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引力子三顶点与引力自能的曲率激发*

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摘要 基于微扰展开,计算了联络场三点 Green 函数及单圈引力子自能对量子 Wilson 圈的贡献.结果表明,引力子三顶点及引力自能将使 Einstein 引力获得定域曲率的激发.

关键词 Einstein 引力 Wilson 圈 曲率激发 引力子三顶点 引力自能

物理可观测量一直是量子引力研究最重要的议题之一.因在目前阶段,在量子引力中定义严格意义上的物理可观测量仍是件比较困难的事.所以在以 Ashtekar 新变量[1,2]作为正则变量的非微扰量子引力中,通常约定将与经典意义下的标量(即任意坐标变换下的不变量)相对应的量作为理论的"预选可观测量"[3,4].

由规范联络定义出的 holonomy 具有规范协变性,而它的矩阵迹,即 Wilson 圈则是一规范不变量[5],可作为量子引力的预选可观测量.另一方面,Wilson 圈又可充当从联络表象到圈表象的表象变换矩阵元[6].而在量子引力的圈表象中,Wilson 圈还是构造物理可观测量(即体积、面积算符)的基础[7,8].此外,通过对具体引力的 Wilson 圈,特别是其量子行为的研究,可为该种引力微观相互作用的机理提供物理上的诠释[9].例如,通过计算 K^2 阶的量子 Wilson 圈贡献,导致了 Einstein 引力不存在定域曲率激发这一不被期望的物理图象[4].在文献[10]的基础上,本文进一步计算了引力联络场的三点 Green 函数及最低阶引力自能对 Einstein 引力的量子 Wilson 圈的贡献.计算结果表明,当计入引力相互作用三顶点的贡献后,该种引力将获得曲率的激发.

1 量子 Wilson 圈

本文将讨论 Einstein 引力的量子 Wilson 圈问题.该引力的作用量由下式给出:

$$S = -\frac{2}{K^2}\int d^4x\sqrt{-g}g^{\rho\sigma}R_{\rho\sigma}, \quad K^2 = 32\pi G. \tag{1}$$

按文献[10]的定义,Christoffel 联络

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$$\Gamma^{\alpha}_{\mu\beta}=\frac{1}{2}g^{\alpha\gamma}(\partial_{\mu}g_{\beta\gamma}+\partial_{\beta}g_{\mu\gamma}-\partial_{\gamma}g_{\mu\beta}),\tag{2}$$

的 Wilson 圈泛函(简称为 Wilson 圈)由下式给出

$$W(l)=-4+\mathrm{Tr}P\exp\left[\oint_{l}\mathrm{d}x^{\mu}\Gamma_{\mu}(x)\right],\tag{3}$$

式中,P 表示矩阵$(\Gamma_{\mu})^{\alpha}_{\beta}=\Gamma^{\alpha}_{\mu\beta}$沿路线 l 的排序算子[11].在经典意义下,该 Wilson 圈的量值反映了矢量沿 Lorentz 流形上一闭合圈 l 平移一周后变化的情况.

熟知,由作用量(1)式给出的引力理论亦可用定域 Lorentz 群不变的规范理论给出.容易证明,由 Lorentz 规范联络 A 给出的 Wilson 圈 W_A 是一规范不变量.且与 Christoffel 联络的 Wilson 圈相等:$W_A=W$[10].因 Christoffel 联络更便于实际计算,所以本文的计算将从(3)式出发.

按 Taylor 展开,(3)式可表示为含 1,2,…个联络场的项:

$$W(l)=W_{(1)}+W_{(2)}+\cdots,\tag{4}$$

式中,$W_{(1)}=\oint_{l}\mathrm{d}x^{\mu}\Gamma^{\alpha}_{\mu\alpha}(x)$, $W_{(2)}=\mathrm{P}\oint_{l}\mathrm{d}x^{\mu}\oint_{l}\mathrm{d}y^{\nu}\Gamma^{\alpha}_{\mu\beta}(x)\Gamma^{\beta}_{\nu\alpha}(y)=$

$$\oint_{l}\mathrm{d}x^{\mu}\int_{o}^{x}\mathrm{d}y^{\nu}\Gamma^{\alpha}_{\mu\beta}(x)\Gamma^{\beta}_{\nu\alpha}(y),\cdots.$$

本文的量子引力场将按通常的平坦背景分解方式来定义

$$h^{\mu\alpha}\equiv K^{-1}(\sqrt{-g}g^{\mu\alpha}-\eta^{\mu\alpha}),\tag{5}$$

式中,$\eta^{\mu\alpha}=(-+++)$为经典平坦背景,引力场 $h^{\mu\alpha}$ 的指标升降将由它来完成.由此,(2)式的联络场可用引力场表示为

$$\Gamma^{\alpha}_{\mu\beta}=-\frac{K}{2}\left[\partial_{\mu}\left(h^{\alpha}_{\beta}-\frac{1}{2}\delta^{\alpha}_{\beta}h^{\gamma}_{\gamma}\right)+\partial_{\beta}h^{\alpha}_{\mu}-\partial^{\alpha}h_{\beta\mu}+\frac{1}{2}(\eta_{\mu\beta}\partial^{\alpha}-\delta^{\alpha}_{\mu}\partial_{\beta})h^{\gamma}_{\gamma}\right]+O(h^{2}).\tag{6}$$

这样,引力作用量(1)式可写成引力场的平方项、立方项….其中平方项将给出 K^0 阶的引力子自由传播子

$$\langle h_{\mu\nu}(x)h_{\alpha\beta}(y)\rangle=\mathrm{i}D_{\mu\nu,\alpha\beta}(x-y)=\frac{\mathrm{i}}{2}(\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\mu\beta}\eta_{\nu\alpha}-\eta_{\mu\nu}\eta_{\alpha\beta})D(x-y),\tag{7}$$

式中 $D(x)=-\int\frac{\mathrm{d}^4p}{(2\pi)^4}p^{-2}\exp(\mathrm{i}p.x)=-\frac{1}{4\pi^2x^2}$ 为无质标量场的传播子,且$\Box D(x)=\delta^4(x)$;而立方项则给出 K^1 阶的引力子三顶点,其动量空间表示为[12]

$$\begin{aligned}&\mathrm{i}U_{\alpha_1\beta_1,\alpha_2\beta_2,\alpha_3\beta_3}(q_1,q_2,q_3)=\\&\mathrm{i}K[q_{2(\alpha_1}q_{3\beta_1)}(\eta_{\alpha_2(\alpha_3}\eta_{\beta_3)\beta_2}-\frac{1}{2}\eta_{\alpha_2\beta_2}\eta_{\alpha_3\beta_3})+2q_{3(\alpha_2}\eta_{\beta_2)(\alpha_1}\eta_{\beta_1)(\alpha_3}q_{2\beta_3)}+\\&q_2\cdot q_3(-\eta_{\alpha_1(\alpha_2}\eta_{\beta_2)(\alpha_3}\eta_{\beta_3)\beta_1}-\eta_{\alpha_1(\alpha_3}\eta_{\beta_3)(\alpha_2}\eta_{\beta_2)\beta_1}+\frac{1}{2}\eta_{\alpha_1(\alpha_2}\eta_{\beta_2)\beta_1}\eta_{\alpha_3\beta_3}+\\&\frac{1}{2}\eta_{\alpha_1(\alpha_3}\eta_{\beta_3)\beta_1}\eta_{\alpha_2\beta_2})+(1,2,3)\text{循环}].\end{aligned}\tag{8}$$

按路径积分量子化程序,量子 Wilson 圈可写成

$$\langle W\rangle=\int[\mathrm{d}h^{\mu\alpha}]W(h)\exp(\mathrm{i}S).\tag{9}$$

由(4)式知,量子 Wilson 圈将由联络场的一点、两点…Green 函数来贡献.按量子引力的观点,联络场在空间中的传播将由引力子的传播来贡献.因此,可借助引力子传播子及引力三顶点等最终求取引力的量子 Wilson 圈的值.

2 联络场三点 Green 函数对量子 Wilson 圈的贡献

按量子场论的观点,非真空破缺场的一点 Green 函数为 0,即$\langle\Gamma\rangle=0$.由此联络场的一点 Green 函数对量子 Wilson 圈的贡献为零: $W_{(1)}=\oint_l \mathrm{d}x^\mu\langle\Gamma^\alpha_{\mu\alpha}(x)\rangle=0$. 所以,量子 Wilson 圈的带头项将由裸传播子贡献

$$\langle W_{(2)}\rangle = P\oint_l \mathrm{d}x^\mu\oint_l \mathrm{d}y^\nu\langle\Gamma^\alpha_{\mu\beta}(x)\Gamma^\beta_{\nu\alpha}(y)\rangle =$$

$$\frac{\mathrm{i}}{4}K^2P\oint_l \mathrm{d}x^\mu\oint_l \mathrm{d}y^\nu[-\partial_\mu\partial_\nu D^{\alpha\beta}_{\alpha\beta}(x-y)+2(\eta^{\alpha\beta}\Box-\partial^\alpha\partial^\beta)D_{\mu\alpha,\nu\beta}(x-y)-$$

$$2\eta^{\alpha\beta}\Box D_{\mu\alpha,\alpha\beta}(x-y)+\eta^{\alpha\beta}\partial_\mu\partial^\lambda D_{\nu\lambda,\alpha\beta}(x-y)+\eta^{\alpha\beta}\partial_\nu\partial^\lambda D_{\mu\lambda,\alpha\beta}(x-y)-$$

$$\frac{1}{2}\eta^{\alpha\beta}\eta^{\lambda\tau}(\partial_\mu\partial_\nu-\eta_{\mu\nu}\Box)D_{\alpha\beta,\lambda\tau}(x-y)]=$$

$$\frac{\mathrm{i}}{4}K^2P\oint_l \mathrm{d}x^\mu\oint_l \mathrm{d}y^\nu[8\partial_\mu\partial_\nu\frac{1}{4\pi^2(x-y)^2}+3\eta_{\mu\nu}\delta^4(x-y)], \tag{10}$$

该结果与文献[10]一致.(10)式中,被积函数为全梯度项,或超定域项(δ 函数),其圈积分的结果为 0.这将给出平坦背景分解方案下 Einstein 引力在 K^2 阶无曲率激发这一不被期望的物理图象[4].

进一步的分析表明,除(10)式给出的 K^2 阶贡献外,量子 Wilson 圈还应包括如下 K^4 阶的贡献:单圈引力子自能对$\langle W_{(2)}\rangle$的贡献;三个引力子传播子和一个 K^1 阶顶点对$\langle W_{(3)}\rangle$的贡献;以及两个传播子对$\langle W_{(4)}\rangle$的贡献.最后,还有如下的 K^6 阶贡献:三个引力子传播子对$\langle W_{(6)}\rangle$的贡献;四个传播子和一个 K^2 阶四顶点对$\langle W_{(4)}\rangle$的贡献;三个传播子和一个单圈引力三顶点对$\langle W_{(3)}\rangle$的贡献;以及双圈引力子自能对$\langle W_{(2)}\rangle$的贡献.

下面将分别给出引力三顶点与最低阶引力子自能对量子 Wilson 圈的 K^4 阶贡献.

由(4)式,联络场三点 Green 函数对 Wilson 圈的贡献为

$$\langle W_{(3)}\rangle = P\oint_l \mathrm{d}x_1^{\mu_1}\oint_l \mathrm{d}x_2^{\mu_2}\oint_l \mathrm{d}x_3^{\mu_3}\langle\Gamma^\alpha_{\mu_1\beta}(x_1)\Gamma^\beta_{\mu_2\gamma}(x_2)\Gamma^\gamma_{\mu_3\alpha}(x_3)\rangle. \tag{11}$$

由(6)式,三个联络的乘积项可用引力场表示如下

$$\Gamma^\alpha_{\mu_1\beta}(x_1)\Gamma^\beta_{\mu_2\gamma}(x_2)\Gamma^\gamma_{\mu_3\alpha}(x_3)=$$

$$-\frac{1}{8}K^3[\partial_{\mu_1}h^\alpha_\beta(x_1)\partial_{\mu_2}h^\beta_\gamma(x_2)\partial_{\mu_3}h^\gamma_\alpha(x_3)+\partial^\alpha h_{\mu_1\gamma}(x_1)\partial^\beta h_{\mu_2\alpha}(x_2)\partial^\gamma h_{\mu_3\beta}(x_3)-$$

$$\partial^\alpha h_{\mu_1\beta}(x_1)\partial^\beta h_{\mu_2\gamma}(x_2)\partial^\gamma h_{\mu_3\alpha}(x_3)+\frac{1}{4}\partial_{\mu_1}h^\alpha_\alpha(x_1)\partial_{\mu_2}h^\beta_\beta(x_2)\partial_{\mu_3}h^\gamma_\gamma(x_3)+$$

$$\frac{1}{8}\partial_{\mu_3}h^\alpha_\alpha(x_1)\partial_{\mu_1}h^\beta_\beta(x_2)\partial_{\mu_2}h^\gamma_\gamma(x_3)-\frac{1}{8}\partial_{\mu_2}h^\alpha_\alpha(x_1)\partial_{\mu_3}h^\beta_\beta(x_2)\partial_{\mu_1}h^\gamma_\gamma(x_3)]-$$

$$\frac{1}{8}K^3\{-\partial_{\mu_1}h^{\alpha\beta}(x_1)(\partial^\gamma h_{\mu_2\alpha}(x_2)\partial_\gamma h_{\mu_3\beta}(x_3)+\partial_\alpha h^\gamma_{\mu_2}(x_2)\partial_\beta h_{\mu_3\gamma}(x_3))+$$

$$\partial_{\mu_1}h^\alpha_\alpha(x_1)\partial^\beta h^\gamma_{\mu_2}(x_2)(\partial_\beta h_{\mu_3\gamma}(x_3)-\partial_\gamma h_{\mu_3\beta}(x_3))-\frac{1}{2}\partial_{\mu_1}h^\alpha_\alpha(x_1)\partial_{\mu_2}h_{\beta\gamma}(x_2)\partial_{\mu_3}h^{\beta\gamma}(x_3)-$$

$$\frac{1}{4}(\partial_{\mu_1}h_{\mu_2\mu_3}(x_1)\partial_\alpha h^\beta_\beta(x_2)+\eta_{\mu_2\mu_3}\partial_{\mu_1}h_{\lambda\alpha}(x_1)\partial^\lambda h^\beta_\beta(x_2))\partial^\alpha h^\gamma_\gamma(x_3)+$$

$$\frac{1}{4}\partial_{\mu_1}h^\alpha_\alpha(x_1)(\eta_{\mu_2\mu_3}\partial^\lambda h^\beta_\beta(x_2)\partial_\lambda h^\gamma_\gamma(x_3)-\partial_{\mu_3}h^\beta_\beta(x_2)\partial_{\mu_2}h^\gamma_\gamma(x_3))+$$

$$[\partial_{\mu_1}h_{\alpha\beta}(x_1)\partial^\gamma h^\beta_{\mu_2}(x_2)\partial^\alpha h_{\mu_3\gamma}(x_3)+\frac{1}{2}(\partial_{\mu_1}h_{\mu_3\alpha}(x_1)\partial_\beta h^\alpha_{\mu_2}(x_2)+$$

$$\partial_{\mu_1}h_{\alpha\beta}(x_1)\partial^\alpha h_{\mu_2\mu_3}(x_2)-\partial_{\mu_1}h_{\mu_3\alpha}(x_1)\partial^\alpha h_{\mu_2\beta}(x_2)-\partial_{\mu_1}h_{\alpha\beta}(x_1)\partial_{\mu_3}h^\alpha_{\mu_2}(x_2))\cdot$$

$$\partial^\beta h^\gamma_\gamma(x_3)+\frac{1}{4}\partial_{\mu_1}h_{\mu_3\alpha}(x_1)\partial^\alpha h^\beta_\beta(x_2)\partial_{\mu_2}h^\gamma_\gamma(x_3)+\frac{1}{2}\partial_{\mu_1}h^\alpha_\alpha(x_1)\partial^\gamma h^\beta_\beta(x_2)\cdot$$

$$(\partial_{\mu_2}h_{\mu_3\gamma}(x_3)-\partial_\gamma h_{\mu_2\mu_3}(x_3))+(2\rightleftharpoons3)]+$$

$$[\partial^\alpha h^\beta_{\mu_1}(x_1)(\partial^\gamma h_{\mu_2\beta}(x_2)\partial^\gamma h_{\mu_3\alpha}(x_3)+\partial_{\mu_2}h^\gamma_\alpha(x_2)\partial_{\mu_3}h_{\beta\gamma}(x_3))+$$

$$\frac{1}{2}\partial^\beta h^\alpha_\alpha(x_1)(\partial_\gamma h_{\mu_2\beta}(x_2)-\partial_\beta h_{\mu_2\gamma}(x_2))\partial^\gamma h_{\mu_1\mu_3}(x_3)+$$

$$\frac{1}{2}\partial^\beta h^\alpha_\alpha(x_1)(\partial_\beta h_{\mu_2\gamma}(x_2)\partial_{\mu_1}h^\gamma_{\mu_3}(x_3)+\partial_{\mu_1}h_{\mu_2\gamma}(x_2)\partial^\gamma h_{\mu_3\beta}(x_3)+$$

$$\partial_{\mu_2}h_{\mu_1\gamma}(x_2)\partial_{\mu_3}h^\gamma_\beta(x_3))+\frac{1}{4}(\partial_{\mu_2}h_{\mu_1\alpha}(x_1)-\partial_\alpha h_{\mu_1\mu_2}(x_1))\partial_{\mu_3}h^\beta_\beta(x_2)\partial^\alpha h^\gamma_\gamma(x_3)+$$

$$\frac{1}{4}(\partial_{\mu_3}h_{\mu_1\mu_2}(x_1)\partial_\alpha h^\beta_\beta(x_2)-\eta_{\mu_2\mu_3}\partial^\lambda h_{\mu_1\alpha}(x_1)\partial_\lambda h^\beta_\beta(x_2))\partial^\alpha h^\gamma_\gamma(x_3)+$$

$$\frac{1}{8}\eta_{\mu_2\mu_3}\partial^\lambda h^\alpha_\alpha(x_1)\partial_\lambda h^\beta_\beta(x_2)\partial_{\mu_1}h^\gamma_\gamma(x_3)-(2\rightleftharpoons3)]+(1,2,3)\text{循环}\},\tag{12}$$

(12)式中各项均为三个引力场的乘积形式:$\partial h(x_1)\cdot\partial h(x_2)\partial h(x_3)$. 所以(11)式的被积函数$\langle\Gamma\Gamma\Gamma\rangle$将由引力子三点 Green 函数$\langle hhh\rangle$来贡献. 由 Feynman 规则,引力子三点 Green 函数最低阶由三个引力子传播子和一个 K^1 阶顶点贡献,即

$$G_{\mu_1\nu_1,\mu_2\nu_2,\mu_3\nu_3}(x_1,x_2,x_3)=\langle h_{\mu_1\nu_1}(x_1)h_{\mu_2\nu_2}(x_2)h_{\mu_3\nu_3}(x_3)\rangle=$$

$$\int d^4y_1d^4y_2d^4y_3D_{\mu_1\nu_1,\alpha_1\beta_1}(x_1-y_1)D_{\mu_2\nu_2,\alpha_2\beta_2}(x_2-y_2)D_{\mu_3\nu_3,\alpha_3\beta_3}(x_3-y_3)\cdot$$

$$U^{\alpha_1\beta_1,\alpha_2\beta_2,\alpha_3\beta_3}(y_1,y_2,y_3)+O(K^3).\tag{13}$$

由(7)、(8)式,求得引力子三点 Green 函数的动量空间表示如下

$$G_{\mu_1\nu_1,\mu_2\nu_2,\mu_3\nu_3}(q_1,q_2,q_3)=$$

$$-K(q_1^2q_2^2q_3^2)^{-1}\Big[\frac{1}{2}q_1^2(\eta_{\mu_1(\mu_2}\eta_{\nu_2)(\mu_3}\eta_{\nu_3)\nu_1}+\eta_{\mu_1(\mu_3}\eta_{\nu_3)(\mu_2}\eta_{\nu_2)\nu_1}+\frac{1}{2}\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3})-$$

$$\frac{1}{4}(q_1^2+q_2^2+q_3^2)\eta_{\mu_1\nu_1}\eta_{\mu_2(\mu_3}\eta_{\nu_3)\nu_2}+2q_{3(\mu_2}\eta_{\nu_2)(\mu_1}\eta_{\nu_1)(\mu_3}q_{2\nu_3)}+$$

$$\eta_{\mu_1\nu_1}(q_{3(\mu_2}\eta_{\nu_2)(\mu_3}q_{3_{\nu_3})}-q_{1(\mu_2}\eta_{\nu_2)(\mu_3}q_{2\nu_3)})+\eta_{\mu_2(\mu_3}\eta_{\nu_3)(\nu_2}q_{2(\mu_1}q_{3\nu_1)}-$$

$$\frac{1}{2}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}q_{1\mu_1}q_{1\nu_1}+(1,2,3)\text{循环}\Big]. \tag{14}$$

由(14)与(12)式可求得动量空间中的联络三点 Green 函数,经傅氏变换得到坐标空间表示后,代入(11)式即可求出$\langle W_{(3)}\rangle$的 K^4 阶贡献.经计算得如下结果

$$\begin{aligned}&\langle W_{(3)}\rangle=\\&\frac{1}{8}K^4P\oint_l \mathrm{d}x_1^{\mu_1}\oint_l \mathrm{d}x_2^{\mu_2}\oint \mathrm{d}x_3^{\mu_3}\{12\partial_{\mu_1}^{x_1}\partial_{\mu_2}^{x_2}\partial_{\mu_3}^{x_3}[D(x_1-x_2)D(x_3-x_1)]+\\&\frac{1}{2}\eta_{\mu_2\mu_3}[4\delta^4(x_2-x_3)\partial_{\mu_1}D(x_1-x_2)-3\delta(x_3-x_1)\partial_{\mu_1}D(x_1-x_2)+\\&3\partial_{\mu_1}\delta(x_1-x_2)D(x_2-x_3)-3\partial_{\mu_1}\delta(x_1-x_2)D(x_3-x_1)]+\\&3D(x_2-x_3)\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}D(x_3-x_1)-3D(x_1-x_2)\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}D(x_3-x_1)+\\&3\partial_{\mu_1}D(x_1-x_2)\partial_{\mu_2}\partial_{\mu_3}D(x_3-x_1)+\partial_{\mu_2}D(x_3-x_1)\partial_{\mu_1}\partial_{\mu_3}D(x_1-x_2)-\\&\partial_{\mu_3}D(x_1-x_2)\partial_{\mu_1}\partial_{\mu_2}D(x_3-x_1)+10\partial_{\mu_1}D(x_1-x_2)\partial_{\mu_2}\partial_{\mu_3}D(x_2-x_3)-\\&3\partial_{\mu_2}D(x_2-x_3)\partial_{\mu_1}\partial_{\mu_3}D(x_3-x_1)-\partial_{\mu_2}D(x_2-x_3)\partial_{\mu_1}\partial_{\mu_3}D(x_1-x_2)+\\&\partial_{\mu_1}D(x_3-x_1)\partial_{\mu_2}\partial_{\mu_3}D(x_2-x_3)-\partial_{\mu_3}D(x_2-x_3)\partial_{\mu_1}\partial_{\mu_2}D(x_3-x_1)+\\&\frac{1}{2}\eta_{\mu_2\mu_3}[\partial^{a}D(x_1-x_2)\partial_{\mu_1}\partial_{a}D(x_3-x_1)+\partial^{a}D(x_3-x_1)\partial_{\mu_1}\partial_{a}D(x_1-x_2)-\\&7\partial^{a}D(x_2-x_3)\partial_{\mu_1}\partial_{a}D(x_1-x_2)+3\partial^{a}D(x_2-x_3)\partial_{\mu_1}\partial_{a}D(x_3-x_1)]+\\&(1,2,3)\text{循环}\}.\end{aligned} \tag{15}$$

(15)式的被积函数中,第 1 项为全梯度项,第 2 至 5 项为超定域项,它们对圈积分的贡献为 0.利用积分公式

$$\int_0^a \mathrm{d}x\int_0^x \mathrm{d}y f(x,y)=\left(\int_0^a \mathrm{d}x\int_0^a \mathrm{d}y-\int_0^a \mathrm{d}y\int_0^y \mathrm{d}x\right)f(x,y) \tag{16}$$

化简(15)式,最终得不为 0 的之结果如下

$$\begin{aligned}&\langle W_{(3)}\rangle=\\&\frac{1}{8}K^4P\oint_l \mathrm{d}x_1^{\mu_1}\oint_l \mathrm{d}x_2^{\mu_2}\oint_l \mathrm{d}x_3^{\mu_3}[\eta_{\mu_1\mu_2}\partial^{a}D(x_3-x_1)\partial_{\mu_3}\partial_{a}D(x_2-x_3)+(1,2,3)\text{循环}]+\\&\frac{1}{8}K^4P\oint_l \mathrm{d}x_1^{\mu_1}\oint_l \mathrm{d}x_2^{\mu_2}[4\eta_{\mu_1\mu_2}\partial^{a}D(x_1-x_2)\partial_{a}D(x_1-x_2)-\\&18\partial_{\mu_1}D(x_1-x_2)\partial_{\mu_2}D(x_1-x_2)].\end{aligned} \tag{17}$$

上式表明,联络三点 Green 函数对量子 Wilson 圈的 K^4 阶有不为 0 的贡献.

3 单圈引力子自能对量子 Wilson 圈的贡献

由文献[12],GR 的单圈引力子自能的贡献为

$$\begin{aligned}&T_{\alpha\beta,\alpha'\beta'}(p)=\\&K^2I[f_1p_{\alpha}p_{\beta}p_{\alpha'}p_{\beta'}+f_2p^4\eta_{\alpha\beta}\eta_{\alpha'\beta'}+f_3p^4(\eta_{\alpha\alpha'}\eta_{\beta\beta'}+\eta_{\alpha\beta'}\eta_{\beta\alpha'})+\end{aligned}$$

$$f_4 p^2(\eta_{\alpha\beta}p_{\alpha'}p_{\beta'}+\eta_{\alpha\beta'}p_\alpha p_\beta)+f_5 p^2(\eta_{\alpha\alpha'}p_\beta p_{\beta'}+\eta_{\alpha\beta'}p_\beta p_{\alpha'}+\eta_{\beta\alpha'}p_\alpha p_{\beta'}+\eta_{\beta\beta'}p_\alpha p_{\alpha'})], \tag{18}$$

式中，$f_1=(2\omega^4-5\omega^3+35\omega^2+16\omega)/8(4\omega^2-1)$，$f_2=(-14\omega^4-7\omega^3+36\omega^2+9\omega)/32(\omega-1)^2(4\omega^2-1)$，$f_3=-f_5=(16\omega^3+18\omega^2-15\omega-8)/32(4\omega^2-1)$，

$f_4=(4\omega^4-10\omega^3+38\omega^2+32\omega+8)/32(\omega-1)(4\omega^2-1)$，$I=\int d^{2\omega}q[q^2(q-p)^2]^{-1}=i\pi^\omega(p^2)^{\omega-2}\Gamma(2-\omega)\Gamma(\omega-1)\Gamma(\omega-1)/\Gamma(2\omega-2)$.

这里的 2ω 为时空维数，$\Gamma(Z)$ 为第二类欧拉积分. 由(18)式，经正规化后，求得引力子二点 Green 函数的辐射修正为

$$G_{\mu\lambda\nu\tau}(p)=-\frac{i}{240}K^2\pi^2\left(\frac{1}{2-\omega}-\ln p^2\right)(328a_{1\mu\lambda\nu\tau}-59a_{2\mu\lambda\nu\tau}+81a_{3\mu\lambda\nu\tau}+104a_{4\mu\lambda\nu\tau}-81a_{5\mu\lambda\nu\tau}), \tag{19}$$

式中，

$$a_{1\mu\lambda\nu\tau}=p^{-4}p_\mu p_\lambda p_\nu p_\tau-\frac{1}{2}p^{-2}\eta_{\mu\lambda}p_\nu p_\tau-\frac{1}{2}p^{-2}\eta_{\mu\lambda}p_\nu p_\tau+\frac{1}{4}\eta_{\mu\lambda}\eta_{\nu\tau},$$

$$a_{2\mu\lambda\nu\tau}=\eta_{\mu\lambda}\eta_{\nu\tau},\ a_{3\mu\lambda\nu\tau}=\eta_{\mu\nu}\eta_{\nu\tau}+\eta_{\mu\tau}\eta_{\nu\lambda},$$

$$a_{4\mu\lambda\nu\tau}=\eta_{\mu\lambda}\eta_{\nu\tau}-p^{-2}\eta_{\mu\lambda}p_\nu p_\tau-p^{-2}\eta_{\nu\tau}p_\mu p_\lambda,$$

$$a_{5\mu\lambda\nu\tau}=\eta_{\mu\lambda}\eta_{\nu\tau}+p^{-2}(\eta_{\mu\nu}p_\lambda p_\tau+\eta_{\mu\tau}p_\nu p_\lambda+\eta_{\lambda\nu}p_\mu p_\tau+\eta_{\lambda\tau}p_\mu p_\nu-2\eta_{\mu\lambda}p_\nu p_\tau-2\eta_{\nu\tau}p_\mu p_\lambda).$$

由(19)及(10)式，即可求得引力子传播子的辐射修正对 $\langle W_{(2)}\rangle$ 的 K^4 阶贡献如下

$$\langle W_{(2)}\rangle_{1-\mathrm{loop}}=\frac{i}{96}K^4\pi^2 P\oint_l dx^\mu\oint_l dy^\nu\left[\frac{1}{2-\omega}(164\partial_\mu\partial_\nu-53\eta_{\mu\nu}\Box)\delta^4(x-y)-164\partial_\mu\partial_\nu Q(x-y)+53\eta_{\mu\nu}\Box Q(x-y)\right], \tag{20}$$

式中，$Q(x)=\int\frac{d^4p}{(2\pi)^4}\ln p^2\exp(ip\cdot x)$. 类似于前面的分析，(20)式右端前3项的圈积分结果为0，但最后一项的贡献不为0. 即引力子自能修正项对 Wilson 圈将有非零的贡献.

4 结论

由(17)，(20)式知，在 K^4 阶，联络三点函数及引力子自能对 Einstein 引力的 Wilson 圈分别有不为零的贡献，且二者不能互相抵消，它们的总贡献，

$$\langle W_{(2)}\rangle_{1-\mathrm{loop}}+\langle W_{(3)}\rangle=\frac{1}{8}K^4P\oint_l dx_1^{\mu_1}\oint_l dx_2^{\mu_2}\oint_l dx_3^{\mu_3}[\eta_{\mu_1\mu_2}\partial^\alpha D(x_3-x_1)\partial_{\mu_3}\partial_\alpha D(x_2-x_3)+(1,2,3)\text{ 循环}]+\frac{1}{8}K^4P\oint_l dx_1^{\mu_1}\oint_l dx_2^{\mu_2}[4\eta_{\mu_1\mu_2}\partial_\alpha D(x_1-x_2)\partial^\alpha D(x_1-x_2)-18\partial_{\mu_1}D(x_1-x_2)\partial_{\mu_2}D(x_1-x_2)+\frac{i53}{12}\pi^2\eta_{\mu\nu}\Box Q(x_1-x_2)], \tag{21}$$

亦不为0. 按文献[4]的讨论，这意味着，计入引力相互作用三顶点和引力自能的贡献后，Einstein 引力在高阶(K^4 阶)将有曲率激发. 这对引力微观相互作用机理的诠释有明显的

意义.

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Curvature Excitation of Three - Graviton Vertex and of Lowest - Order Graviton Self - energy

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Abstract Based on perturbative expansion, the contributions to quantum Wilson loop from three - connection field Green functions and from the lowest - order graviton self - energy are calculated respectively. The results show there exists excitation with localized curvature in Einstein gravity when the contributions of three - graviton vertex are considered.

Key words Einstein gravity, Wilson loop, curvature excitation, three - graviton vertex, Lowest - order graviton self - energy

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