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参数空间十六极振动的量子化*

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摘要 利用核十六极形变最普遍的参数化形式，通过正则量子化程序，导出了参数空间十六极振动的量子化哈密顿量，分析了十六极形变和振动对于核结构研究的重要意义.

关键词 十六极形变 参数化 量子化

1 引言

众所周知，原子核内普遍存在着形变，其中四极形变是主要的，八极形变和十六极形变也是两种重要的形变模式. 为了具体表述原子核的形状，需要在内禀坐标系下，对核的形变进行参数化，用能明显表述原子核形状和空间方位的参数描述原子核的形变. 玻尔通过对四极形变的参数化[1]，使玻尔参数具有如下的性质:

(1) β_2为原子核的总形变，具有 O_h群变换不变性.

(2) γ_2在 O_h群变换下具有简单的变化性质：不变、增加 $2\pi/3$倍角、改变方向，所以γ_2的变化只须在$(0, 2\pi/3)$范围之间.

玻尔通过上述参数化后，导出了参数空间量子化的哈密顿量，即著名的玻尔哈密顿量. 玻尔哈密顿量能够把振动和转动联系起来讨论，不少人在这方面做过工作[2—4]. 然而，随着实验的进展，由于玻尔哈密顿量只考虑四极形变，已很难解释核超形变状态下出现的新现象. 象全同带、$\Delta I=4$ 的 staggering 等，staggering 现象的发现，增加了人们对于多极形变研究的兴趣. 目前，staggering 现象的物理机制尚不清楚，一些研究认为，它可能起源于原子核的 C_4对称性[5]，为了进一步分析这些问题，需要研究原子核的多极形变和振动. 近年来，八极–十六极形变已被用来分析实验，象核裂变，质量的形状依赖性，高自旋态的超形变[6]等. 十六极形变和十六极相互作用也已被用来讨论超形变核态的新现象，象$\Delta I=4$ 的颤动[7]等. 文献 [8]和 [9]已分别给出了八极形变和十六极形变最一般的参数

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化形式，出现在模型中的参数具有同玻尔参数相似的特性．我们已经在文献[8]的基础上分析了八极形变的参数化，并在参数空间导出了八极形变量子化的哈密顿量．本文进一步在文献[9]的基础上分析十六极形变的参数化及参数空间的量子化，具体导出十六极形变量子化的哈密顿量，这种哈密顿量能够讨论十六极形变中任何一个或几个参数的振动，对研究原子核高自旋和超形变状态下出现的新现象，可能具有重要的意义．下面第二部分给出了十六极形变的参数化程式，第三部分导出了参数空间量子化哈密顿量．第四部分为结论．

2 十六极形变的参数化

对于平衡形变为球形的原子核，半径可写为

$$R = R_0\left(1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi)\right). \tag{1}$$

(1)式中的$\lambda = 2$对应四极形变，$\lambda = 3$对应八极形变，$\lambda = 4$对应十六极形变．在取四极形变的对称轴为主轴的坐标系下，原子核的半径可表为：

$$R(\theta,\phi) = R_0\left[1 + a_{20}Y_{20}(\theta,\phi) + a_{22}Y_{22}^{(+)}(\theta,\phi) + \sum_{\lambda\neq 2}\left(a_{\lambda 0}Y_{\lambda 0}(\theta,\phi) + \sum_{\mu>0}\left[a_{\lambda\mu}Y_{\lambda\mu}^{(+)} + b_{\lambda\mu}Y_{\lambda\mu}^{(-)}(\theta,\phi)\right]\right)\right],$$

其中

$$Y_{\lambda\mu}^{(+)} = \frac{1}{\sqrt{2}}[Y_{\lambda\mu} + (-)^{\mu}Y_{\lambda-\mu}],$$

$$Y_{\lambda\mu}^{(-)} = \frac{1}{\mathrm{i}\sqrt{2}}[Y_{\lambda\mu} - (-)^{\mu}Y_{\lambda-\mu}].$$

分别为球谐函数的实部和虚部．

当只考虑十六极形变时

$$R(\theta,\phi) = R_0\left[a_{40}Y_{40}(\theta,\phi) + \sum_{\mu>0}\left[a_{4\mu}Y_{4\mu}^{(+)}(\theta,\phi) + b_{4\mu}Y_{4\mu}^{(-)}(\theta,\phi)\right]\right]. \tag{2}$$

十六极张量$a_{4\mu}$，$b_{4\mu\neq 0}$构成O_h群的一个一维不可约表示，一个二维不可约表示和两个三维不可约表示的基：

$$a_4 = \sqrt{\frac{7}{12}}a_{40} + \sqrt{\frac{5}{12}}a_{44},$$

$$e_{40} = \sqrt{\frac{5}{12}}a_{40} - \sqrt{\frac{7}{12}}a_{44}, \quad e_{42} = -a_{42},$$

$$f_{4x}=\sqrt{\frac{7}{8}}b_{41}+\sqrt{\frac{1}{8}}b_{43},\qquad f_{4y}=-\sqrt{\frac{7}{8}}a_{41}+\sqrt{\frac{1}{8}}a_{43},\qquad f_{4z}=b_{44},$$

$$g_{4x}=\sqrt{\frac{1}{8}}b_{41}-\sqrt{\frac{7}{8}}b_{43},\qquad g_{4y}=\sqrt{\frac{1}{8}}a_{41}+\sqrt{\frac{7}{8}}a_{43},\qquad g_{4z}=b_{42}. \tag{3}$$

a_4是 O_h不变的，(e_{40}, e_{42})在 O_h变换下和(a_{20}, a_{22})相同[9]．f_{4v}和 $g_{4v}(v=x,y,z)$在 Bohr 转动 R_i下的变换规律和矢量相似[9]．逆变换 I和 Bohr 转动 R_i 对矢量的变换性质见参考文献[1,9]

将上述十六极张量参数化，使其具有 Bohr 参数相似的性质：

$$\begin{aligned}
&a_4=\beta\cos\varepsilon\cos\delta,\qquad e_{40}=\beta\cos\varepsilon\sin\delta\cos\gamma,\qquad e_{42}=\beta\cos\varepsilon\sin\delta\sin\gamma,\\
&f_{4x}=\beta\sin\varepsilon\sin\theta\cos\varphi\cos\xi,\quad f_{4y}=\beta\sin\varepsilon\sin\theta\sin\varphi\cos\eta,\quad f_{4z}=\beta\sin\varepsilon\cos\theta\cos\zeta,\\
&g_{4x}=\beta\sin\varepsilon\sin\theta\cos\varphi\sin\xi,\quad g_{4y}=\beta\sin\varepsilon\sin\theta\sin\varphi\sin\eta,\quad g_{4z}=\beta\sin\varepsilon\cos\theta\sin\zeta.
\end{aligned} \tag{4}$$

$\beta^2=\sum_{m=0}^{4}a_{4m}^2+\sum_{m=1}^{4}b_{4m}^2$ 为原子核的总形变，在 O_h变换下不变，其它参数的物理意义见参考文献 [9].

3　量子化的哈密顿量

当原子核作多极振动，势能可写为

$$V=\frac{1}{2}\sum_{\lambda\mu}C_\lambda\left|\alpha_{\lambda\mu}\right|^2, \tag{5}$$

动能为

$$T=\frac{1}{2}\sum_{\lambda\mu}B_\lambda\left|\dot{\alpha}_{\lambda\mu}\right|^2, \tag{6}$$

正则动量为

$$\pi_{\lambda\mu}=\frac{\partial T}{\partial\dot{\alpha}_{\lambda\mu}}=B_\lambda\dot{\alpha}^*_{\lambda\mu}, \tag{7}$$

哈密顿量为

$$H_s=T+V=\sum_{\lambda\mu}\left[\frac{1}{2B_\lambda}\left|\pi_{\lambda\mu}\right|^2+\frac{C_\lambda}{2}\left|\alpha_{\lambda\mu}\right|^2\right]. \tag{8}$$

对于十六极振动，

$$T_4=\frac{1}{2}B\left[\dot{a}_{40}^2+\sum_{\mu>0}\left(\dot{a}_{4\mu}^2+\dot{b}_{4\mu}^2\right)\right],$$

$$V_4=\frac{C}{2}\left[\sum_{\mu\geqslant0}\left|a_{4\mu}\right|^2+\sum_{\mu>0}\left|b_{4\mu}\right|^2\right], \tag{9}$$

量子化的哈密顿量 H为

$$H = -\frac{1}{2B}\left[\frac{\partial^2}{\partial a_{40}^2} + \sum_{\mu>0}\left(\frac{\partial^2}{\partial a_{4\mu}^2} + \frac{\partial^2}{\partial b_{4\mu}^2}\right)\right] + V. \tag{10}$$

(10)式为原子核作十六极振动的量子化哈密顿量,其中的参数与原子核的形状没有明显的关系,为了给出与原子核的形变直接相联系的哈密顿量,使其具有 Bohr 哈密顿量类似的特点,我们将其转化到参数空间

利用(3)式和下面的关系

$$\frac{\partial}{\partial a_{40}} = \frac{\partial a_4}{\partial a_{40}}\frac{\partial}{\partial a_4} + \frac{\partial e_{40}}{\partial a_{40}}\frac{\partial}{\partial e_{40}} + \frac{\partial e_{42}}{\partial a_{40}}\frac{\partial}{\partial e_{42}} + \frac{\partial f_{4x}}{\partial a_{40}}\frac{\partial}{\partial f_{4x}} + \frac{\partial f_{4y}}{\partial a_{40}}\frac{\partial}{\partial f_{4y}} + \frac{\partial f_{4z}}{\partial a_{40}}\frac{\partial}{\partial f_{4z}} + \frac{\partial g_{4x}}{\partial a_{40}}\frac{\partial}{\partial g_{4x}} + \frac{\partial g_{4y}}{\partial a_{40}}\frac{\partial}{\partial g_{4y}} + \frac{\partial g_{4z}}{\partial a_{40}}\frac{\partial}{\partial g_{4z}}. \tag{11}$$

相似的对于 a_{41}, a_{42}, a_{43}, a_{44}, b_{41}, b_{42}, b_{43}, b_{44},得哈密顿量 H 如下

$$H = -\frac{1}{2B}\left[\frac{\partial^2}{\partial a_4^2} + \frac{\partial^2}{\partial e_{40}^2} + \frac{\partial^2}{\partial e_{42}^2} + \frac{\partial^2}{\partial f_{4x}^2} + \frac{\partial^2}{\partial f_{4y}^2} + \frac{\partial^2}{\partial f_{4z}^2} + \frac{\partial^2}{\partial g_{4x}^2} + \frac{\partial^2}{\partial g_{4y}^2} + \frac{\partial^2}{\partial g_{4z}^2}\right] + V. \tag{12}$$

出现在 H 中的参数构成 O_h 群四个不可约表示的生成元,它们和原子核的形变参数的关系如(4)式所示,利用

$$\frac{\partial}{\partial a_4} = \frac{\partial\beta}{\partial a_4}\frac{\partial}{\partial\beta} + \frac{\partial\varepsilon}{\partial a_4}\frac{\partial}{\partial\varepsilon} + \frac{\partial\delta}{\partial a_4}\frac{\partial}{\partial\delta} + \frac{\partial\gamma}{\partial a_4}\frac{\partial}{\partial\gamma} + \frac{\partial\theta}{\partial a_4}\frac{\partial}{\partial\theta} + \frac{\partial\varphi}{\partial a_4}\frac{\partial}{\partial\varphi} + \frac{\partial\xi}{\partial a_4}\frac{\partial}{\partial\xi} + \frac{\partial\eta}{\partial a_4}\frac{\partial}{\partial\eta} + \frac{\partial\zeta}{\partial a_4}\frac{\partial}{\partial\zeta} = \cos\varepsilon\cos\delta\frac{\partial}{\partial\beta} - \frac{\sin\varepsilon\cos\delta}{\beta}\frac{\partial}{\partial\varepsilon} - \frac{\sin\delta}{\beta\cos\varepsilon}\frac{\partial}{\partial\delta}, \tag{13.1}$$

$$\frac{\partial}{\partial e_{40}} = \frac{\partial\beta}{\partial e_{40}}\frac{\partial}{\partial\beta} + \frac{\partial\varepsilon}{\partial e_{40}}\frac{\partial}{\partial\varepsilon} + \frac{\partial\delta}{\partial e_{40}}\frac{\partial}{\partial\delta} + \frac{\partial\gamma}{\partial e_{40}}\frac{\partial}{\partial\gamma} + \frac{\partial\theta}{\partial e_{40}}\frac{\partial}{\partial\theta} + \frac{\partial\varphi}{\partial e_{40}}\frac{\partial}{\partial\varphi} + \frac{\partial\xi}{\partial e_{40}}\frac{\partial}{\partial\xi} + \frac{\partial\eta}{\partial e_{40}}\frac{\partial}{\partial\eta} + \frac{\partial\zeta}{\partial e_{40}}\frac{\partial}{\partial\zeta} = \cos\varepsilon\sin\delta\cos\gamma\frac{\partial}{\partial\beta} - \frac{\sin\varepsilon\sin\delta\cos\gamma}{\beta}\frac{\partial}{\partial\varepsilon} + \frac{\cos\delta\cos\gamma}{\beta\cos\varepsilon}\frac{\partial}{\partial\delta} - \frac{\sin\gamma}{\beta\cos\varepsilon\sin\delta}\frac{\partial}{\partial\gamma}, \tag{13.2}$$

$$\frac{\partial}{\partial e_{42}} = \frac{\partial\beta}{\partial e_{42}}\frac{\partial}{\partial\beta} + \frac{\partial\varepsilon}{\partial e_{42}}\frac{\partial}{\partial\varepsilon} + \frac{\partial\delta}{\partial e_{42}}\frac{\partial}{\partial\delta} + \frac{\partial\gamma}{\partial e_{42}}\frac{\partial}{\partial\gamma} + \frac{\partial\theta}{\partial e_{42}}\frac{\partial}{\partial\theta} + \frac{\partial\varphi}{\partial e_{42}}\frac{\partial}{\partial\varphi} + \frac{\partial\xi}{\partial e_{42}}\frac{\partial}{\partial\xi} + \frac{\partial\eta}{\partial e_{42}}\frac{\partial}{\partial\eta} + \frac{\partial\zeta}{\partial e_{42}}\frac{\partial}{\partial\zeta} = \cos\varepsilon\sin\delta\sin\gamma\frac{\partial}{\partial\beta} - \frac{\sin\varepsilon\sin\delta\sin\gamma}{\beta}\frac{\partial}{\partial\varepsilon} + \frac{\cos\delta\sin\gamma}{\beta\cos\varepsilon}\frac{\partial}{\partial\delta} - \frac{\cos\gamma}{\beta\cos\varepsilon\sin\delta}\frac{\partial}{\partial\gamma}, \tag{13.3}$$

$$\frac{\partial}{\partial f_{4x}} = \frac{\partial\beta}{\partial f_{4x}}\frac{\partial}{\partial\beta} + \frac{\partial\varepsilon}{\partial f_{4x}}\frac{\partial}{\partial\varepsilon} + \frac{\partial\delta}{\partial f_{4x}}\frac{\partial}{\partial\delta} + \frac{\partial\gamma}{\partial f_{4x}}\frac{\partial}{\partial\gamma} + \frac{\partial\theta}{\partial f_{4x}}\frac{\partial}{\partial\theta} + \frac{\partial\varphi}{\partial f_{4x}}\frac{\partial}{\partial\varphi} + \frac{\partial\xi}{\partial f_{4x}}\frac{\partial}{\partial\xi} +$$

$$\frac{\partial\eta}{\partial f_{4x}}\frac{\partial}{\partial\eta}+\frac{\partial\zeta}{\partial f_{4x}}\frac{\partial}{\partial\zeta}=\sin\varepsilon\sin\theta\cos\varphi\cos\xi\frac{\partial}{\partial\beta}-\frac{\cos\varepsilon\sin\theta\cos\varphi\cos\xi}{\beta}\frac{\partial}{\partial\varepsilon}+$$
$$\frac{\cos\theta\cos\varphi\cos\xi}{\beta\sin\varepsilon}\frac{\partial}{\partial\theta}-\frac{\sin\varphi\cos\xi}{\beta\sin\varepsilon\sin\theta}\frac{\partial}{\partial\varphi}-\frac{\sin\xi}{\beta\sin\varepsilon\sin\theta\cos\varphi}\frac{\partial}{\partial\xi}, \tag{13.4}$$

$$\frac{\partial}{\partial f_{4y}}=\frac{\partial\beta}{\partial f_{4y}}\frac{\partial}{\partial\beta}+\frac{\partial\varepsilon}{\partial f_{4y}}\frac{\partial}{\partial\varepsilon}+\frac{\partial\delta}{\partial f_{4y}}\frac{\partial}{\partial\delta}+\frac{\partial\gamma}{\partial f_{4y}}\frac{\partial}{\partial\gamma}+\frac{\partial\theta}{\partial f_{4y}}\frac{\partial}{\partial\theta}+\frac{\partial\varphi}{\partial f_{4y}}\frac{\partial}{\partial\varphi}+\frac{\partial\xi}{\partial f_{4y}}\frac{\partial}{\partial\xi}+$$
$$\frac{\partial\eta}{\partial f_{4y}}\frac{\partial}{\partial\eta}+\frac{\partial\zeta}{\partial f_{4y}}\frac{\partial}{\partial\zeta}=\sin\varepsilon\sin\theta\sin\varphi\cos\eta\frac{\partial}{\partial\beta}+\frac{\cos\varepsilon\sin\theta\sin\varphi\cos\eta}{\beta}\frac{\partial}{\partial\varepsilon}+$$
$$\frac{\cos\theta\sin\varphi\cos\eta}{\beta\sin\varepsilon}\frac{\partial}{\partial\theta}+\frac{\cos\varphi\cos\eta}{\beta\sin\varepsilon\sin\theta}\frac{\partial}{\partial\varphi}-\frac{\sin\eta}{\beta\sin\varepsilon\sin\theta\sin\varphi}\frac{\partial}{\partial\eta}, \tag{13.5}$$

$$\frac{\partial}{\partial f_{4z}}=\frac{\partial\beta}{\partial f_{4z}}\frac{\partial}{\partial\beta}+\frac{\partial\varepsilon}{\partial f_{4z}}\frac{\partial}{\partial\varepsilon}+\frac{\partial\delta}{\partial f_{4z}}\frac{\partial}{\partial\delta}+\frac{\partial\gamma}{\partial f_{4z}}\frac{\partial}{\partial\gamma}+\frac{\partial\theta}{\partial f_{4z}}\frac{\partial}{\partial\theta}+\frac{\partial\varphi}{\partial f_{4z}}\frac{\partial}{\partial\varphi}+\frac{\partial\xi}{\partial f_{4z}}\frac{\partial}{\partial\xi}+$$
$$\frac{\partial\eta}{\partial f_{4z}}\frac{\partial}{\partial\eta}+\frac{\partial\zeta}{\partial f_{4z}}\frac{\partial}{\partial\zeta}=\sin\varepsilon\cos\theta\cos\zeta\frac{\partial}{\partial\beta}+\frac{\cos\varepsilon\cos\theta\cos\zeta}{\beta}\frac{\partial}{\partial\varepsilon}-$$
$$\frac{\sin\theta\cos\zeta}{\beta\sin\varepsilon}\frac{\partial}{\partial\theta}-\frac{\sin\zeta}{\beta\sin\varepsilon\cos\theta}\frac{\partial}{\partial\zeta}, \tag{13.6}$$

$$\frac{\partial}{\partial g_{4x}}=\frac{\partial\beta}{\partial g_{4x}}\frac{\partial}{\partial\beta}+\frac{\partial\varepsilon}{\partial g_{4x}}\frac{\partial}{\partial\varepsilon}+\frac{\partial\delta}{\partial g_{4x}}\frac{\partial}{\partial\delta}+\frac{\partial\gamma}{\partial g_{4x}}\frac{\partial}{\partial\gamma}+\frac{\partial\theta}{\partial g_{4x}}\frac{\partial}{\partial\theta}+\frac{\partial\varphi}{\partial g_{4x}}\frac{\partial}{\partial\varphi}+\frac{\partial\xi}{\partial g_{4x}}\frac{\partial}{\partial\xi}+$$
$$\frac{\partial\eta}{\partial g_{4x}}\frac{\partial}{\partial\eta}+\frac{\partial\zeta}{\partial g_{4x}}\frac{\partial}{\partial\zeta}=\sin\varepsilon\sin\theta\cos\varphi\sin\xi\frac{\partial}{\partial\beta}+\frac{\cos\varepsilon\sin\theta\cos\varphi\sin\xi}{\beta}\frac{\partial}{\partial\varepsilon}+$$
$$\frac{\cos\theta\cos\varphi\sin\xi}{\beta\sin\varepsilon}\frac{\partial}{\partial\theta}-\frac{\sin\varphi\sin\xi}{\beta\sin\varepsilon\sin\theta}\frac{\partial}{\partial\varphi}+\frac{\cos\xi}{\beta\sin\varepsilon\sin\theta\cos\varphi}\frac{\partial}{\partial\xi}, \tag{13.7}$$

$$\frac{\partial}{\partial g_{4y}}=\frac{\partial\beta}{\partial g_{4y}}\frac{\partial}{\partial\beta}+\frac{\partial\varepsilon}{\partial g_{4y}}\frac{\partial}{\partial\varepsilon}+\frac{\partial\delta}{\partial g_{4y}}\frac{\partial}{\partial\delta}+\frac{\partial\gamma}{\partial g_{4y}}\frac{\partial}{\partial\gamma}+\frac{\partial\theta}{\partial g_{4y}}\frac{\partial}{\partial\theta}+\frac{\partial\varphi}{\partial g_{4y}}\frac{\partial}{\partial\varphi}+\frac{\partial\xi}{\partial g_{4y}}\frac{\partial}{\partial\xi}+$$
$$\frac{\partial\eta}{\partial g_{4y}}\frac{\partial}{\partial\eta}+\frac{\partial\zeta}{\partial g_{4y}}\frac{\partial}{\partial\zeta}=\sin\varepsilon\sin\theta\sin\varphi\sin\eta\frac{\partial}{\partial\beta}+\frac{\cos\varepsilon\sin\theta\sin\varphi\sin\eta}{\beta}\frac{\partial}{\partial\varepsilon}+$$
$$\frac{\cos\theta\sin\varphi\sin\eta}{\beta\sin\varepsilon}\frac{\partial}{\partial\theta}+\frac{\cos\varphi\sin\eta}{\beta\sin\varepsilon\sin\theta}\frac{\partial}{\partial\varphi}+\frac{\cos\eta}{\beta\sin\varepsilon\sin\theta\sin\varphi}\frac{\partial}{\partial\eta}, \tag{13.8}$$

$$\frac{\partial}{\partial g_{4z}}=\frac{\partial\beta}{\partial g_{4z}}\frac{\partial}{\partial\beta}+\frac{\partial\varepsilon}{\partial g_{4z}}\frac{\partial}{\partial\varepsilon}+\frac{\partial\delta}{\partial g_{4z}}\frac{\partial}{\partial\delta}+\frac{\partial\gamma}{\partial g_{4z}}\frac{\partial}{\partial\gamma}+\frac{\partial\theta}{\partial g_{4z}}\frac{\partial}{\partial\theta}+\frac{\partial\varphi}{\partial g_{4z}}\frac{\partial}{\partial\varphi}+\frac{\partial\xi}{\partial g_{4z}}\frac{\partial}{\partial\xi}+$$
$$\frac{\partial\eta}{\partial g_{4z}}\frac{\partial}{\partial\eta}+\frac{\partial\zeta}{\partial g_{4z}}\frac{\partial}{\partial\zeta}=\sin\varepsilon\cos\theta\sin\zeta\frac{\partial}{\partial\beta}+\frac{\cos\varepsilon\cos\theta\sin\zeta}{\beta}\frac{\partial}{\partial\varepsilon}-$$
$$\frac{\sin\theta\sin\zeta}{\beta\sin\varepsilon}\frac{\partial}{\partial\theta}+\frac{\cos\zeta}{\beta\sin\varepsilon\cos\theta}\frac{\partial}{\partial\zeta}, \tag{13.9}$$

将(13,1—9)式代入(12)，经过冗长的计算，最后整理得

$$H = -\frac{1}{2B}\left[\frac{\partial^2}{\partial\beta^2} + \frac{1}{\beta^2}\frac{\partial^2}{\partial\varepsilon^2} + \frac{1}{\beta^2\cos^2\varepsilon}\frac{\partial^2}{\partial\delta^2} + \frac{1}{\beta^2\sin^2\varepsilon}\frac{\partial^2}{\partial\theta^2} + \frac{1}{\beta^2\cos^2\varepsilon\sin^2\delta}\frac{\partial^2}{\partial\gamma^2} + \right.$$
$$\frac{1}{\beta^2\sin^2\varepsilon\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{1}{\beta^2\sin^2\varepsilon\cos^2\theta}\frac{\partial^2}{\partial\zeta^2} + \frac{1}{\beta^2\sin^2\varepsilon\sin^2\theta\cos^2\varphi}\frac{\partial^2}{\partial\xi^2} +$$
$$\frac{1}{\beta^2\sin^2\varepsilon\sin^2\theta\sin^2\varphi}\frac{\partial^2}{\partial\eta^2} + \frac{8}{\beta}\frac{\partial}{\partial\beta} + \frac{5\cos^2\varepsilon - 2\sin^2\varepsilon}{\beta^2\sin\varepsilon\cos\varepsilon}\frac{\partial}{\partial\varepsilon} + \frac{\cos\delta}{\beta^2\cos^2\varepsilon\sin\delta}\frac{\partial}{\partial\delta} +$$
$$\left.\frac{3\cos^2\theta - \sin^2\theta}{\beta^2\sin^2\varepsilon\sin\theta\cos\theta}\frac{\partial}{\partial\theta} + \frac{\cos^2\varphi - \sin^2\varphi}{\beta^2\sin^2\varepsilon\sin^2\theta\sin\varphi\cos\varphi}\frac{\partial}{\partial\varphi}\right] + V. \tag{14}$$

(14)式即为参数空间量子化的哈密顿量，β表示原子核的总形变，$\varepsilon, \theta, \gamma, \delta, \varphi, \zeta, \xi, \eta$八个参数确定了原子核的表面形状和空间方位．文献[9]讨论了每个参数的物理意义，量子化的哈密顿量(14)式可用来讨论一个或几个形变参数的振动，对研究原子核高自旋，超形变以及极端条件下的性质具有重要的意义．

4 结论

首次在参数空间，导出了原子核作十六极振动量子化哈密顿量．其中的参数具有明显的物理意义．不仅给出了 O_h不变性的原子核总形变参数β，也给出了包含四极形变在内的其它形变参数，对研究原子核不同模式下的振动有重要的意义，可用来分析原子核高自旋，超形变状态下所出现的新现象．

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Quantization of Nuclear Hexadecapole Vibration in the Space of Parameter*

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Abstract The quantized Hamiltonian is derived from the general parametral form of the hexadecapole Vibration in the space of parameter, which maybe important to analyze properties of nuclear structure.

Key words hexadecapole deformation, parametrization, quantization

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