Random Interactions in Nuclei and Extension of 0^+ Dominance in Ground States to Irreps of Group Symmetries

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Abstract　Random one plus two-body hamiltonians invariant with respect to $O(\mathcal{N}_1)\oplus O(\mathcal{N}_2)$ symmetry in the group-subgroup chains $U(\mathcal{N})\supset U(\mathcal{N}_1)\oplus U(\mathcal{N}_2)\supset O(\mathcal{N}_1)\oplus O(\mathcal{N}_2)$ and $U(\mathcal{N})\supset O(\mathcal{N})\supset O(\mathcal{N}_1)\oplus O(\mathcal{N}_2)$ of a variety of interacting boson models are used to investigate the probability of occurrence of a given $(\omega_1\omega_2)$ irreducible representation (irrep) to be the ground state in even-even nuclei; $[\omega_1]$ and $[\omega_2]$ are symmetric irreps of $O(\mathcal{N}_1)$ and $O(\mathcal{N}_2)$ respectively. Employing a 500 member random matrix ensemble for N boson systems (with $N = 10 - 25$), it is found that for $\mathcal{N}_1, \mathcal{N}_2 \geqslant 3$ the $(\omega_1\omega_2) = (00)$ irrep occurs with $\sim 50\%$ and $(\omega_1\omega_2) = (N0)$ and $(0N)$ irreps each with $\sim 25\%$ probability. Similarly, for $\mathcal{N}_1 \geqslant 3, \mathcal{N}_2 = 1$, for even N the $\omega_1 = 0$ occurs with $\sim 75\%$ and $\omega_1 = N$ with $\sim 25\%$ probability and for odd N, $\omega_1 = 0$ occurs with $\sim 50\%$ and $\omega_1 = 1, N$ each with $\sim 25\%$ probability. An extended Hartree-Bose mean-field analysis is used to explain all these results.

Key words　random interactions, nuclei, interacting boson models, group symmetries, $O(\mathcal{N}_1)\oplus O(\mathcal{N}_2)$, ground state irreps, probabilities, mean-field analysis

1　Introduction

Two-body random matrix ensembles (TBRE) defined over Hilbert spaces of various nuclear models led to the discovery that many of the regular features observed in low-lying levels and near the yrast line in nuclei can arise due to random interactions (with rotational symmetry) and this is opposed to the conventional ideas of using regular (or coherent) interactions like pairing etc. in the nuclear hamiltonian. For the first time this result is found by Bertsch et al.[1] using the shell model who showed that with random interactions ground states in even-even nuclei will be 0^+ with very high probability and they also generate odd-even staggering in binding energies, the seniority pairing gap etc. Similarly, Bijker and Frank[2] using the interacting boson model showed that random interactions generate vibrational and rotational structures with high probability. These unexpected results gave rise to a new field of research activity with random interactions in nuclei (they go beyond the TBRE applications for smoothed (with respect to energy) state densities, strength sums, transition matrix elements, information entropy in wavefunctions

etc. in nuclei and other finite quantum systems; see [3] and references therein). In particular: (i) Zelevinsky and collaborators proposed the idea of geometric chaos for describing regular features generated by TBRE's; (ii) Arima's group introduced a variety of prescriptions, for predicting the probabilities, for simple systems such as single j-shell for fermions, single l-shell for bosons and some of their extensions; (iii) Bijker and Frank used a mean-field analysis with projective coherent states for interacting boson systems. For details of these studies we refer the readers to two recent reviews on this subject[4,5].

A very important aspect of TBRE's is that they admit group symmetries[6]. With m particles (fermions or bosons) in $\mathcal{N}$ single particle states there is a $U(\mathcal{N})$ spectrum generating algebra (SGA). In all the shell model/Interacting boson model studies reviewed in [4,5] one plus two-body hamiltonians that are rotational scalars are considered, i.e. all the $O(3)$ scalars in $U(\mathcal{N})\supset O(3)$ with one and two particle matrix elements of the one and two-body parts respectively chosen to be random variables (in some studies the one body part is dropped). Immediately one sees that TBRE's can be

extended to scalars of various subgroups of $U(\mathcal{N})$, i.e. scalars of G in $U(\mathcal{N}) \supset G \supset \dots \supset O(3)$. The purpose of the present paper is to consider such an extension to $O(\mathcal{N}_1)$ $\oplus O(\mathcal{N}_2)$, $\mathcal{N}_1 + \mathcal{N}_2 = \mathcal{N}$ which appears in a very large class of interacting boson models (IBM's) used in nuclear structure and address the question of with what probability a given $(\omega_1 \omega_2)$ irreducible representation (irrep) will be the ground state in even-even nuclei; note that $[\omega_1]$ and $[\omega_2]$ are symmetric irreps of $O(\mathcal{N}_1)$ and $O(\mathcal{N}_2)$ respectively. In Section 2 given are the random one plus two-body hamiltonians with $O(\mathcal{N}_1) \oplus O(\mathcal{N}_2)$ symmetry in interacting boson models. Section 3 gives the results of numerical TBRE calculations and their understanding using an extended Hartree-Bose mean-field analysis. Finally Section 4 gives conclusions and future outlook.

2 Random interactions with $O(\mathcal{N}_1) \oplus O(\mathcal{N}_2)$ symmetry in IBM's

Large class of interacting boson models (IBM's) of nuclei admit $U(\mathcal{N}) \supset U(\mathcal{N}_1) \oplus U(\mathcal{N}_2) \supset O(\mathcal{N}_1) \oplus O(\mathcal{N}_2)$ and $U(\mathcal{N}) \supset O(\mathcal{N}) \supset O(\mathcal{N}_1) \oplus O(\mathcal{N}_2)$ group-subgroup chains; $\mathcal{N} = \mathcal{N}_1 + \mathcal{N}_2$. Examples (all for even-even nuclei) are: (i) sp IBM or nuclear vibron model[7] with $U(4)$ SGA and $(\mathcal{N}_1, \mathcal{N}_2) = (3,1)$; (ii) sd IBM for quadrupole collective states[8] with $U(6)$ SGA and $(\mathcal{N}_1, \mathcal{N}_2) = (5, 1)$; (iii) spd IBM for GDR states[9] with $U(9)$ SGA and $(\mathcal{N}_1, \mathcal{N}_2) = (8,1), (6,3), (5,4)$; (iv) sdg IBM for quadrupole plus hexadecupole states[10] with $U(15)$ SGA and $(\mathcal{N}_1, \mathcal{N}_2) = (14, 1), (9, 6), (10, 5)$; (v) $sdpf$ IBM for octupole states[11] with $U(16)$ SGA and $(\mathcal{N}_1, \mathcal{N}_2) = (15,1), (10,6)$ etc.; (vi) $sdgpf$ IBM[12] with $U(25)$ SGA and $(\mathcal{N}_1, \mathcal{N}_2) = (24,1), (15,10)$ etc; (vii) spp' IBM or the $U(7)$ model for 3-body clusters in nuclei[13] with $U(7)$ SGA and $(\mathcal{N}_1, \mathcal{N}_2) = (6,1), (4,3)$; (viii) IBM-3 or the isospin (T) invariant sd IBM (here the bosons carry $T = 1$ degree of freedom)[14] with $U(18)$ SGA and $(\mathcal{N}_1, \mathcal{N}_2) = (15,3)$; (ix) IBM-4 or the spin-isospin (S, T) invariant sd IBM (here the bosons carry $(ST) = (10) \oplus (01)$ degree of freedom)[15] with $U(36)$ SGA giving examples with $(\mathcal{N}_1, \mathcal{N}_2) = (30,6), (3,3), (18, 18), (15, 15)$; (x) IBM-2 or proton-neutron IBM[16] with $U(12)$ SGA and $(\mathcal{N}_1, \mathcal{N}_2) = (10,2)$. In this paper, for simplicity, we consider group chains with $\mathcal{N}_1 \geqslant 3, \mathcal{N}_2 = 1$ and $\mathcal{N}_1 \geqslant 3, \mathcal{N}_2 \geqslant 3$, i.e. $\mathcal{N}_2 = 2$ situations (as in (x) above) are not

considered.

Group chains, for symmetric $U(\mathcal{N})$ irreps $\{\mathcal{N}\}$ the irrep labels for other group algebras in the chains and their reductions for the $\mathcal{N}_1 \geqslant 3$ and $\mathcal{N}_2 = 1$ situation (hereafter called I) are,

$$\left| \begin{matrix} U(\mathcal{N}) & \supset & U(\mathcal{N}_1 = \mathcal{N} - 1) & \supset & O(\mathcal{N}_1) & \supset & K \\ \{N\} & & \{n_1\} & & [\omega_1] & & \alpha \end{matrix} \right\rangle,$$
$$n_1 = 0,1,2,\dots,N, \quad \omega_1 = n_1, n_1 - 2, \dots, 0 \text{ or } 1 \quad (1)$$

and

$$\left| \begin{matrix} U(\mathcal{N}) & \supset & O(\mathcal{N}) & \supset & O(\mathcal{N}_1) & \supset & K \\ \{N\} & & [\omega] & & [\omega_1] & & \alpha \end{matrix} \right\rangle,$$
$$\omega = N, N - 2, \dots, 0 \text{ or } 1, \quad \omega_1 = 0,1,2,\dots,\omega. \quad (2)$$

In (1,2), label(s) α for the irreps of K need not be specified as the algebra K do not play any role in the present work. Note that $U(\mathcal{N}) \supset U(\mathcal{N}_1 = N - 1) \oplus U(\mathcal{N}_2 = 1)$ and the $U(\mathcal{N}_2 = 1)$ and its irreps $\{n_2\}$, $n_2 = N - n_1$ are not shown in (1). The general one plus two-body $O(\mathcal{N}_1)(O(\mathcal{N}_2 = 1)$ will not exist) scalar hamiltonian built out of the Casimir operators of the group algebras in the chains (1,2) is given by

$$H^{\text{I}} = \frac{1}{N}[\alpha_1 C_1(U(\mathcal{N}_1)) + \alpha_2 C_1(U(\mathcal{N}_2 = 1))] +$$
$$\frac{1}{N(N-1)}[\alpha_3 C_2(U(\mathcal{N}_1)) + \alpha_4 C_2(U(\mathcal{N}_2 = 1)) +$$
$$\alpha_5 C_1(U(\mathcal{N}_1)) C_1(U(\mathcal{N}_2 = 1)) +$$
$$\alpha_6 C_2(O(\mathcal{N})) + \alpha_7 C_2(O(\mathcal{N}_1))]. \quad (3)$$

Similarly for the $\mathcal{N}_1 \geqslant 3$ and $\mathcal{N}_2 \geqslant 3$ situation (hereafter called II), the group chains, for symmetric $U(\mathcal{N})$ irreps $\{\mathcal{N}\}$ the irrep labels for other group algebras in the chains and their reductions are,

$$\left| \begin{matrix} U(\mathcal{N}) & \supset & U(\mathcal{N}_1) & \oplus & U(\mathcal{N}_2) & \supset & O(\mathcal{N}_1) & \oplus & O(\mathcal{N}_2) & \supset & K \\ \{N\} & & \{n_1\} & & \{n_2\} & & [\omega_1] & & [\omega_2] & & \alpha \end{matrix} \right\rangle,$$
$$n_1 = 0,1,2,\dots,N; n_2 = n - n_1,$$
$$\omega_1 = n_1, n_1 - 2, \dots, 0 \text{ or } 1, \omega_2 = n_2, n_2 - 2, \dots, 0 \text{ or } 1 \quad (4)$$

and

$$\left| \begin{matrix} U(\mathcal{N}) & \supset & O(\mathcal{N}) & \supset & O(\mathcal{N}_1) & \oplus & O(\mathcal{N}_2) & \supset & K \\ \{N\} & & [\omega] & & [\omega_1] & & [\omega_2] & & \alpha \end{matrix} \right\rangle,$$
$$\omega = N, N - 2, \dots, 0 \text{ or } 1, \omega_1 + \omega_2 = \omega, \omega - 2, \dots, 0 \text{ or } 1. \quad (5)$$

The general one plus two-body $O(\mathcal{N}_1) \oplus O(\mathcal{N}_2)$ scalar hamiltonian built out of the Casimir operators of the group algebras in the chains (4,5) is given by

$$H^{\text{II}} = \frac{1}{N}[\alpha_1 C_1(U(\mathcal{N}_1)) + \alpha_2 C_1(U(\mathcal{N}_2))] +$$

$$\frac{1}{N(N-1)}[\,\alpha_3 C_2(U(\mathcal{N}_1)) + \alpha_4 C_2(U(\mathcal{N}_2)) +$$

$$\alpha_5 C_1(U(\mathcal{N}_1)) C_1(U(\mathcal{N}_2)) +$$

$$\alpha_6 C_2(O(\mathcal{N})) + \alpha_7 C_2(O(\mathcal{N}_1)) +$$

$$\alpha_8 C_2(O(\mathcal{N}_2))\,]. \tag{6}$$

In $(3,6)$, C_1 and C_2 are linear and quadratic Casimir invariants and their matrix elements for example are $\langle C_1(U(\mathcal{N}_1))\rangle^{\{n_1\}} = n_1$, $\langle C_2(U(\mathcal{N}_1))\rangle^{\{n_1\}} = n_1(n_1 + \mathcal{N}_1 - 1)$ and $\langle C_2(O(\mathcal{N}_1))\rangle^{[\omega_1]} = \omega_1(\omega_1 + \mathcal{N}_1 - 2)$. Given N bosons, in the $|Nn_1\omega_1\rangle$ basis for I and $|N(n_1 n_2)(\omega_1\omega_2)\rangle$ basis for II, the many boson H matrix for H^{I} and H^{II} respectively will be always tridiagonal; the α_6 terms in Eqs. $(3,6)$ generate off-diagonal matrix elements. In particular I is a generalization of the spIBM analyzed before[17,18] while II is completely new. For each allowed ω_1 in I and $(\omega_1\omega_2)$ in II the H matrices are constructed using the transformation brackets, given in [19], between the chains (1) and (2) for I and (4) and (5) for II. The matrices are diagonalized, for each member of a 500 member TBRE, for boson numbers $N = 10 - 25$. Thus, in the calculations the parameters in Eqs. $(3,6)$ are chosen to be independent Gaussian variables with zero mean and unit variance and 500 samples of the same are considered. Now we will discus the results.

3　Results for $\mathcal{N}_1 \geqslant 3, \mathcal{N}_2 = 1$ and $\mathcal{N}_1 \geqslant 3, \mathcal{N}_2 \geqslant 3$ systems and their mean-field analysis

3.1　Results of numerical calculations

Fig.1(a) and 1(b) give the probabilities, in the situation $\mathcal{N}_1 \geqslant 3$ and $\mathcal{N}_2 = 1$, i.e. for I as $\mathcal{N}_1$ is varied. In Fig.1(a) shown are the results for $\omega_1 = 0$, N for even boson number $N(N = 10)$ and in Fig.1(b) for $\omega_1 = 0, 1$, N for odd $N(N = 15)$ to be ground states. Fig.1(c) shows the same results but as a function of the boson number N for $\mathcal{N}_1 = 14$. It should be noted that the probabilities are negligiblly small for the $[\omega_1]$ irreps not shown in the figures. In general for even N, $\omega_1 = 0$ is ground state irrep with $\sim 65\%$—74% and $\omega_1 = N$ with $\sim 25\%$—32% probability. Similarly for odd N, $\omega_1 = 0$ is ground state irrep with $\sim 46\%$—50%, $\omega_1 = 1$ with $\sim 18\%$—24% and $\omega_1 = N$ with $\sim 24\%$—30% probability. They reproduce the spIBM results known before[5,17,18] and provide a test of the present calculations. They are also close to the sdIBM results known before(al-

though in these studies $K = O(3)$ is chosen and H^{I} to be a $O(3)$ scalar, note that $\omega_1 = 1$ gives $L = 1$ in spIBM and $L = 2$ in sdIBM)[5,18,20].
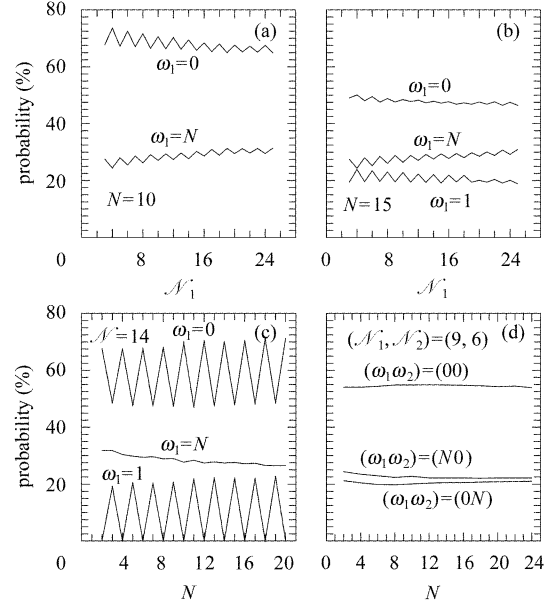

Fig.1. Probabilities for various group irreps to be ground states. (a) and (b) give the variation as a function of $\mathcal{N}_1$ for fixed boson number and (c) and (d) as a function of the boson number N. (a), (b) and (c) are for I and (d) is for II. See text for details.

In the situation $\mathcal{N}_1 \geqslant 3, \mathcal{N}_2 \geqslant 3$(i.e. II), Fig.1(d)(for various N values with $(\mathcal{N}_1, \mathcal{N}_2) = (9, 6)$) and Table 1(for various $(\mathcal{N}_1, \mathcal{N}_2)$ with $N = 10, 20$) give the results for the irreps $(\omega_1\omega_2) = (00), (N0), (0N)$. Here only even N is considered. It is seen that in general $(\omega_1\omega_2) = (00)$ is ground state with $\sim 50\%$—55% and $(\omega_1\omega_2) = (0N)$ and $(N0)$ each with $\sim 20\%$—24% probability. The probabilities for other $(\omega_1\omega_2)$ irreps to be ground states is negligiblly small. Before giving a mean-field analysis of these results some remarks are in order. In sdgIBM with $(\mathcal{N}_1, \mathcal{N}_2) = (9, 6)$, for γ-soft nuclei the $(\omega_1\omega_2) = (0N)$ is expected to be the ground state but, as seen from Fig.1(d), random interactions give this irrep only with $\sim 20\%$ probability. Therefore random interactions are not good for sdgIBM for $(\mathcal{N}_1, \mathcal{N}_2) = (9, 6)$. However for $(\mathcal{N}_1, \mathcal{N}_2) = (14, 1)$(this system is used recently in phase transition studies[21]), the $U(14)$ irrep $[0]$ occurs, for even N, as ground state with $\sim 70\%$ and thus random interactions may be useful here. Another example is, in IBM-4 with $(\mathcal{N}_1, \mathcal{N}_2) = (3, 3)$ the $(\omega_1\omega_2) = (ST) = (00)$ irrep is ground state with $\sim 50\%$ probability. However in real

nuclei this irrep is expected to be the ground state and therefore in IBM-4 one can use random interactions but a regular part enhancing the probability for $(ST) = (00)$ should be added.

3.2 Mean-field analysis

Bijker and Frank[18,20] carried out a mean-field analysis of sp and sd IBM's to give a quantitative understanding of the probabilities, with random interactions in these models, for a given L to be the ground state. We will follow this approach with suitable extensions to describe the results found in Fig. 1 and Table. 1. We begin with I, i. e. $\mathcal{N}_1 \geqslant 3, \mathcal{N}_2 = 1$. The $\mathcal{N}_2 = 1$ gives s bosons (with angular momentum $l = 0$). Similarly $\mathcal{N}_1 \geqslant 3$ gives bosons carrying $\mathcal{N}_1$ degrees of freedom and they can be thought of as bosons with $l = l_1, l_2, \ldots, l_r$, $\sum_{i=1}^{r} (2l_i + 1) = \mathcal{N}_1$. Just as in [20], a one parameter H is considered (with α_1 and α_6 terms in(3)),

$$H = \frac{1}{N}\cos\chi \hat{n}_1 + \frac{1}{N(N-1)}\sin\chi S_+ S_- , S_+ = s^\dagger s^\dagger -$$
$$\sum_{i=1}^{r} b_{l_i}^\dagger \cdot b_{l_i}^\dagger , S_- = (S_+)^\dagger . \qquad (7)$$

In $(7) - \pi/2 < \chi \leqslant 3\pi/2$ so that all attractive and repulsive interactions are included. The ground state shapes and hence the ground state ω_1 are determined by minimizing the energy functional for the axially symmetric coherent state (CS),

$$|N\alpha\rangle = \frac{1}{\sqrt{N!}}(\cos\alpha s^\dagger + \sin\alpha x_0^\dagger)^N |0\rangle ,$$

$$x_0^\dagger = \frac{1}{\sqrt{r}}\sum_{i=1}^{r} b_{l_i}^\dagger , 0 . \qquad (8)$$

In Eq. (8), $-\pi/2 < \alpha \leqslant \pi/2$ and it gives for example correctly the CS used for sdgIBM in the past[10]. The energy functional $E(\alpha) = \langle N\alpha | H | N\alpha \rangle = \cos\chi\sin^2\alpha + \frac{1}{4}\sin\chi\cos^2 2\alpha$. The minima of $E(\alpha)$ and the corresponding shape parameters α of the CS divide into three classes: (i) $\alpha = 0$ for $-\pi/2 < \chi \leqslant \pi/4$, i.e. in a $3\pi/4$ range; (ii) $\alpha = \pi/2$ for $3\pi/4 \leqslant \chi \leqslant 3\pi/2$, i.e. in a $3\pi/4$ range; (iii) α such that $\cos 2\alpha = \cot\chi$ for $\pi/4 \leqslant \chi \leqslant 3\pi/4$, i.e. in a $\pi/2$ range. It is seen from (8) that $\alpha = 0$ gives s boson condensate and hence here $\omega_1 = 0$ and it occurs with $(3\pi/4(2\pi)) \times 100\% = 37.5\%$ probability. Similarly $\alpha = \pi/2$ gives x boson condensate and here, apart from a constant factor, $E = -\sin\chi\omega_1(\omega_1 + \mathcal{N}_1 - 2)$ where $\omega_1 = n_1, n_1 - 2, \ldots, 0$ or 1. Then clearly, $\sin\chi$ positive (this happens $\pi/4$ times) gives $\omega_1 = N$ to be lowest with 12.5\% probability and $\sin\chi$ negative (this happens $\pi/2$ times) gives $\omega_1 = 0$ to be lowest for even N and $\omega_1 = 1$ for odd N with 25\% probability. For $\cos 2\alpha = \cot\chi$ the condensate is deformed with both s and x bosons. It is plausible to argue that the condensate here gives a band with $L = \kappa\omega_1, \omega_1 = 0, 1, 2, \ldots, N$ and $\kappa = 1$ for spIBM, $\kappa = 2$ for sdIBM, $\kappa = 4$ for sdgIBM etc. The moment of inertia $(\mathcal{I})$ of these bands should follow from $O(\mathcal{N}_1)$ cranking (a method for this may be possible via the results in [22]). As yet there is no theory for this and therefore we assume that the $O(3)$ cranking formula given in [18,20] is

Table 1. Probabilities (in percentage) for $(\omega_1\omega_2)$ to be ground state irrep.

model	$\mathcal{N}_1$	$\mathcal{N}_2$	N	$(\omega_1\omega_2) = (00)$	$(\omega_1\omega_2) = (N0)$	$(\omega_1\omega_2) = (0N)$
$U(7)$	4	3	10	55.4	21.4	20.5
			20	54	21.2	20.6
spdIBM	6	3	10	55	22.4	19.8
			20	53.5	21.9	20.3
sdgIBM	10	5	10	55.3	22.9	19.2
			20	53.6	22.8	20.4
$spdf$IBM	10	6	10	49.3	24.6	21.9
			20	49.3	23.8	22
$sdgpf$IBM	15	10	10	53.8	22.9	20.3
			20	54.4	22.4	20.5
IBM-3	15	3	10	49	27.1	19.8
			20	49	25.6	20.7
IBM-4	3	3	10	49.2	22.8	22.8
			20	48.8	23.2	23.2
	30	6	10	50.1	28.6	18.6
	18	18	10	50	23.5	23.5
	15	15	10	49.6	23.6	23.6

valid here to within a constant. Then (with $\mathcal{F} = (\sin\chi - \cos\chi)/\sin\chi\cos\chi$) it is easily seen that for $\cos 2\alpha = \cot\chi$, $\omega_1 = 0$ is lowest with 12.5% probability and $\omega_1 = N$ is lowest with 12.5% probability. Combining all these results will give for I for the ground state probabilities: (i) $\omega_1 = 0$ with 75% and $\omega_1 = N$ with 25% for even N; (ii) $\omega_1 = 0$ with 50%, $\omega_1 = 1$ with 25% and $\omega_1 = N$ with 25% for odd N. They give a good description of the results in Figs. 1(a), 1(b) and 1(c).

Now we will consider the mean-field analysis for II, i.e. for $\mathcal{N}_1$, $\mathcal{N}_2 \geqslant 3$ and the discussion will be restricted to even N. Just as the $x^\dagger$ operator in Eq. (8), let us introduce $y^\dagger$ and $z^\dagger$ operators, $y_0^\dagger = \frac{1}{\sqrt{p}} \sum_{i=1}^{p} b_{l_i,0}^\dagger$, $\sum_{i=1}^{p} (2l_i + 1) = \mathcal{N}_1$ and $z_0^\dagger = \frac{1}{\sqrt{q}} \sum_{j=1}^{q} b_{l_j',0}^\dagger$, $\sum_{j=1}^{q} (2l_j' + 1) = \mathcal{N}_2$. Then the hamiltonian, CS and $E(\alpha)$ are,

$$H = \frac{1}{N}\cos\chi \hat{n}_2 + \frac{1}{N(N-1)}\sin\chi S_+ S_-,$$

$$S_+ = S_+(1) - S_+(2) = \sum_{i=1}^{p} b_{l_i}^\dagger \cdot b_{l_i}^\dagger - \sum_{j=1}^{q} b_{l_j'}^\dagger \cdot b_{l_j'}^\dagger,$$

$$S_- = (S_+)^\dagger, \qquad (9)$$

$$\left| N\alpha \right\rangle = \frac{1}{\sqrt{N!}}(\cos\alpha y_0^\dagger + \sin\alpha z_0^\dagger)^N \left| 0 \right\rangle,$$

$$E(\alpha) = \cos\chi \sin^2\alpha + \frac{1}{4}\sin x \cos^2 2\alpha.$$

The H in (9) is same as H^{II} defined in Eq. (6) but only with the α_2 and α_6 terms. The equilibrium shapes correspond to $\alpha = 0$, $\alpha = \pi/2$ and $\cos 2\alpha = \cot\chi$ with the range of $\chi's$ just as before. The $\alpha = 0$ gives y-boson condensate with energy $E(\alpha = 0) \propto -\sin\chi\omega_1(\omega_1 + \mathcal{N}_1 - 2)$. Then the ground state irreps are $(\omega_1\omega_2) = (00)$ with 25% and $(\omega_1\omega_2) = (N0)$ with 12.5% probability. Similarly $\alpha = \pi/2$ gives z-boson condensate with energy $E(\alpha = \pi/2) \propto -\sin\chi\omega_2(\omega_2 + \mathcal{N}_2 - 2)$ and then the ground state irreps are $(\omega_1\omega_2) = (00)$ with 25% and $(\omega_1\omega_2) = (0N)$ with 12.5% probability. In the situation $\cos 2\alpha = \cot\chi$, cranking has to be done with respect to both $O(\mathcal{N}_1)$ and $O(\mathcal{N}_2)$. Evaluating moment of inertias as before gives E to be, to within a constant,

$$E = \frac{\omega_1(\omega_1 + \mathcal{N}_1 - 2)}{A_+} + \frac{\omega_2(\omega_2 + \mathcal{N}_2 - 2)}{A_-},$$

$$A_\pm = \mp \frac{(\sin\chi \pm \cos\chi)}{\cos\chi \sin\chi}. \qquad (10)$$

With $\pi/4 \leqslant \chi \leqslant 3\pi/4$ here, it is seen that A_+ is +ve and A_- is −ve for $\pi/4 \leqslant \chi \leqslant \pi/2$ and A_+ is −ve and A_- is +ve for $\pi/2 \leqslant \chi \leqslant 3\pi/4$. Therefore, here $(N0)$ and $(0N)$ irreps

will be ground states each with 12.5% probability. Combining all the results give for II, $(\omega_1\omega_2) = (00)$, $(N0)$ and $(0N)$ irreps to be ground states with 50%, 25% and 25% probability. These numbers clearly describe the results in Table 1 and Fig. 1(d).

In summary, the mean-field approach of [18,20] with proper extensions gives a good understanding of the results in Fig. 1 and Table 1 although all the results are obtained using a constant probability for χ in Eqs. (7,9). Extension of the analysis to odd N for II (here the lowest $(\omega_1\omega_2)$ are (10) and (01)) and also the calculations for $\mathcal{N}_2 = 2$ will be given elsewhere.

4 Conclusions and future outlook

In this paper for the first time TBRE's preserving irreps of group symmetries (other than $O(3)$), for boson systems, are introduced and showed that the 0^+ dominance observed in ground states extends to group irreps. An extended mean-field analysis is shown to give good description of the numerical results obtained for a variety of interacting boson models. The mean-field analysis in Section 3 is restricted to the simple mixing Hamiltonians given by Eqs. (7,9) and in a future paper this will be extended to the full Hamiltonians given by Eqs. (3,6). Similarly, in a future publication we will consider $O(\mathcal{N})$, $\mathcal{N} > 3$ cranking so that the application of $O(3)$ cranking to I and II in Section 3 can be validated. At present the justification for using $O(3)$ cranking comes from the good agreement between the mean-field results and those in Fig. 1 and Table 1. It should be added that Kusnezov's analysis for spIBM[17], based on random polynomials, can be applied to H^I and H^{II} as the matrices here are tridiagonal and this will be done elsewhere. Finally, it will be interesting to extend the present work to other general classes of group-subgroup chains in IBM's (see[14,15] for examples) and also to group chains for fermion systems (as they appear for example in the shell model[23]). It is plausible that the results of these extensions will give deeper understanding of geometric chaos and regularities generated by random interactions.

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原子核中的无规相互作用以及基态 0^+ 主导向群表示论的推广

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摘要　在相互作用玻色子模型框架下，使用 $U(\mathcal{N}) \supset U(\mathcal{N}_1) \oplus U(\mathcal{N}_2) \supset O(\mathcal{N}_1) \oplus O(\mathcal{N}_2)$ 和 $U(\mathcal{N}) \supset O(\mathcal{N}) \supset O(\mathcal{N}_1) \oplus O(\mathcal{N}_2)$ 群链下具有 $O(\mathcal{N}_1) \oplus O(\mathcal{N}_2)$ 对称性的无规单体及两体哈密顿量来研究给定的不可约表示 $(\omega_1\omega_2)$ 成为偶偶核基态的出现几率，其中 $[\omega_1]$ 和 $[\omega_2]$ 分别为 $O(1)$ 和 $O(2)$ 的对称表示．采用 500 个无规矩阵系综对具有 10—25 个玻色子的体系进行研究后发现，当 $\mathcal{N}_1, \mathcal{N}_2 \geqslant 3$ 时，$(\omega_1\omega_2) = (00)$ 出现的几率 $\sim 50\%$；而 $(0N)$ 和 $(N0)$ 出现的几率为 25%．类似地，当 $\mathcal{N}_1 \geqslant 3, \mathcal{N}_2 = 1$ 时，对 N 对偶数情形，$\omega_1 = 0$ 出现的几率约为 75%；$\omega_1 = N$ 出现的几率为 25%；对 N 为奇数情形，$\omega_1 = 0$ 出现的几率约为 50%，而 $\omega_1 = 1$ 或 N 出现的几率都约为 25%．采用了一种推广的 Hatree-Bose 平均场理论来解释所有的这些结果．

关键词　无规相互作用　原子核　相互作用玻色子模型　群对称性 $O(\mathcal{N}_1) \oplus O(\mathcal{N}_2)$　基态的不可约表示　几率　平均场分析