

推导赝标 Goldstone 玻色子有效拉氏量

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摘要 从 QCD 出发,未作近似推导出了赝标量 Goldstone 玻色子的有效手征拉氏量(ECL)理论.并以 QCD 中格林函数的形式给出了直到 p^4 阶的 ECL 的系数的定义.

关键词 量子色动力学(QCD) 赝标量 Goldstone 玻色子 有效手征拉氏量(ECL)

在低能强子物理的研究中,选择物理的粒子作为自由度要比选择夸克和胶子更为方便.以物理赝标量 Goldstone 玻色子为自由度的理论是由 S. Weinberg^[1]提出,由 Gasser 和 Leutwyler^[2]发展完善的有效手征拉氏量(ECL).这些理论已被广泛应用,但它们仅以对称性为依据,至于能否由基本理论 QCD 导出,却仍需探讨.本文讨论了赝标量 Goldstone 玻色子的情况,给出从 QCD 推导出 ECL 的一种简洁方法.在另外一些文章中^[3,4],通过更复杂的处理,我们还给出了如何数值计算出 ECL 的系数的途径.

1 从 Gasser 和 Leutwyler^[2]的生成泛函出发:

$$Z[J] = \int Dq D\bar{q} D\psi D\bar{\psi} DA_\mu \exp \left\{ i \int dx \left[L_{m_q=0}^{\text{QCD}}(q, \bar{q}, \psi, \bar{\psi}, A_\mu) + \bar{q} J q \right] \right\}, \quad (1)$$

其中 $q_a^{a\xi}$ 和 $\psi_a^{a\xi}$ 分别为轻夸克和重夸克场, a, a, ξ 分别为色,味,洛仑兹指标; A_μ^i 为胶子场; $L_{m_q=0}^{\text{QCD}}(q, \bar{q}, \psi, \bar{\psi}, A_\mu)$ 为 QCD 手征极限下的裸拉氏量; $J(x)$ 为外源:

$$J(x) = \psi(x) + d(x)\gamma_5 - s(x) + ip(x)\gamma_5. \quad (2)$$

为得到有效手征拉氏量(ECL),需要引入描述赝标量 Goldstone 玻色子的场 $U(x)$.我们先定义厄密场 $\sigma(x)$ 和么正场 $\Omega(x)$:

$$(\Omega' \sigma \Omega' \pm \Omega'^+ \sigma \Omega'^+)^{ab}(x) \equiv \left(\frac{1}{\gamma_5} \right)_{\text{qf}} \bar{q}(x)^{(b\xi)} q(x)^{(a\xi)}, \quad (3)$$

令 $U'(x) = \Omega'^2(x)$ 及 $e^{i\theta(x)} = \det U'(x)$.进一步可从 $U'(x)$ 中提出 $U(1)$ 因子:定义

$U(x) = e^{-\frac{i\vartheta(x)}{N_f}} U'(x)$, 使 $\det U = 1$. 现在 $U(x)$ 是 $SU(N_f)_R \times SU(N_f)_L$ 的非线性表示, 可用来描述赝标 Goldstone 玻色子. 厄密场 $\sigma(x)$ 是我们不需要的中间场, 可在 (3) 中消去:

$$e^{-\frac{i\vartheta(x)}{N_f}} \Omega^+(x) \text{tr}_1 [P_R(\bar{q}q)^T(x, x)] \Omega^+(x) = e^{\frac{i\vartheta(x)}{N_f}} \Omega(x) \text{tr}_1 [P_L(\bar{q}q)^T(x, x)] \Omega(x), \quad (4)$$

$$e^{2i\vartheta(x)} = \frac{\det[\text{tr}_1 [P_R(\bar{q}q)^T(x, x)]]}{\det[\text{tr}_1 [P_L(\bar{q}q)^T(x, x)]]}, \quad (5)$$

其中 $(\bar{q}q)^T(x, y)^{(a^e)(b^e)} = \bar{q}(y)^{b^e} q(x)^{a^e}$, 记号 T 表示对一切指标及空间坐标转置.

注意到对任意满足 $\det O(x) = \det O^+(x)$ 的算符 O , 均有恒等式:

$$\int DU \delta(U^+ U - 1) \delta(\det U - 1) \delta(\Omega O^+ \Omega - \Omega^+ O \Omega^+) F(O) = \text{const}, \quad (6)$$

其中 $\frac{1}{F(O)} = \det O \int D\sigma \delta(O^+ O - \sigma^+ \sigma) \delta(\sigma - \sigma^+)$.

上式的证明见文献[3]. 取 $O(x) = e^{-\frac{i\vartheta(x)}{N_f}} \text{tr}_1 [P_R(\bar{q}q)^T(x, x)]$, 则(6)式中由 δ 函数所限定的场 $U(x)$ 就是我们期望引进的赝标量 Goldstone 玻色子场的非线性表示. 将(6)插入(1), 得到:

$$\begin{aligned} Z[J] &= \int DU \delta(U^+ U - 1) \delta(\det U - 1) e^{iS_{\text{eff}}[U, J]}, \\ e^{iS_{\text{eff}}[U, J]} &= \int Dq D\bar{q} D\psi D\bar{\psi} DA_\mu \delta(e^{-\frac{i\vartheta}{N_f}} \Omega^+ \text{tr}_1 [P_R(\bar{q}q)^T] \Omega^+ - e^{\frac{i\vartheta}{N_f}} \Omega \text{tr}_1 [P_L(\bar{q}q)^T] \Omega) \times \\ &\quad \exp\left\{i\Gamma_I[(\bar{q}q)^T] + i \int dx [L_{m_q=0}^{\text{QCD}}(q, \bar{q}, \psi, \bar{\psi}, A_\mu) + \bar{q}Jq]\right\}, \quad (7) \\ \exp\{-i\Gamma_I[(\bar{q}q)^T]\} &= \prod_x \frac{1}{F[O(x)]} = \prod_x \left[\sqrt{\det[\text{tr}_1 [P_R(\bar{q}q)^T]} \det[\text{tr}_1 [P_R(\bar{q}q)^T]} \right] \\ &\quad \int D\sigma \delta(\text{tr}_1 [P_R(\bar{q}q)^T] \text{tr}_1 [P_R(\bar{q}q)^T] - \sigma^+ \sigma) \delta(\sigma - \sigma^+). \end{aligned}$$

手征变换

$$q_n(x) = [P_R \Omega^+(x) + P_L \Omega(x)] q(x),$$

$$J_n(x) = [P_R \Omega(x) + P_L \Omega^+(x)] (J(x) + i\phi) [P_R \Omega(x) + P_L \Omega^+(x)]. \quad (8)$$

本理论在该变换下具有对称性, 但还出现反常项. 反常项的一般结构已由诸多文献讨论过^[2,5,6], 本文不再考虑. 注意 $\vartheta_n(x) = \vartheta(x)$, $\Gamma_I[(\bar{q}q)_n^T(x, x)] = \Gamma_I[(\bar{q}q)^T(x, x)]$. 现在

$$\begin{aligned} e^{iS_{\text{eff}}[U, J]} &= \int Dq D\bar{q} D\psi D\bar{\psi} DA_\mu \delta\left(\bar{q}^a(x) \left(-i \sin \frac{\vartheta(x)}{N_f} + \gamma_5 \cos \frac{\vartheta(x)}{N_f}\right) q^b(x)\right) \times \\ &\quad \exp\left\{i\Gamma_I[(\bar{q}q)^T] + i \int dx [L_{m_q=0}^{\text{QCD}}(q, \bar{q}, \psi, \bar{\psi}, A_\mu) + \bar{q}J_n q + \text{anormal y terms}]\right\}. \quad (9) \end{aligned}$$

我们还可以从 J_n 中消除赝标量部分. 在 $\delta\left(\bar{q}^a \left(-i \sin \frac{\vartheta}{N_f} + \gamma_5 \cos \frac{\vartheta}{N_f}\right) q^b\right)$ 之后可插

入下述因子而不引起任何改变: $\exp\left\{\left(p_n/\cos\frac{\vartheta}{N_f}\right)\bar{q}^a\left(-i\sin\frac{\vartheta}{N_f}+\gamma_5\cos\frac{\vartheta}{N_f}\right)q^b\right\}$. 因此:

$$e^{iS_{\text{eff}}[U,J]} = \int DqD\bar{q}D\psi D\bar{\psi}DA_\mu\delta\left(\bar{q}^a\left(-i\sin\frac{\vartheta}{N_f}+\gamma_5\cos\frac{\vartheta}{N_f}\right)q^b\right)\times \\ \exp\left\{i\Gamma_I[(\bar{q}q)^T] + i\int dx[L_{m_q=0}^{\text{QCD}}(q,\bar{q},\psi,\bar{\psi},A_\mu) + \bar{q}\left(\psi_n + d_n\gamma_5 - s_n - p_n\tan\frac{\vartheta}{N_f}\right)q]\right\}. \quad (10)$$

其中已略去反常项. 上式是不包含反常项的 ECL 理论的完全精确的表达式. 将其按动量 p^2 的幂次展开, 可以得到任意阶 ECL 的形式及其系数的表达式. 被冻结的自由度 $\bar{q}^a\left(-i\sin\frac{\vartheta}{N_f}+\gamma_5\cos\frac{\vartheta}{N_f}\right)q^b$ 可经过一个 $U_A(1)$ 变换变成 $\bar{q}^a\gamma_5q^b$. $\bar{q}^a\gamma_5q^b$ 自由度被冻结是因为赝标 Goldstone 玻色子场 $U(x)$ 已包含它了. 这说明我们对场 $U(x)$ 的选择是合理的.

3 将(10)按 p^2 的幂次展开, 以得到 ECL 的具体形式

首先展开到 p^2 阶:

$$S_{\text{eff}}|_{p^2} = F_0^2\int dx\text{tr}_f[a_n^2 + B_0s_n^2], \quad (11)$$

$$S_{\text{eff}}|_{p^2} = F_0^2\int dx\text{tr}_f\left[\frac{1}{4}(\nabla^\mu U^\dagger)(\nabla_\mu U) + \frac{1}{2}B_0[U(s - ip) + U^\dagger(s + ip)]\right], \quad (12)$$

(12)式中 ∇^μ 定义同文献[2]. (12)式正是文献[2]中 p^2 阶 ECL 的准确结果. 注意(11)式中没有出现 $\text{tr}_f[v_n^2]$. 这是因为存在一个 hidden symmetry: $s_n \rightarrow h^+ s_n h$, $p_n \rightarrow h^+ p_n h$, $a_n^a \rightarrow h^+ a_n^a h$, $v_n^a \rightarrow h^+ v_n^a h + h^+ i\partial^\mu h$ 时理论不变. v_n^a 只有和微商 $i\partial^\mu$ 构成协变微商一起出现才能保持这个对称性. 宇称守恒则要求 $\text{tr}_f[p_n]$ 不出现. 系数 F_0^2, B_0 定义如下:

$$F_0^2 B_0 = -\frac{1}{N_f}\langle\bar{q}q\rangle, \\ F_0^2 = \frac{i}{8(N_f^2 - 1)}\int dx[\delta^{ab}\delta^{bc} - \frac{1}{N_f}\delta^{ab}\delta^{cd}]\langle[\bar{q}^a(0)\gamma^\mu\gamma_5q^b(0)\bar{q}^c(x)\gamma_\mu\gamma_5q^d(x)]\rangle_C, \quad (13)$$

这里 $\langle O \rangle_C$ 表示 $\langle O \rangle$ 的连通部分. 而 $\langle O \rangle$ 定义为:

$$\langle O \rangle = \frac{\int DqD\bar{q}D\psi D\bar{\psi}DA_\mu O\delta\left(\bar{q}^a\left(-i\sin\frac{\vartheta}{N_f}+\gamma_5\cos\frac{\vartheta}{N_f}\right)q^b\right)\exp\left\{i\Gamma_I[(\bar{q}q)^T] + i\int dxL_{m_q=0}^{\text{QCD}}(q,\bar{q},\psi,\bar{\psi},A_\mu)\right\}}{\int DqD\bar{q}D\psi D\bar{\psi}DA_\mu\delta\left(\bar{q}^a\left(-i\sin\frac{\vartheta}{N_f}+\gamma_5\cos\frac{\vartheta}{N_f}\right)q^b\right)\exp\left\{i\Gamma_I[(\bar{q}q)^T] + i\int dxL_{m_q=0}^{\text{QCD}}(q,\bar{q},\psi,\bar{\psi},A_\mu)\right\}}. \quad (14)$$

同样, 将(10)式展开到 p^4 阶:

$$S_{\text{eff}}|_{p^4} = \int dx\text{tr}_f[-K_1[d_\mu a^\mu]^2 - K_2[d^\mu a_n^\nu - d^\nu a_n^\mu][d_\mu a_{n\mu} - d_\nu a_{n\nu}] + \\ K_3[a_n^2]^2 + K_4 a_n^a a_n^b a_{n\mu} a_{n\mu} + K_5 a_n^2 \text{tr}_f[a_n^2] + K_6 a_n^a a_n^b \text{tr}_f[a_{n\mu} a_{n\mu}] +$$

$$K_7 s_\Omega^2 + K_8 s_\Omega \text{tr}_f[s_\Omega] + K_9 p_\Omega^2 + K_{10} p_\Omega \text{tr}_f[p_\Omega] + K_{11} s_\Omega a_\Omega^2 + K_{12} s_\Omega \text{tr}_f[a_\Omega^2] - K_{13} V_\Omega^\mu V_{\Omega\mu} + iK_{14} V_\Omega^\mu a_{\Omega\mu} a_\Omega + K_{15} p_\Omega d^\mu a_{\Omega\mu}], \quad (15)$$

这里 $d^\mu a_\Omega^\nu \equiv \partial^\mu a_\Omega^\nu - i\nu_\Omega^\mu a_\Omega^\nu + ia_\Omega^\mu \nu_\Omega^\nu$, $V_\Omega^\mu \equiv \partial^\mu \nu_\Omega^\nu - \partial^\nu \nu_\Omega^\mu - i\nu_\Omega^\mu \nu_\Omega^\nu + i\nu_\Omega^\nu \nu_\Omega^\mu$. 各系数分别为:

$$\begin{aligned} & \frac{i}{4} \int dx x^\mu x^\nu \langle [\bar{q}^a(0) \gamma^\mu \gamma_5 q^b(0)] [\bar{q}^c(x) \gamma^\nu \gamma_5 q^d(x)] \rangle_c = \\ & \left[\left(\frac{1}{2} K_1 - K_2 \right) (g^{\mu\nu} g^{\nu\mu} + g^{\nu\mu} g^{\mu\nu}) + 2K_2 g^{\mu\nu} g^{\nu\mu} \right] \delta^{ad} \delta^{bc} + \dots, \\ - \frac{i}{24} \int dx dy dz & \langle [\bar{q}^a(0) \gamma^\mu \gamma_5 q^b(0)] [\bar{q}^c(x) \gamma^\nu \gamma_5 q^d(x)] [\bar{q}^e(y) \gamma^\lambda \gamma_5 q^f(y)] [\bar{q}^g(z) \gamma^\rho \gamma_5 q^h(z)] \rangle_c = \\ & \frac{1}{6} \left\{ \delta^{ad} \delta^{cf} \delta^{eh} \delta^{gb} \left[\frac{1}{2} (g^{\mu\nu} g^{\lambda\rho} + g^{\nu\mu} g^{\rho\lambda}) K_3 + g^{\mu\lambda} g^{\nu\rho} K_4 \right] + \right. \\ & \delta^{ad} \delta^{ch} \delta^{ef} \delta^{gb} \left[\frac{1}{2} (g^{\mu\nu} g^{\lambda\rho} + g^{\nu\mu} g^{\rho\lambda}) K_3 + g^{\nu\mu} g^{\rho\lambda} K_4 \right] + \\ & \delta^{af} \delta^{ad} \delta^{ch} \delta^{gb} \left[\frac{1}{2} (g^{\mu\lambda} g^{\nu\rho} + g^{\nu\mu} g^{\rho\lambda}) K_3 + g^{\nu\mu} g^{\rho\lambda} K_4 \right] + \\ & \delta^{af} \delta^{ch} \delta^{ef} \delta^{gb} \left[\frac{1}{2} (g^{\mu\nu} g^{\lambda\rho} + g^{\nu\mu} g^{\rho\lambda}) K_3 + g^{\mu\nu} g^{\rho\lambda} K_4 \right] + \\ & \delta^{ad} \delta^{cb} \delta^{eh} \delta^{fg} \left[g^{\mu\nu} g^{\lambda\rho} 2K_5 + (g^{\mu\lambda} g^{\nu\rho} + g^{\nu\mu} g^{\rho\lambda}) K_6 \right] + \\ & \delta^{af} \delta^{cb} \delta^{ch} \delta^{ef} \left[g^{\mu\lambda} g^{\nu\rho} 2K_5 + (g^{\nu\mu} g^{\rho\lambda} + g^{\mu\nu} g^{\rho\lambda}) K_6 \right] + \\ & \left. \delta^{ch} \delta^{cb} \delta^{cf} \delta^{ad} \left[g^{\nu\mu} g^{\rho\lambda} 2K_5 + (g^{\mu\nu} g^{\lambda\rho} + g^{\nu\mu} g^{\rho\lambda}) K_6 \right] \right\} + \dots, \quad (16) \end{aligned}$$

$$\begin{aligned} & \frac{i}{2} \int dx \langle [\bar{q}^a(0) q^b(0)] [\bar{q}^c(x) q^d(x)] \rangle_c = K_7 \delta^{ad} \delta^{bc} + K_8 \delta^{ab} \delta^{cd} \\ & \frac{i}{2} \int dx \langle [\bar{q}^a(0) q^b(0)] [\bar{q}^c(x) q^d(x)] \tan^2 \frac{\theta(x)}{N_f} \rangle_c = K_9 \delta^{ad} \delta^{bc} + K_{10} \delta^{ab} \delta^{cd} \\ & \frac{1}{8} \int dx dy \langle [\bar{q}^a(0) q^b(0)] [\bar{q}^c(x) \gamma^\mu \gamma_5 q^d(x)] [\bar{q}^e(y) \gamma_\mu \gamma_5 q^f(y)] \rangle_c = \\ & \quad \cdot \frac{1}{2} K_{11} (\delta^{ad} \delta^{cf} \delta^{eb} + \delta^{af} \delta^{cd} \delta^{eb}) + K_{12} \delta^{ab} \delta^{cf} \delta^{eb} + \dots, \end{aligned}$$

$$\begin{aligned} 144K_{13} &= \frac{i}{4(N_f^2 - 1)} \int dx (5g_{\mu\nu} g_{\mu\nu} - 2g_{\mu\nu} g_{\nu\mu}) x^\mu x^\nu \times \\ & \left[\langle [\bar{q}^a(0) \gamma^\mu q^b(0)] [\bar{q}^c(x) \gamma^\mu q^d(x)] \rangle_c - \frac{1}{N_f} \langle [\bar{q}^a(0) \gamma^\mu q^a(0)] [\bar{q}^b(x) \gamma^\mu q^b(x)] \rangle_c \right] \\ - i36K_{14} &= (2g_{\mu\nu} g_{\mu\nu} + 2g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho}) (T_A^{\mu\nu\lambda\rho} - T_B^{\mu\nu\lambda\rho}), \\ K_{15} &= \frac{i}{4(N_f^2 - 1)} \int dx x^\mu \left[\langle \bar{q}^a(0) q^b(0) \tan \frac{\theta(0)}{N_f} \bar{q}^b(x) \gamma_\mu \gamma_5 q^a(x) \rangle_c \right. \\ & \quad \left. - \frac{1}{N_f} \langle \bar{q}^a(0) q^a(0) \tan \frac{\theta(0)}{N_f} \bar{q}^b(x) \gamma_\mu \gamma_5 q^b(x) \rangle_c \right] \end{aligned}$$

其中 $-\frac{1}{2} \int dx dy x^\mu \langle [\bar{q}^a(0) \gamma^\mu q^b(0)] [\bar{q}^c(x) \gamma^\nu \gamma_5 q^d(x)] [\bar{q}^e(y) \gamma^\nu \gamma_5 q^f(y)] \rangle_c =$

$$\delta^{ad} \delta^{cf} \delta^{eb} T_A^{\mu\nu\lambda\rho} + \delta^{af} \delta^{cd} \delta^{eb} T_B^{\mu\nu\lambda\rho} + \dots$$

(15)式同样可进一步化成文献[2]中 p^4 阶 ECL 的形式,得到文献[2]中系数 $L_1, \dots, L_{10}, H_1, H_2$ 与此处 K_1, \dots, K_{15} 的关系为:

$$\begin{aligned}
 L_1 &= \frac{1}{32}K_4 + \frac{1}{16}K_5 + \frac{1}{16}K_{13} - \frac{1}{32}K_{14}, & L_2 &= \frac{1}{16}K_4 + \frac{1}{16}K_6 + \frac{1}{8}K_{13} - \frac{1}{16}K_{14}, \\
 L_3 &= \frac{1}{16}K_3 - \frac{1}{8}K_4 - \frac{3}{8}K_{13} + \frac{3}{16}K_{14}, & L_4 &= \frac{1}{16B_0}K_{12}, \\
 L_5 &= \frac{1}{16B_0}K_{11}, & L_6 &= \frac{1}{16B_0^2}K_8, \\
 L_7 &= -\frac{1}{16N_f}K_1 - \frac{1}{16B_0^2}K_{10} - \frac{K_{15}}{16B_0N_f}, & L_8 &= \frac{1}{16}K_1 + \frac{1}{16B_0^2}K_7 - \frac{1}{16B_0^2}K_9 + \frac{1}{16B_0}K_{15}, \\
 L_9 &= \frac{1}{2}K_{13} - \frac{1}{8}K_{14}, & L_{10} &= +\frac{1}{2}K_2 - \frac{1}{2}K_{13}, \\
 H_1 &= \frac{1}{4}K_2 - \frac{1}{4}K_{13}, & H_2 &= -\frac{1}{8}K_1 + \frac{1}{8B_0^2}K_7 + \frac{1}{8B_0^2}K_9 - \frac{1}{8B_0}K_{15}.
 \end{aligned} \tag{17}$$

其中略去了反常项的贡献.

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Derivation of Pseudoscalar Goldstone Boson Effective Lagrangian

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Abstract The Effective Chiral Lagrangian (ECL) for the Pseudoscalar Goldstone Bosons is derived from QCD without making approximations. The coefficients up to p^4 order in the ECL are expressed in terms of certain Green's function in QCD.

Key words quantum chromodynamics (QCD), pseudoscalar Goldstone bosons, Effective chiral lagrangian (ECL)