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Relation of Dawson Furnstahl's Empty Sea Ansatz to Dirac's Hole Theory *

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Abstract There are two methods to take account of the contribution of negative energy states in the relativistic random phase approximation. One is due to Dawson and Furnstahl. They made the ansatz that the Dirac sea is empty. The other is the Dirac hole theory, which postulated that the sea should be fully occupied. The two methods seem contradictory concerning the way through which the negative-energy states contribute. The relation between the two theories is studied and the conditions under which the Dawson-Furnstahl theory is a good approximation to the Dirac theory are enunciated.

Key words Dawson-Furnstahl's empty VS Dirac's filled sea

There are two methods to take account of the contribution of the negative-energy states in the relativistic random phase approximation (RRPA). One is the wellknown Dirac hole theory and the other is due to Dawson and Furnstahl (D-F) [1, 2]. They pointed out that it is of vital importance to take the negative-energy (NE) states into account, because only then will the Dirac single-particle basis become complete. They found that in this way the law of current conservation can be preserved and the spurious $J^{\pi} = 1^{-1}$ state fully separated out. Since in the relativistic mean-field theory (RMFT)[2, 3], no-sea approximation is made, they suggested that on the basis of RMFT one may assume the NE sea is empty and thus besides the positive energy (PE) particle-hole pairs one should further consider pairs formed from a particle at one of the NE states and a hole in the occupied PE states (referred to as ah pairs as in Ref. [4]). Since RMFT based on the method of effective Lagrangians has now achieved a remarkable success in describing the ground-state properties of nuclei quantitatively, it is natural to ask whether such Lagrangian can also give a good description of nuclear excited states. Ma et al.[5] found that the D-F method can indeed improve the no-sea approximation significantly. Recently, Ring et al.[4] have made a detailed study of the etffects of the ah pairs and found that they are indeed important. However, according to the Dirac hole theory, which observes the energy principle and assumes the negative-energy states are fully occupied, the particle-hole pairs one should consider are formed from a particle at one of the unoccupied PE states and a hole either in the occupied PE states or in the fully filled NE states (the latter hole, as wellknown, is the antiparticle). So, how are the ah and particle-antiparticle pairs related with each other, as they differ qualitatively?

We would like to point out that the D-F method can be understood from another point of view. It is closely related with the Dirac hole theory. Their relation and the conditions under which the D-F method can be regarded as a good approximation to the Dirac theory will be discussed in some detail in the following.

For simplicity of description we shall restrict our discussion to nuclear matter. For our purpose it suffices to consider the correlation function

$$C(A, B; x_1, x_2) = \langle T[A(x_1)B(x_2)]\rangle - \langle A(x_1)\rangle\langle B(x_2)\rangle, \quad (1)$$

where $\langle O\rangle = \langle \Psi_0 | O | \Psi_0\rangle$, $K(x) = \bar{\psi}(x)\Gamma_{\kappa}\psi(x)$ ($K = A$ or B), $\psi(x)$ is the nucleon field operator, whereas operator Γ_{κ} is field- and time-independent. For instance, for the

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isovector multipole operator we have $\Gamma_\kappa = \tau_3 r^\lambda Y_{\lambda\mu}(\theta,\varphi)$ and for the bilinear Dirac current and scalar $\Gamma_\kappa = \gamma_\mu (\kappa = \mu = 1,2,3,4)$ and $\Gamma_\kappa = 1$, respectively, etc. In the lowest order approximation Eq. (1) has the form

$$C^0(A,B;x_1,x_2) = -\mathrm{Tr}[\Gamma_A(\boldsymbol{x}_1)G^0(x_1-x_2)\times \Gamma_B(\boldsymbol{x}_2)G^0(x_2-x_1)], \quad (2)$$

where $G^0(x)$ is the relativistic Hartree approximation to the nucleon propagator

$$G_{\alpha\beta}(x = x_1 - x_2) = \langle T[\psi_\alpha(x_1)\bar{\psi}_\beta(x_2)]\rangle \quad (3)$$

and $x \equiv x_\mu = (\boldsymbol{x}, \mathrm{i}x_0)$ with $x_0 = t$. If Γ_κ is further independent of $\boldsymbol{x}$(indicated by κ taking a small letter), the Fourier transform of Eq. (2) is given by

$$C^0(a,b;kk') = -(2\pi)^4\delta^{(4)}(k-k')\int\frac{\mathrm{d}^4q}{(2\pi)^4}\times \mathrm{Tr}[\Gamma_a G^0(q+k)\Gamma_b G^0(q)] \equiv (2\pi)^4\delta^{(4)}(k-k')C^0(a,b;k), \quad (4)$$

where $C^0(a,b;k)$ is just the expression for the polarization tensor in the σ-ω model, if $\Gamma_\kappa = \{\gamma_\mu, 1\}(\kappa = 1 \text{ to } 5)$.

In the following, we will give the detailed formula of $C^0(a,b;k)$ for Dirac's hole theory and D-F's method, and show their differences. Firstly, in the Dirac hole theory, using the relation $G^0(p) = G^0_{\mathrm{F}}(p) + G^0_{\mathrm{D}}(p)$, where

$$G^0_{\mathrm{F}}(p) = (\gamma_\mu p_\mu + \mathrm{i}M^*)\frac{1}{2|E_{\mathrm{p}}|}\left\{\frac{\theta(E_{\mathrm{p}})}{p_0 - E_{\mathrm{p}} + \mathrm{i}\varepsilon} + \frac{\theta(-E_{\mathrm{p}})}{p_0 - E_{\mathrm{p}} - \mathrm{i}\varepsilon}\right\}, \quad (5a)$$

$$G^0_{\mathrm{D}}(p) = (\gamma_\mu p_\mu + \mathrm{i}M^*)\frac{\mathrm{i}\pi}{E_{\mathrm{p}}}\theta(p_0)\theta(k_{\mathrm{F}} - |\boldsymbol{p}|)\delta(p_0 - E_{\mathrm{p}}), \quad (5b)$$

$E_{\mathrm{p}} = \pm[\boldsymbol{p}^2 + M^{*2}]^{1/2}$, M^* is the effective mass and k_{F} denotes the Fermi momentum, we may rewrite $C^0(a,b;k)$ as

$$C^0(a,b;k) = C^0_{\mathrm{FF}}(a,b;k) + C^0_m(a,b;k) + C^0_{\mathrm{DD}}(a,b;k), \quad (6a)$$

$$C^0_m(a,b;k) = C^0_{\mathrm{DF}}(a,b;k) + C^0_{\mathrm{FD}}(a,b;k). \quad (6b)$$

On the right-hand side of Eq. (6) the subscript D or F indicates G^0 in Eq. (4) is G^0_{D} or G^0_{F} and the three parts will be referred to as FF-, m- and DD-part, respectively.

Secondly, in D-F method, it makes an alternative ansatz for G^0, which will be denoted $\tilde{G}^0$. It accomplishes the MFT prescription by shifting the negative-energy poles to the lower-half planes, as if the Dirac sea were empty. In particular,

$$\tilde{G}^0_{\mathrm{F}}(p) = (\gamma_\mu p_\mu + \mathrm{i}M^*)\frac{1}{2|E_{\mathrm{p}}|}\left\{\frac{\theta(E_{\mathrm{p}})}{p_0 - E_{\mathrm{p}} + \mathrm{i}\varepsilon} + \frac{\theta(-E_{\mathrm{p}})}{p_0 - E_{\mathrm{p}} + \mathrm{i}\varepsilon}\right\} \quad (7)$$

and $\tilde{G}^0_{\mathrm{D}}(p) = G^0_{\mathrm{D}}(p)$. Clearly, comparing Eq. (5) and Eq. (7), the only difference between $G^0(p)$ and $\tilde{G}^0(p)$ consists in their F (Feynman) part. Since all the poles of $\tilde{G}^0_{\mathrm{F}}(p)$ are in the lower-half plane, substituting Eq. (7) in Eq. (4), as pointed out in Ref.[1], we get $\tilde{C}^0_{\mathrm{FF}}(a,b;k) = 0$. Besides, we find $\tilde{C}^0_{\mathrm{DD}}(a,b;k) = C^0_{\mathrm{DD}}(a,b;k)$ and

$$\tilde{C}^0(a,b;k) = \tilde{C}^0_m(a,b;k) + C^0_{\mathrm{DD}}(a,b;k), \quad (8a)$$

$$\tilde{C}^0_m(a,b;k) = C^0_m(a,b;k) + \Delta C^0_m. \quad (8b)$$

Comparing Eq. (8) with Eq. (6), one notes the assumption of empty NE sea is equivalent to the assertion that C^0_{FF} can be neglected and $\tilde{C}^0_m$ differs from C^0_m by ΔC^0_m. Thus, if they are small, the D-F theory will be a good approximation to the Dirac hole theory. Moreover, Eq. (8) shows why the D-F method can give a better result than the no-sea approximation, because the latter only takes account of C^0_{DD} and a part of C^0_m given by the first term in Eqs. (5a) or (7), while the former has further correctly considered the full C^0_m with even a correction ΔC^0_m(see below).

In order to expose the physical implication of Eq. (8) more clearly, consider, for instance, Eq. (4) with $\Gamma_a = \gamma_\eta$ and $\Gamma_b = \gamma_\lambda$. Substituting $G^0_{\mathrm{F}}(p)$ and $\tilde{G}^0_{\mathrm{F}}(p)$ in Eqs. (6b) and (8b), we obtain

$$\mathrm{Im}[\mathrm{i}C^0_m(\eta,\lambda;k)] = M_1(\eta,\lambda;k) + M_2(\eta,\lambda;k), \quad (9a)$$

$$\mathrm{Im}[\mathrm{i}\tilde{C}^0_m(\eta,\lambda;k)] = M_1(\eta,\lambda;k) - M_2(\eta,\lambda;k), \quad (9b)$$

$$M_1(\eta,\lambda;k) = -\frac{1}{8\pi^2}\int\mathrm{d}^3q\,\frac{\theta(k_{\mathrm{F}} - |\boldsymbol{q}|)}{E_q}\times \left\{\frac{t_{\eta\lambda}(q+k)}{E_{q+k}}\delta(k_0 + E_q - E_{q+k}) + \frac{t_{\eta\lambda}(q-k)}{E_{q-k}}\delta(k_0 - E_q + E_{q-k})\right\}, \quad (9c)$$

$$M_2(\eta,\lambda;k) = -\frac{1}{8\pi^2}\int\mathrm{d}^3q\,\frac{\theta(k_{\mathrm{F}} - |\boldsymbol{q}|)}{E_q}\times \left\{\frac{t_{\eta\lambda}(q+k)}{E_{q+k}}\delta(k_0 + E_q + E_{q+k}) + \frac{t_{\eta\lambda}(q-k)}{E_{q-k}}\delta(k_0 - E_q - E_{q-k})\right\}, \quad (9d)$$

where $E_q = [\boldsymbol{q} + M^{*2}]^{1/2}$ and

$$t_{\eta\lambda}(q \pm k) = [2q_\eta q_\lambda \pm q_\eta k_\lambda \pm q_\lambda k_\eta \mp q\cdot k\delta_{\eta\lambda}]_{q_0 = E_q}. \quad (10)$$

Since $\Delta C^0_m(\eta,\lambda;k) = -2M_2(\eta,\lambda;k)$, one concludes that $\tilde{C}^0_m$ and C^0_m differ from each other only in the sign before M_2. If we require $k_0 > 0$, it is seen from Eq. (9d) that the first term in M_2 is zero and only the second term will con-

tribute. Chin noted[6] that this term represents the decay mechanism of the collective mode caused by the particle-antiparticle pair creation. We shall show that the D-F ansatz gives a correct sign of the relevant damping width, though the latter is now related with the ah pair. The δ-function in Eq.(9d) shows M_2 becomes effective only if $k_0 > 2M^{*2}$, which is about 1.1GeV for $M^* \simeq 0.6M$, i.e. if the excitation energy is not too high, we have $\tilde{C}^0_m \simeq C^0_m$. Since $M_2 = 0$ if $0 < k_0 < 2M^{*2}$, in the energy region where for instance, giant multipole resonances are studied we even expect $\tilde{C}^0_m = C^0_m$. Clearly the above derivation also applies to the other components of $C^0(a,b;k)$ (see Eq. (4)) and similar results obtain. The detailed formulae and the corresponding renormalization procedure will not be written down here, as they are known[6—8] and space-consuming.

Sofar we have only considered the lowest order approximation to Eq. (1). The parameters used are $\bar{g}_s^2 = 0.6942$, $\bar{g}_v^2 = 1.2059$ ($\bar{g}^2 = g^2/16\pi^2$), whereas $k_F = 280$, $M = 939$, $m_s = 520$, $m_v = 783$ (all in MeV) and $M^* = 0.6M$. To illustrate the effects of C^0_{FF} and ΔC^0_m (see Eqs. (6) and (8)) as well as the difference in results between the Dirac and D-F theories, we shall, as an example, discuss $C(4) = \mathrm{Im}[iC(4,4;k)]$, which is real and nonnegative and closely related to the longitudinal response function. In Fig. 1, it shows the dependence of $C^0(4)$ on the energy transfer k_0. Note its dimension is $[mass]^2$ as indicated. Indeed, in the small k_0 region we have $\tilde{C}^0(4) = C^0(4)$, because both $C^0_{FF}(4)$ and $\Delta C^0_m(4)$ will not be zero only if $k_0 > 2M^*$. In the large k_0 region since $C^0_{DD}(4) = \tilde{C}^0_{DD}(4) = 0$ (see Eq. (A. 24) in Ref. [7]) and according to Eq. (9b) $M_1(4,4;k) = 0$ for timelike k_μ[6], one gets $\tilde{C}^0(4) = \tilde{C}^0_m(4)$ and $C^0(4) = C^0_m(4) + C^0_{FF}(4)$. From Fig. 1 it is seen that the sign of $\tilde{C}^0_m(4)$ is correct, while $C^0_m(4) = -\tilde{C}^0_m(4)$ has a wrong sign. However, in the Dirac theory the relevant effect is represented by $C^0(4)$ whose sign is again correct owing to the contribution of $C^0_{FF}(4)$. It is interesting to find that ΔC^0_m included in $\tilde{C}^0_m$ in Eq. (8b) actually means a correction rather than a defect. It somehow tries to correct the drawback of neglecting C^0_{FF}, though the effect is not yet sufficient as shown by the figure.

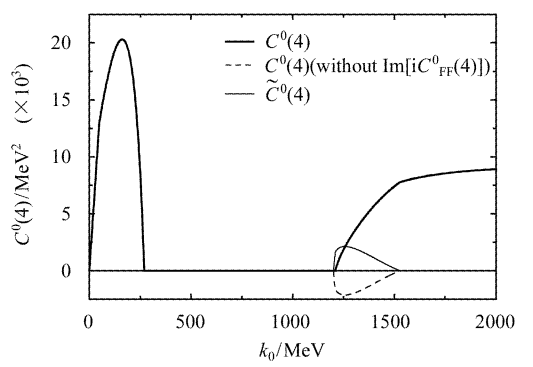

Fig.1. Imaginary part of the lowest order approximation to the correlation function $C(4,4;k)$: $C^0(4) \equiv \mathrm{Im}[iC^0(4,4;k)]$.

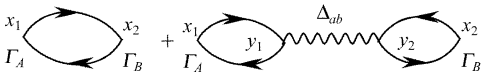

Fig.2. Graphical representation of RRPA for the calculation of the correlation function $C(a,b;x_1,x_2)$.

In order to gain some idea about the accumulative effect, we have further made a RRPA calculation of $C(a,b;x_1,x_2)$ with C^0 given by Eq. (4). In the perturbation expansion of $C(a,b;x_1,x_2)$ we take only the ring diagrams and sum them to all orders. This is equivalent to the RPA. The summation of ring diagrams leads to the integral equation for the correlation function,

$$C(a,b;k) = C^0(a,b;k) + C^0(a,c;k)\bar{\Delta}_{cd}(k)C(d,b;k), \qquad (11)$$

where $\bar{\Delta}^0_{ab}(k) = \begin{pmatrix} (ig_v)^2\Delta^0_v\delta_{\mu\nu} & 0 \\ 0 & g_s^2\Delta_s \end{pmatrix}$ and $\Delta^0_k(k) = [k^2 + m_\kappa^2]^{-1}$ ($\kappa = s$ or v). Fig.2 shows Eq. (11) diagrammatically. Two calculation schemes are considered. One is the ω-scheme, in which only the vector meson is taken into account. In this case $\bar{\Delta}_{ab}$ reduces to $(ig_v)^2\Delta^0_v\delta_{\mu\nu}$. The other is the μ-scheme, where the scalar and vector mesons as well as their mixing are considered, i.e. the full $C(a,b;k)$ is used (see Ref.[6] and Eq. (19)). $C(4,\omega)$ and $C(4,\mu)$ will be used to denote results calculated according to the ω-scheme and μ-scheme, respectively. We have solved Eq. (11) and depicted our calculated RRPA results in Fig.3. Comparing with the lowest order approximation one notes the accumulative effect is quite important. It changes not only the magnitude but also the behavior of $C^0(4)$ palpably. Since according to Eq. (11) the real as well as the imaginary part of $C^0(a,b;k)$ is effective, it is interesting to ask whether C^0_{FF} may now cause some marked difference between $C(4)$ and $\tilde{C}(4)$. Consider first the ω-scheme. It is seen that in the region of small k_0 we still have $\tilde{C}(4,\omega) \simeq C(4,\omega)$ for small as well as large momentum transfer $k_a \equiv |\boldsymbol{k}|$. In the large k_0 region their magnitudes

become much smaller and they differ widely. Consider now the μ-scheme. Though in the region of large k_0 we have $C(4,\omega)\simeq C(4,\mu)$ and $\tilde{C}(4,\omega)\simeq\tilde{C}(4,\mu)$ for both small and large k_a, there is an unexpected effect of the mixed polarization insertion (MPI) in the small k_0 region. Clearly the difference between $C(4,\mu)$ and $C(4,\omega)$ as well as that between $\tilde{C}(4,\mu)$ and $\tilde{C}(4,\omega)$ is mainly caused by MPI,

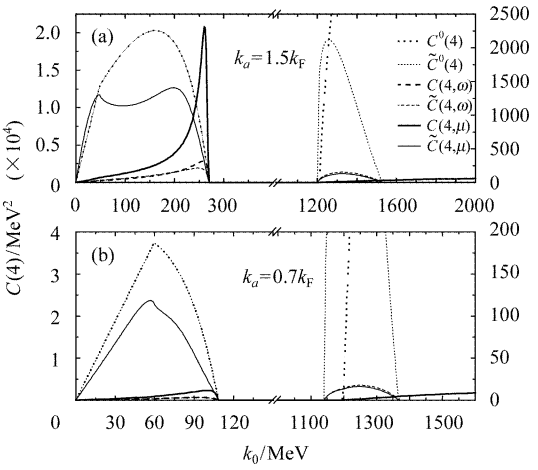

Fig.3. Curves represent $C(4)\equiv\mathrm{Im}[\mathrm{i}C(4,4;k)]$ in RRPA. (a) Momentum transfer $k_a\equiv|\boldsymbol{k}|=1.5k_F$, where $C(4,\omega)\simeq C(4,\mu)$ if k_0 is large; (b) $k_a=0.7k_F$, where $\tilde{C}(4,\omega)\simeq C(4,\mu)$ if k_0 is small, while $\tilde{C}(4,\omega)\simeq\tilde{C}(4,\mu)$ and $C(4,\omega)\simeq C(4,\mu)$ if k_0 is large.

while the reason that $\tilde{C}(4,\mu)$ differs from $C(4,\mu)$ and $\tilde{C}(4,\omega)$ from $C(4,\omega)$ is chiefly due to the accumulative effect of the FF-part of the polarization tensor (AEFF). Fig.3 shows both MPI and AEFF affect $\tilde{C}(4,\mu)$ and there is a significant difference between $\tilde{C}(4,\mu)$ and $C(4,\mu)$. Thus, it is difficult to regard $\tilde{C}(4,\mu)$ as a good approximation to $C(4,\mu)$. However, this does not mean that the D-F method cannot be successfully applied to this k_0 region, which is important for the study of giant resonances and quasielastic electron scattering, because AEFF may still be taken into account effectively by readjusting the relevant parameters. In Fig.4, we have plotted the results of $\tilde{C}(4,\mu)$ by adjusting $\bar{g}_s^2$ for $k_a=2k_F$, $k_a=1.5k_F$ and $k_a=0.7k_F$, respectively. We keep $C(4,\mu)$ the same as given in Fig.3, because we presume it is the correct one. In Fig.4(b), it is seen that $\tilde{C}(4,\mu)$ can be made to approach $C(4,\mu)$, and in Fig.4c, $\tilde{C}(4,\mu)$ for $\bar{g}_s^2=0.233$ and $C(4,\mu)$ are closed to each other. However, from Fig.4(a) to 4(c), we have found that the fitting is quite difficult for large k_a, though it is easier if k_a is smaller. It suggests AEFF is still worthy of study. Clearly the readjustment can be achieved more effectively, if the model contains more adjustable parameters. For large k_0, as can be seen from Fig.3, to try to fit $\tilde{C}(4,\mu)$ with $C(4,\mu)$ is not meaningful. Fig.4 discloses $\tilde{C}(4,\mu)$ changes noticeably with model parameters. It hints, as point out in Ref. [4], the related small amplitude oscillations might only be quasi-stable. However, according to our calculation the change of $C(4,\mu)$ with $\bar{g}_s^2$ proceeds gradually (see Fig.5).

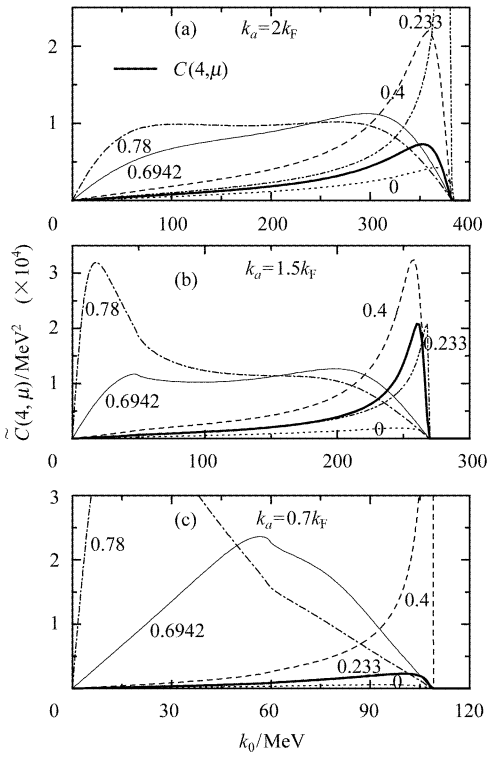

Fig.4. $\tilde{C}(4,\mu)$ for different values of $\bar{g}_s^2$: 0 (dotted line), 0.233 (dash-dot-dotted line), 0.4 (dashed line), 0.6942 (light solid line) and 0.78 (dash-dotted line). (a) $k_a=2k_F$; (b) $k_a=1.5k_F$; (c) $k_a=0.7k_F$.

In summary, we have shown the D-F ansatz is not in

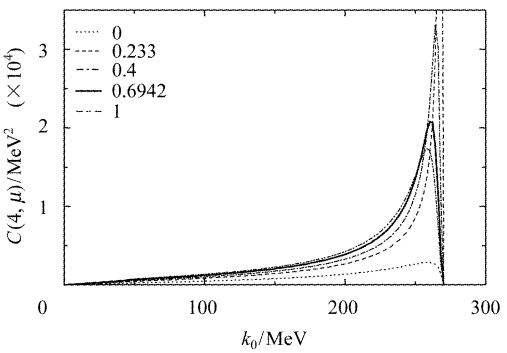

Fig.5. $C(4,\mu)$ for different values of $\bar{g}_s^2$.

contradiction with Dirac's hole theory. It is better than the no-sea approximation, because in addition to the DD-part, it has correctly taken the m-part into account and besides, it even contains a correction to the approximation where one simply neglects the FF part in the Dirac theory, though the correction is generally not yet sufficient. For the calculation of response functions our results confirm the previous conclusion [1, 4,5, 6—8] that a RRPA calculation is necessary. They further indicate that both MPI and AEFF are important.

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Dawson-Furnstahl 计算方案与 Dirac 空穴理论的联系*

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摘要　在相对论无规位相近似中,有两种顾及负能态贡献的方法. 一种是由 Dawson 和 Furnstahl(DF)提出的计算方案. 他们假定 Dirac 海是空的. 另外一种就是 Dirac 空穴理论,它认为负能海的全充满的. 这两种方法在顾及负能态贡献上看似完全对立. 文章中考察了这两种方法的关系,并给出了 DF 方法的适用范围.

关键词　Dawson-Furnstahl 计算方案　Dirac 空穴理论

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