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Influence of Zero-Point Energy Correction on Effective Nucleon Mass in Derivative Coupling Models

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Abstract In the light of derivative coupling models, we argue that the zero-point energy of the vacuum could not be simply thrown away at high temperature. So here the finite contribution which is temperature dependent has been separated from it and the influence of this correction on effective nucleon mass in nuclear matter has been studied.

Key words ZM model, zero-point energy of vacuum, effective nucleon mass

In recent years, the properties of hadronic matter at finite temperature remain an active area[1-3]. The QHD (quantum hadrodynamics) model[4], proposed by Walecka in early days, is the main theory in this area. It has led many interesting and important theory results in describing finite nuclei and the properties of nuclear matter at both zero temperature and finite temperature. For example, binding energy, saturation density and phase transition of nuclear matter. It also gives good predictions in experiments, such as, noncentral spin-orbit splitting in finite nuclei. However, this model has its shortcomings. For instance, the effective mass of nucleon in nuclear matter at moderately high density and/or temperature becomes very small, or even negative if Δ particles are included[5]. In order to avoid this problem, Zimanyi and Moszkowski have proposed models (ZM) for hadronic matter differing from the Walecka model only in the form of the coupling of the nucleon to the scalar meson[6]. After that, improved ZM models or derivative coupling models have been developed[7,8]. These models give better results than Walecka model. Recently, variants of the ZM models have already been applied to investigate many physical problems, such as, multilambda matter properties, neutron star, Δ-excited nuclear matter and some thermodynamical properties of nuclear matter[1,9].

In the study of nuclear matter at finite temperature, the effective nucleon mass is an important quantity. By applying ZM models, many authors have discussed the temperature and density dependence of the effective nucleon mass at low and high temperature[1,7,9]. As in Ref.[1], the temperature has been extended to 400MeV. However, they all neglected the zero-point energy of the vacuum. As it is an infinite energy shift of vacuum which is not measurable, this energy shift has been regularized to zero. That is to say, it has been thrown away. But we find that there is temperature dependent part in the zero-point energy, it could not been simply thrown away. We argue that the finite part relating to temperature should be separated from the zero-point energy. And we think that this part has nontrivial contribution at high temperature. In this paper, we name this finite part as the zero-point energy correction. The purpose of our paper is to study how this zero-point energy correction influences the effective nucleon mass in ZM models. As we know, the ZM models are not renormalizable. The zero-point energy correction here is not acquired through renormalization. The procedure is first to subtract the pure vacuum contribution, then to separate the finite piece which is temperature dependent as the zero-point energy correction.

Since ZM models have been discussed in detail in past literature[6,7], here we will only present the Lagrangian obtained after the proper rescaling of the fields[1]:

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$$\mathscr{L}_R = \bar{\psi} i \gamma_\mu \partial^\mu \psi + m^{*\alpha}\left(-g_\omega \bar{\psi}\gamma_\mu \psi \omega^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu\right) - \bar{\psi}(M - m^{*\beta} g_\sigma \sigma)\psi + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2), \qquad (1)$$

where

$$m^* = \left[1 + \frac{g_\sigma \sigma}{M}\right]^{-1} \qquad (2)$$

ψ, σ and ω are the fields of nucleon, σ and ω mesons respectively; M is the nucleon mass and $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$; α and β have the following values for the different models: Walecka, $\alpha = 0, \beta = 0$; ZM, $\alpha = 0, \beta = 1$; ZM2, $\alpha = 1, \beta = 1$; ZM3, $\alpha = 2, \beta = 1$. Here for simplicity, we just consider two of them: the ZM and ZM3 models.

The energy density ε of the system can be obtained as usual by the average of the energy-momentum tensor. If the vacuum contribution is neglected, after mean field approximation, we can write

$$\varepsilon_0 = \frac{C_\omega^2}{2M^2} m^{*\alpha} \rho^2 + \frac{M^4}{2C_\sigma^2}\left(\frac{1 - m^*}{m^{*\beta}}\right)^2 + \frac{\gamma}{(2\pi)^3}\int d^3 k E^*(k)(n_k + \bar{n}_k). \qquad (3)$$

This is the usual result given in past literature[1,7]. Here γ is the degeneracy factor (for nuclear matter $\gamma = 4$); n_k and $\bar{n}_k$ stand for the Fermi-Dirac distribution for baryons and antibaryons respectively. $E^*(k)$ is given by $E^*(k) = \sqrt{k^2 + M^{*2}}$ with the effective nucleon mass $M^* = M - m^* g_\sigma \bar{\sigma}$. ρ is the net baryon density. We have introduced $C_\sigma^2 = g_\sigma^2 M^2 / m_\sigma^2$ and $C_\omega^2 = g_\omega^2 M^2 / m_\omega^2$. Now we want to consider the vacuum contribution. Then the pure vacuum contribution of zero temperature must first be subtracted off. Thus,

$$\varepsilon = \varepsilon_0 + \Delta\varepsilon_{zp}, \qquad (4)$$

where

$$\Delta\varepsilon_{zp} = -\frac{\gamma}{(2\pi)^3}\int d^3 k\left[\sqrt{k^2 + M^{*2}} - \sqrt{k^2 + M^2}\right]. \qquad (5)$$

$\Delta\varepsilon_{zp}$ is the zero-point energy[11], it represents the difference of energy of a filled negative energy Fermi sea of baryons with mass M^* and that of a filled negative energy Fermi sea of baryons of mass M. If $\Delta\varepsilon_{zp}$ is regularized to zero, we recover the usual result in past literature. But it is obvious that the first term of $\Delta\varepsilon_{zp}$ in Eq.(5) is temperature dependent. Simply throwing it away is not proper. So we want to separate the finite contribution of $\Delta\varepsilon_{zp}$, which is temperature dependent. From Ref. [11], we know that the original form of $\Delta\varepsilon_{zp}$ could be written as

$$\Delta\varepsilon_{zp} = -\frac{i\mathrm{tr}}{(2\pi)^4}\int d^4 k \gamma^0 k^0\left[\frac{1}{\gamma_\mu k^\mu - M^*} - \frac{1}{\gamma_\mu k^\mu - M}\right], \qquad (6)$$

here tr indicates a trace over the matrix indices. If we introduce a dimensionless variable $x = k/M$ and consider,

$$\frac{M^*}{M} = 1 - \frac{m^* g_\sigma \sigma}{M} \equiv 1 - \eta, \qquad (7)$$

then,

$$\Delta\varepsilon_{zp} = -\frac{i\mathrm{tr}}{(2\pi)^4}\int d^4 x \gamma^0 x^0\left[\frac{1}{\gamma_\mu x^\mu - 1 + \eta} - \frac{1}{\gamma_\mu x^\mu - 1}\right]. \qquad (8)$$

As η is small, the term in square brackets could be expanded in a power series,

$$\left[\frac{1}{\gamma_\mu x^\mu - 1 + \eta} - \frac{1}{\gamma_\mu x^\mu - 1}\right] = \sum_{p=1}^{\infty}\frac{(-\eta)^p}{(\gamma_\mu x^\mu - 1)^{p+1}}. \qquad (9)$$

Insert it back to the Eq.(8) and notice that,

$$\mathrm{tr}\left[\gamma^0 x^0 \sum_{p=1}^{\infty}\frac{(-\eta)^p}{(\gamma_\mu x^\mu - 1)^{p+1}}\right] = -\mathrm{tr}\left[x^0 \frac{\partial}{\partial x^0}\sum_{p=1}^{\infty}\frac{(-\eta)^p}{p(\gamma_\mu x^\mu - 1)^p}\right]. \qquad (10)$$

Then a partial integration on x^0 will reduce the integral in Eq.(8) to the form

$$\Delta\varepsilon_{zp} = -\frac{i\mathrm{tr}}{(2\pi)^4}\int d^4 x \sum_{p=1}^{\infty}\frac{(-\eta)^p}{p(\gamma_\mu x^\mu - 1)^p}. \qquad (11)$$

Through dimensional regularization, it could be seen that only the first four terms in the sum over p are ultraviolet divergent, and we treat this part as really unobservable infinite energy shift of vacuum. The terms with $p \geq 5$ have enough powers of x downstairs for convergence. This is the finite energy shift which is temperature dependent. So we just take this finite part as our zero-point energy correction. Through calculation, we find that the final expression of this correction is formally the same as the result derived by Ref. [10], which is, however, in the light of the Walecka model. So the zero-point energy correction can be written as

$$\Delta\varepsilon = -\frac{\gamma}{16\pi^2}M^4\left[\left(\frac{M^*}{M}\right)^4 \log\frac{M^*}{M} - \frac{1}{4} + \frac{4}{3}\frac{M^*}{M} - 3\left(\frac{M^*}{M}\right)^2 + 4\left(\frac{M^*}{M}\right)^3 - \frac{25}{12}\left(\frac{M^*}{M}\right)^4\right]. \qquad (12$$

But one must motice here, for ZM models, $M^* = M -$

$m^* g_\sigma \bar{\sigma} = m^* M$. Thus the resulting mean-field equation of state, including the zero-point energy correction, is

$$\varepsilon = \frac{C_\omega^2}{2M^2} m^{*\alpha} \rho^2 + \frac{M^4}{2C_\sigma^2}\left(\frac{1-m^*}{m^{*\beta}}\right)^2 + \frac{\gamma}{(2\pi)^3}\int d^3 k E^*(k)(n_k + \bar{n}_k) + \Delta\varepsilon, \quad (13)$$

$$\Delta\varepsilon = -\frac{\gamma}{16\pi^2} M^4 \Big[m^{*4}\log m^* - \frac{1}{4} + \frac{4}{3}m^* - 3m^{*2} + 4m^{*3} - \frac{25}{12}m^{*4}\Big]. \quad (14)$$

In order to get the effective nucleon mass, one can minimize ε with respect to m^*. Thus we obtain the self-consistent equation of m^*,

$$1 - m^* - \frac{\gamma C_\sigma^2}{2\pi^2} m^{*3\beta+1}\int \frac{x^2 dx}{\sqrt{x^2 + m^{*2}}}(n_x + \bar{n}_x) - \frac{\alpha}{2}\frac{C_\sigma^2 C_\omega^2}{M^6} m^{*\alpha+2\beta}\rho^2 + \frac{\gamma m^{*3} C_\sigma^2}{16\pi^2}\Big[4m^{*3}\log m^* + \frac{4}{3} - 6m^* + 12m^{*2} - \frac{22}{3}m^{*3}\Big] \quad (15)$$

where the dimensionless variable $x = \frac{k}{M}$ has been used. This equation can be solved at given temperature and chemical potential to determine M^*. Here, for ZM model, $C_\sigma^2 = 169.2$, $C_\omega^2 = 59.1$; for ZM3, $C_\sigma^2 = 443.3$, $C_\omega^2 = 305.5$[1]. Then we can study temperature and density dependence of M^* with the zero-point energy correction, and compare them to those without this correction.

In Fig. 1 we show M^* as a function of T at zero net baryon density for ZM and ZM3 models. The correction acts to raise the value of M^* at given T. At moderately high temperature, the changing effect is remarkable. For ZM3 model, when $T = 250\text{MeV}$, $\Delta M^* = M_c^* - M^* \approx 20\text{MeV}$. (The subscript c means "with correction".) And the separation of ZM3 model is more remarkable than that of ZM. For both models, the higher the temperature is, the more obvious the separation is. This means the vacuum

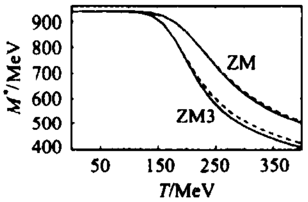

Fig. 1. The effective nucleon mass in nuclear matter as a function of the temperature at $\rho = 0$. (Dashing line stands for with correction; solid line without correction.)

contribution is more effective at high temperature. It is accordance with our understanding of the vacuum.

In Fig. 2 we show the behavior of the effective nucleon mass with net baryon density at different temperature for ZM and ZM3 models. When temperature is low, the curves with corrections are not so different from those without corrections. When temperature is high, the correction acts to move the whole curve upward. It is not difficult to understand. As in Eq. (8), $\Delta\varepsilon$ is positive definite, and the whole energy spectrum is shifted upward. So is the mass spectrum. At given temperature, the mass shift in ZM3 model is more remarkable than that of ZM. Moreover, the mass shift is density dependent. But it is interesting that, at low temperature, the mass shift increases with density increasing; when temperature is raised, the mass shift at low density increases more rapidly than that of high density. Over some critical temperature, the mass shift will decrease with density increasing. Such as Fig. 2(c), for ZM3 model at $T = 250\text{MeV}$, the two curves get closer to each other when density increases. Through some numerical evaluation, we find that the related critical temperature is about 319MeV for ZM model and is about 232MeV for ZM3 model.

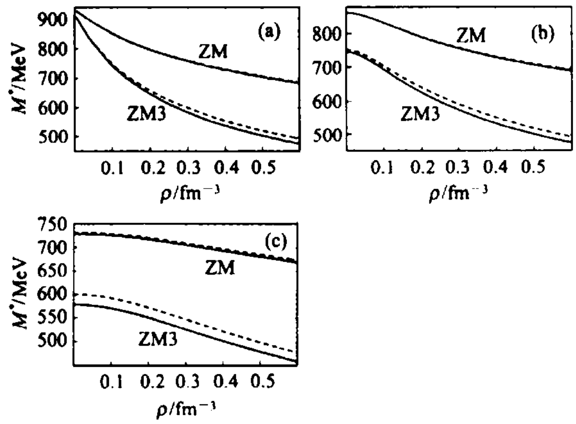

Fig. 2. (a) The effective nucleon mass as a function of ρ at $T = 150\text{MeV}$. (b) Same as (a) for $T = 200\text{MeV}$. (c) Same as (a) for $T = 250\text{MeV}$. (Dashing line stands for with correction and solid line without correction for all three cases.)

In summary, in this paper we have separated the finite zero-point energy correction in ZM models, and discussed how this correction influences the effective nucleon mass in nuclear matter. We find that at low temperature this correction has little contribution, while at high temperature it increases the effective nucleon mass remarkably. When ZM models are extended to study the thermodynamical properties at high temperature, the zero-point energy correction should be included.

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ZM模型中零点能修正对核子有效质量的影响

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摘要 基于ZM模型,讨论了真空零点能的贡献.认为真空零点能不能简单地丢掉,所以零点能中与温度相关的有限部分被分离了出来,并具体讨论了在核物质中这一零点能修正对核子有效质量的影响,发现在高温时它有着非平庸的贡献.

关键词 ZM模型 真空零点能 核子有效质量

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