψ into (PV) and into (PP) can be factorized by a very simple and general consideration described as follows[4]. The decays proceed through a strongly interacting three-gluon (ggg) intermediate state and through electromagnetic interaction mediated by one photon γ and (γgg) states. In the gluonic case, there are two $I=0$ transitions $c\bar{c}\to ggg\to\frac{(u\bar{u}+d\bar{d})}{\sqrt{2}}$ and $c\bar{c}\to ggg\to s\bar{s}$, which are proportional to the amplitude A and $\frac{\lambda A}{\sqrt{2}}$ respectively. The parameter λ accounts for flavor-$SU(3)$ breaking effect. The electromagnetic transitions generate the $I=1$ state $\left(\frac{(u\bar{u}-d\bar{d})}{\sqrt{2}}\right)$ and two $I=0$ states $\left(\frac{(u\bar{u}+d\bar{d})}{\sqrt{2}}\text{ and }s\bar{s}\right)$. Their amplitude are proportional to

1) $\theta=0$ in $J/\psi\to(\rho\text{ or }\omega)\pi^0\to\pi^+\pi^-2\pi^0$ and $\theta=\pi/2$ in $J/\psi\to(\rho\text{ or }\omega)\pi^0\to\pi^+\pi^-\pi^0$ in the paper of A. Bramon and J. Casulleras, Phys. Lett., 1986, 173B:97.

$3a$, a and $-\sqrt{2}\lambda a$ respectively. The flavor space wave functions for P including η and η' read

$$P = \lambda^a\Phi^a = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}, \tag{6}$$

where $\eta_8 = \eta\cos\theta + \eta'\sin\theta$ and $\eta_1 = \eta'\cos\theta - \eta\sin\theta$ with θ as η-η' mixing angle, and the V-wave functions including ϕ's read

$$V = \sqrt{2}\begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \tag{7}$$

With the above, the decay amplitudes of (J/ψ→PP, PV) are as follows,

$$A(\pi^+\pi^-) = 3a, \tag{8}$$

$$A(K^+K^-) = \frac{1}{2}(1-\lambda)A + (2+\lambda)a, \tag{9}$$

$$A(K^0\bar{K}^0) = \frac{1}{2}(1-\lambda)A - (1-\lambda)a, \tag{10}$$

$$A(\rho^0\pi^0) = f_V(A+a), \tag{11}$$

$$A(K^{*+}K^-) = f_V\left[\frac{1}{2}(1+\lambda)A + (2-\lambda)a\right], \tag{12}$$

$$A(K^{*0}\bar{K}^0) = f_V\left[\frac{1}{2}(1+\lambda)A - (1+\lambda)a\right], \tag{13}$$

$$A(\omega\pi^0) = f_V(3a), \tag{14}$$

$$A(\rho\eta') = f_V(3aX_{\eta'}), \tag{15}$$

$$A(\omega\eta') = f_V[(A+a)X_{\eta'} + \sqrt{2}rA(\sqrt{2}X_{\eta'} + Y_{\eta'})], \tag{16}$$

$$A(\rho\eta) = f_V(3aX_\eta), \tag{17}$$

$$A(\omega\eta) = f_V[(A+a)X_\eta + \sqrt{2}rA(\sqrt{2}X_\eta + Y_\eta)], \tag{18}$$

$$A(\phi\eta) = f_V\left[(A-2a)\lambda Y_\eta + rA(\sqrt{2}X_\eta + Y_\eta)\frac{(1+\lambda)}{2}\right], \tag{19}$$

$$A(\phi\eta') = f_V\left[(A-2a)\lambda Y_{\eta'} + rA(\sqrt{2}X_{\eta'} + Y_{\eta'})\frac{(1+\lambda)}{2}\right], \tag{20}$$

where $X_\eta = \sqrt{\frac{1}{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta$, $X_{\eta'} = \sqrt{\frac{1}{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta$, $Y_\eta = -X_{\eta'}$, $Y_{\eta'} = X_\eta$. The additional parameter r is the relative weight of the disconnected diagram to connected diagram for the decays involving the final state η or η'[5,6]. The Eqs.(8)—(14) are same as ones in Ref.[4], and others are new.

The corresponding branching ratios of these decays are following:

$$Br(\pi^+\pi^-) = 9a^2, \tag{21}$$

$$Br(K^+K^-) = \left|\frac{1}{2}(1-\lambda)A + (2+\lambda)ae^{i\phi}\right|^2, \tag{22}$$

$$Br(K^0\bar{K}^0) = \left|\frac{1}{2}(1-\lambda)A - (1-\lambda)ae^{i\phi}\right|^2, \tag{23}$$

$$Br(\rho^0\pi^0) = f_V^2|(A + ae^{i\phi})|^2, \tag{24}$$

$$Br(K^{*+}K^-) = f_V^2\left|\frac{1}{2}(1+\lambda)A + (2-\lambda)ae^{i\phi}\right|^2, \tag{25}$$

$$Br(K^{*0}\bar{K}^0) = f_V^2\left|\frac{1}{2}(1+\lambda)A - (1+\lambda)ae^{i\phi}\right|^2, \tag{26}$$

$$Br(\omega(\rho^0)\pi^0) = (1+\epsilon+\epsilon^2)f_V^2 9a^2, \tag{27}$$

$$Br(\rho\eta') = f_V^2|3aX_{\eta'}|^2, \tag{28}$$

$$Br(\omega\eta') = f_V^2|(A + ae^{i\phi})X_{\eta'} + \sqrt{2}rA(\sqrt{2}X_{\eta'} + Y_{\eta'})|^2, \tag{29}$$

$$Br(\rho\eta) = f_V^2|3aX_\eta|^2, \tag{30}$$

$$Br(\omega\eta) = f_V^2|(A + ae^{i\phi})X_\eta + \sqrt{2}rA(\sqrt{2}X_\eta + Y_\eta)|^2, \tag{31}$$

$$Br(\phi\eta) = f_V^2\left|(A - 2ae^{i\phi})\lambda Y_\eta + rA(\sqrt{2}X_\eta + Y_\eta)\frac{1+\lambda}{2}\right|^2, \tag{32}$$

$$Br(\phi\eta') = f_V^2\left|(A - 2ae^{i\phi})\lambda Y_{\eta'} + rA(\sqrt{2}X_{\eta'} + Y_{\eta'})\frac{1+\lambda}{2}\right|^2, \tag{33}$$

where ϕ is their relative phase between A and a, and the parameter A and a are real. In the Eq.(24) the ρ-ω interference effect is subtracted from the branching ratio of J/ψ→ρπ⁰. In the Eq.(27), a magnificent factor $1+\epsilon+\epsilon^2$ has been added due to Eq.(5) in order to taking the ρ-ω interference effects into account. Actually, through directly detecting the data of $J/\psi \to 4\pi$ one can only get $Br(\omega(\rho^0)\pi^0)$ rather than $Br(\omega\pi^0)$ which is equal to $f_V^2 9a^2$.

In the branching ratio formulae of Eqs. (21)—(33) we do not write out the corresponding phase-space factors explicitly which are proportional to the cube of the final momenta in two-body decays. They will be taken into account in the practical phenomenological fit later.

3 Datum fit to obtain $Br(\rho \to \pi^+\pi^-\pi^0)$

Let us now use the data in PDG-2002[2] to perform the fit to all branching ratios of Eqs. (21)—(33). This fit will lead to determining the ϵ and $Br(\rho \to \pi^+\pi^-\pi^0)$. The experimental branching ratio data of PDG-2002 are listed in the second column of Table 1.

Firstly, following Ref. [4], we use the branch ratio data of ($J/\psi \to PP$, $V(\pi^0, K)$) only to perform a fit to Eqs. (21)—(27) (call it as (PP, $V(\pi^0, K)$)-fit hereafter). In this case, there are seven equations with six adjustable free parameters a, A, λ, ϕ, f_V, ϵ, and hence it is an over-determination problem with potential of predictions. The fit with minimum $\chi^2 = 0.46$ leads to the values of the parameters and seven corresponding branching ratios listed in the third column of Table 1, in which $(\omega\pi^0)_{uncor}$ and $(\omega\pi^0)_{cor}$ represent $Br(J/\psi \to \omega(\rho)\pi^0 \to 4\pi)$ and $Br(J/\psi \to \omega\pi^0 \to 4\pi)$ respectively, i.e.,

$$Br(J/\psi \to \omega\pi^0)_{cor} = f_V^2 9a^2. \qquad (34)$$

In the fit (see Table 1), we have $(\omega\pi^0)_{uncor} = (4.2 \pm 0.61) \times 10^{-4}$, $a = 0.21 \pm 0.02$, $A = 2.94 \pm 0.72$ and the interference factor $\epsilon = 0.71 \pm 0.58$, then we obtain $(\omega\pi^0)_{cor} = f_V^2 9a^2 = (1.89 \pm 0.83) \times 10^{-4}$. In other hand, from Eqs. (2), (11), (14), ϵ reads

$$\epsilon = \frac{|A + ae^{i\phi}|}{|3a|} \times \sqrt{\frac{Br(\rho \to 3\pi)}{Br(\omega \to 3\pi)}}. \qquad (35)$$

Then the branching ratio of $\rho \to \pi^+\pi^-\pi^0$ is predicted as follows,

$$Br(\rho \to 3\pi) = Br(\omega \to 3\pi) \times \left(\frac{3|a|}{|A + ae^{i\phi}|}\right)^2 \epsilon^2. \qquad (36)$$

Substituting the a, A and ϕ values obtained from the fit (see the third column of Table 1) and experiment data of $Br(\omega \to 3\pi)$ into Eq. (34) and Eq. (36), we then obtain the (PP, $V(\pi^0, K)$)-fit's results as follows,

$$Br(J/\psi \to \omega\pi^0)_{cor}|_{(PP,V(\pi^0,K))} = (1.89 \pm 0.83) \times 10^{-4}, \qquad (37)$$

$$Br(\rho \to \pi^+\pi^-\pi^0)|_{(PP,V(\pi^0,K))} = (2.0 \pm 1.64) \times 10^{-2}. \qquad (38)$$

Table 1. The second column displayed the experimental values for the branch ratios of J/ψ→PP and J/ψ→PV in PDG-2000 datum. The results of a fit to the first seven branching ratios is listed in the third column. The results of a fit to the total thirteen branching ratios is listed in the forth column.

J/ψ decay	PDG-2002 ($\times 10^{-4}$)	a partial fit 1 ($\times 10^{-4}$)	a global fit 2 ($\times 10^{-4}$)
1. $\pi^+\pi^-$	1.47 ± 0.23	1.44 ± 0.23	1.92 ± 0.05
2. K^+K^-	2.37 ± 0.31	2.45 ± 0.28	2.04 ± 0.08
3. $K^0\bar{K}^0$	1.08 ± 0.14	1.06 ± 0.14	0.87 ± 0.05
4. $\rho^0\pi^0$	42.0 ± 5	43.04 ± 4.48	41.97 ± 0.68
5. $K^{*+}K^-$	25.0 ± 2.0	24.11 ± 1.41	23.64 ± 0.5
6. $K^{*0}K^0$	21.0 ± 2.0	21.66 ± 1.7	24.21 ± 0.49
7. $(\omega\pi^0)_{uncor}$	4.2 ± 0.6	4.2 ± 0.61	4.2 ± 0.2
8. $(\rho\eta')$	1.05 ± 0.18		0.7 ± 0.05
9. $(\omega\eta')$	1.67 ± 0.25		1.73 ± 0.11
10. $(\rho\eta)$	1.93 ± 0.23		1.82 ± 0.08
11. $(\omega\eta)$	15.8 ± 1.6		18.32 ± 0.36
12. $(\phi\eta)$	6.5 ± 0.7		5.85 ± 0.23
13. $(\phi\eta')$	3.3 ± 0.4		2.55 ± 0.23
χ^2		0.46/1	21.4/5
EDM		0.45×10^{-6}	0.69×10^{-6}
fit a		0.21 ± 0.02	0.24 ± 0.012
A		2.94 ± 0.72	2.69 ± 0.17
λ		0.6 ± 0.1	0.62 ± 0.03
ϕ		1.37 ± 0.14	1.6 ± 0.11
f_V		1.26 ± 0.36	1.38 ± 0.1
ϵ		0.71 ± 0.58	0.3 ± 0.16
θ			−0.343 ± 0.026
r			−0.144 ± 0.001
$(\omega\pi^0)_{cor}$		$(1.89 \pm 0.83) \times 10^{-4}$	$(3.02 \pm 0.2) \times 10^{-4}$
$\rho \to 3\pi$		$(2.0 \pm 1.64) \times 10^{-2}$	$(0.59 \pm 0.315) \times 10^{-2}$

Secondly, we perform more complete datum fit in which the processes of $J/\psi \to V\eta$ and $J/\psi \to V\eta'$ are included. In this case, there are 13 Eqs. ((21)—(33)) and eight free parameters: a, A, λ, ϕ, f_V, ϵ, θ, r. And hence it is an over-determination problem with more constraints, and will be called as (PP, $V(\pi^0, K, \eta, \eta')$)-fit hereafter. The results are as follows,

$$Br(J/\psi \to \omega\pi^0)_{cor}|_{(PP,V(\pi^0,K,\eta,\eta'))} = (3.02 \pm 0.2) \times 10^{-4}, \qquad (39)$$

$$Br(\rho\to\pi^+\pi^-\pi^0)|_{(\mathrm{PP,V}(\pi^0,\mathrm{K},\eta,\eta'))} = (0.59\pm0.315)\times10^{-2}, \tag{40}$$

$$\theta = -0.343\pm0.026 = -19.68°\pm1.49°, \tag{41}$$

where η-η' mixing angle θ is agreement with one in Refs. [5,6], and both $Br(\mathrm{J}/\psi\to\omega\pi^0)_{\mathrm{cor}}$ and $Br(\rho\to\pi^+\pi^-\pi^0)$ are reasonable agreement with the results Eqs. (37), (38) obtained by (PP, V(π^0,K))-fit within the errors.

The parameter λ is the constituent quark mass ratio m_u/m_s which should be about 0.6[6—8] due to light flavor $SU(3)$-breaking. The results of $\lambda\approx0.6\pm0.1$ for (PP, V(π^0, K))-fit and $\lambda\approx0.62\pm0.03$ for (PP, V(π^0, K, η,η'))-fit indicate the fits meet this requirement, and, hence, the results yielded by them are rather reliable.

4 Large isospin breaking effect in decay $\rho^0\to\pi^+\pi^-\pi^0$

In this section, following Ref. [1], we provide a theoretical estimation to $Br(\rho^0\to\pi^+\pi^-\pi^0)$. Using Feynman propagators method, the on-shell amplitude[1] of the decay $\rho\to\pi^+\pi^-\pi^0$ is determined by

$$\mathcal{M}_{\rho^0\to3\pi} = \left(f_{\rho3\pi} + \frac{\Pi_{\rho\omega}(p^2)f_{\omega3\pi}}{p^2 - m_\omega^2 + \mathrm{i}m_\omega\Gamma_\omega}\right)\Bigg|_{p^2=m_\rho^2}, \tag{42}$$

where, the momentum-dependent ρ^0-ω interference amplitude $\Pi_{\rho\omega}(p^2)$ is defined by the ρ-ω interaction Lagrangian $\mathscr{L}_{\rho\omega}$ as follows,

$$\mathscr{L}_{\rho\omega} = \int\frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}p\cdot x}\Pi_{\rho\omega}(p^2)\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\omega_\mu(p)\rho_\nu^0(x). \tag{43}$$

The first and second term of expression (42) correspond to the contributions of direct coupling (ρ^0-3π) and ω resonance exchange respectively. Because $m_\rho\approx m_\omega$ and Γ_ω is small, the denominator of the second term is small. Therefore, contribution from ω resonance exchange is large. This is called "hidden isospin symmetry breaking effect" according to Ref. [1]. This effect brings a significant contribution and plays an essential role in the decay $\rho\to\pi^+\pi^-\pi^0$. So when we deal with the decay of $\rho\to\pi^+\pi^-\pi^0$, the process $\rho\to\omega\to\pi^+\pi^-\pi^0$ must be considered.

In fact, the contributions of ρ^0-ω interference are dominant and the direct coupling can be omitted. The direct coupling $f_{\rho3\pi}\propto(m_d - m_u)$, therefore, it is very small. In order to be sure of this point, we derive this quantity in a practical model called as $U(2)_L\times U(2)_R$ chiral theory of mesons[9] in follows. Denoting the direct vertices of ρ^0-3π as $\mathscr{L}_{\rho3\pi} = f_{\rho3\pi}\epsilon^{\mu\nu\alpha\beta}\epsilon_{ijk}\rho_\mu^0\,\partial_\nu\pi^i\,\partial_\alpha\pi^j\,\partial_\beta\pi^k$, then $f_{\rho3\pi}$ can be calculated in this theory[9] and has the form

$$f_{\rho3\pi} = -\frac{m_d - m_u}{\pi^2 gf_\pi^3 m}\left(1 - \frac{16c}{3g} + \frac{6c^2}{g^2} - \frac{8c^3}{3g^3}\right)\approx -2\times10^{-11}\,\mathrm{MeV}^{-3}, \tag{44}$$

where the values of model's parameters m, g, c determined in Ref. [9] have been used. To the second term in the parentheses of expression (42), $\Pi_{\rho\omega}(m_\rho^2)$ has be determined to approximate $-4\times10^3\,\mathrm{MeV}^2$[10,11]. $\omega\to3\pi$ is the dominant channel for ω-decays, and hence $f_{\omega3\pi}$ can be estimated by using the width $\Gamma_{\omega\to3\pi} = 7.5\mathrm{MeV}$. It is approximate value is about $3\times10^{-7}\,\mathrm{MeV}^{-3}$. Thus the typical value of the second term in expression (42) is $(5+2\mathrm{i})\times10^{-8}\,\mathrm{MeV}^{-3}$ approximately. Comparing it with expression (44), we can see that the direct coupling $f_{\rho3\pi}$ is indeed very small, and it is ignorable. Therefore, discarding $f_{\rho3\pi}$ in expression (42), we have, approximately,

$$\Gamma_{\rho^0\to3\pi} = \left|\frac{\Pi_{\rho\omega}(m_\rho^2)}{m_\rho^2 - m_\omega^2 + \mathrm{i}m_\omega\Gamma_\omega}\right|^2\Gamma_{\omega\to3\pi}. \tag{45}$$

This equation means that the contributions due to ρ-ω interference to $Br(\rho^0\to3\pi)$ are dominate, or the hidden isospin-breaking effects introduced in Ref. [1] are dominate for the process $\rho^0\to3\pi$. From expression (45) we obtain desired result as follows,

$$Br(\rho^0\to3\pi)\approx0.2\times10^{-2}. \tag{46}$$

Our experiment datum fitting result (40) is consistent with this theoretical estimation result. This fact indicates that both (PP, V(π^0, K))-fit and (PP, V(π^0, K, η, η'))-fit are reasonable even though the resulting $Br(\rho^0\to3\pi)$ is much larger than one in Ref. [3] and rather closes the upper limit for it in Ref. [12].

5 Discussion

Through the study presented in the above, we conclude that ρ-ω interference effects can be detected in the J/$\psi\to\pi^+\pi^-\pi^0\pi^0$ decay, which receives a contribution from the $\rho^0\to\pi^+\pi^-\pi^0$ decay mode. J/ψ decays offer an almost unique opportunity for observing $\rho^0\to\pi^+\pi^-\pi^0$, where the smallness of $Br(\rho^0\rightarrow\pi^+\pi^-\pi^0)/Br(\omega\rightarrow\pi^+\pi^-\pi^0)$ is compensated by the large ratio $A(J/\psi\rightarrow\rho\pi^0)/A(J/\psi\rightarrow\omega\pi^0)$ between a (simply Zweig-forbidden) strong amplitude over an EM one. This is the key point for the practical determining $Br(\rho^0\rightarrow\pi^+\pi^-\pi^0)$ through employing J/ψ decay branching radios. Our results for 2 datum-fits are $Br(\rho\rightarrow\pi^+\pi^-\pi^0)|_{(PP,V(\pi^0,K))}=(2.0\pm1.64)\times10^{-2}$ and $Br(\rho\rightarrow\pi^+\pi^-\pi^0)|_{(PP,V(\pi^0,K,\eta,\eta'))}=(0.59\pm0.315)\times10^{-2}$ respectively, which are anomalously large and match each other within the errors.

In order to pursue whether these anomalously large results of $Br(\rho\rightarrow\pi^+\pi^-\pi^0)$ are reasonable or not, a theoretical estimation for ρ-ω interference effects to the process of $(\rho\rightarrow\pi^+\pi^-\pi^0)$ has also been discussed in this paper. Following Ref. [1], we found that the contributions due to so called hidden isospin-breaking effects are dominate for the process $\rho\rightarrow\pi^+\pi^-\pi^0$. The theoretical prediction is $Br(\rho^0\rightarrow3\pi)\approx0.2\times10^{-2}$ which is in good agreement with our datum-fit results. Then, considering this fact and noting that both result of η-η′ angle θ and the result of constituent quark ratio $\lambda=m_u/m_s$ obtained by the fits are also reasonable, we conclude that $Br(\rho\rightarrow\pi^+\pi^-\pi^0)\approx10^{-3}—10^{-2}$ is reliable.

Finally, we like to argue that in order to reduce the error-bar of $Br(\rho\rightarrow\pi^+\pi^-\pi^0)$, more precisely experimental measurements to $(J/\psi\rightarrow PP,PV)$ are expected. The high quality data for J/ψ in the future BESⅢ would be useful.

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在J/ψ衰变与$\rho\rightarrow\pi^+\pi^-\pi^0$衰变中的ρ-ω干涉*

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摘要 通过分析$J/\psi\rightarrow\pi^+\pi^-\pi^0\pi^0$来研究ρ-ω干涉.利用描述J/ψ衰变的一般的唯象模型对PDG-2002关于J/ψ衰变到PP和PV的实验数据做了拟合(P表示赝标介子,V表示矢量介子),得到$\rho^0\rightarrow\pi^+\pi^-\pi^0$反常大的分支比$\sim10^{-3}—10^{-2}$.理论的分析结果也与这一反常大的分支比相符合.同时,得到了合理的η-η′混合角和组分夸克质量比:$\theta=-19.68°\pm1.49°$, $m_u/m_s\approx0.6$.

关键词 J/ψ衰变 ρ-ω干涉 $SU(3)$破缺效应 η-η′混合

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