第23卷 第12期 高能物理与核物理 Vol.23, No.12
1999 年12月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS Dec., 1999

$A_{n-1}^{(1)}$面模型反射方程的多参数解

石康杰 李广良 范 桁 侯伯宇

(西北大学现代物理研究所 西安 710069)

摘要 利用面型因式化 L 算子，通过 $A_{n-1}^{(1)}$ 面模型反射方程的对角解，构造了一个含有 $n+1$ 个任意参数的非对角解.

关键词 $A_{n-1}^{(1)}$面模型 面型因式化 L 算子 反射方程 非对角解

1 引言

在二维完全可解晶格模型中，非周期边界条件是一类重要的问题. 自从 Sklyanin 提出反射方程来解决这个问题以后，有关反射方程的解已有不少工作[1-9]. 人们利用反射方程的解来构造可解模型[1,10]，因此，寻找反射方程的新解是一件有意义的工作. 在文献[11]中，R.E.Behrend 和 P.A.Pearce 给出了一种由已知的面型反射方程的解来得到一个新解的方法，他们是利用聚合的面权通过面型反射方程的一个已知解来构造一个增加了任意参数的新解。本文利用面型因式化 L 算子，通过 M.T.Batchelor 等人[6]所给出的 $A_{n-1}^{(1)}$ 面模型反射方程的对角解，得到了一个含有 $n+1$ 个任意参数的非对角解.

2 $A_{n-1}^{(1)}$面模型的反射方程及因式化 L 算子

先定义 $A_{n-1}^{(1)}$ 面模型的面权 $W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z)$[12]. 其定义为

$$W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z) = \frac{\sigma_0(z+w)}{\sigma_0(w)} \qquad (b=d=a+\hat{\mu},\ c=a+2\hat{\mu})\ ,$$

$$W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z) = \frac{\sigma_0(-z+a_{\mu\nu}w)}{\sigma_0(a_{\mu\nu}w)} \qquad (b=d=a+\hat{\mu},\ c=a+\hat{\mu}+\hat{\nu})\ , \tag{1}$$

$$W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z) = \frac{\sigma_0(z)\,\sigma_0((1+a_{\mu\nu})w)}{\sigma_0(w)\sigma_0(a_{\mu\nu}\,w)} \qquad (b=a+\hat{\mu},\ d=a+\hat{\nu},\ c=a+\hat{\mu}+\hat{\nu})\ ,$$

1998-10-13收稿

$$W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z) = 0 \qquad\qquad (其它情况).$$

式中 $a = (a_0, a_1, \cdots, a_{n-1}) \in Z^n$, $\hat{\mu} = (0, 0, \cdots, 0, 1, 0, \cdots, 0)$(第$\mu$个位置为 1,其余位置为零),$\bar{a}_\mu = a_\mu - \frac{1}{n}\sum_\nu a_\nu + \delta_\mu, \mu, \nu \in Z_n$, δ_μ是一些一般的复数,$a_{\mu\nu} = \bar{a}_\mu - \bar{a}_\nu$, w 为交叉参数,$\sigma_0(z) = \theta\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}(z,\tau)$,$\theta\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}(z,\tau)$定义为

$$\theta\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}(z,\tau) = \sum_{m\in Z} e^{\pi\sqrt{-1}(m+c_1)^2\tau + 2\pi\sqrt{-1}(m+c_1)(z+c_2)}, \tag{2}$$

c_1,c_2为任意的参数. 面权 $W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z)$满足 star-triangle方程

$$\sum_s W\begin{bmatrix} d & c \\ s & b \end{bmatrix}(z_1 - z_2) W\begin{bmatrix} e & d \\ f & s \end{bmatrix}(z_1 - z_3) W\begin{bmatrix} f & s \\ a & b \end{bmatrix}(z_2 - z_3) =$$

$$\sum_s W\begin{bmatrix} e & d \\ s & c \end{bmatrix}(z_2 - z_3) W\begin{bmatrix} s & c \\ a & b \end{bmatrix}(z_1 - z_3) W\begin{bmatrix} e & s \\ f & a \end{bmatrix}(z_1 - z_2) \ . \tag{3}$$

$A_{n-1}^{(1)}$面模型中的反射方程为

$$\sum_{sf} W\begin{bmatrix} e & d \\ s & c \end{bmatrix}(z_1 - z_2) K\begin{pmatrix} s & {}^{f}_{e} \end{pmatrix}(z_1) W\begin{bmatrix} f & s \\ b & c \end{bmatrix}(z_1 + z_2)\, K\begin{pmatrix} b & {}^{a}_{f} \end{pmatrix}(z_2) =$$

$$\sum_{sf} K\begin{pmatrix} d & {}^{f}_{e} \end{pmatrix}(z_2) W\begin{bmatrix} f & d \\ s & c \end{bmatrix}(z_1 + z_2) K\begin{pmatrix} s & {}^{a}_{f} \end{pmatrix}(z_1) W\begin{bmatrix} a & s \\ b & c \end{bmatrix}(z_1 - z_2) \ . \tag{4}$$

式中 $K\begin{pmatrix} s & {}^{f}_{e} \end{pmatrix}(z)$称为边界权(boundary weight),如果它满足面型反射方程(4),则称其为面型反射方程(4)的一个解. 利用面型反射方程的解以及其对偶方程的解,可以构造对不同谱互相对易的转移(transfer)矩阵,从而得到可解模型.

为了构造面型因式化 L 算子,再引入 Z_n模型与 $A_{n-1}^{(1)}$面模型面-顶角对应的 intertwiner[12],它是一个列矢量$\phi_{a,q+\hat{\mu}}(z)$,其分量为

$$\phi_{a,a+\hat{\mu}}^{(j)}(z) = \theta^{(j)}(z - nw\bar{a}_\mu, n\tau),$$

$$\theta^{(j)}(z,n\tau) = \theta\begin{bmatrix} \frac{1}{2} - \frac{j}{n} \\ \frac{1}{2} \end{bmatrix}(z, n\tau) \ , \tag{5}$$

$j\in Z_n$. 对于 $b \neq a+\hat{\mu}$($\hat{\mu}$为任意 n 维单位行矢量)，定义

$$\phi_{a,b}^{(j)}(z)=0. \tag{6}$$

利用 intertwiner，面–顶角对应可以写为

$$R_{12}(z_1-z_2)\phi_{a,b}(z_1)\otimes\phi_{b,c}(z_2)=\sum_d W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z_1-z_2)\phi_{d,c}(z_1)\otimes\phi_{a,d}(z_2)\ . \tag{7}$$

$R_{12}(z)$为 Z_n Belavin 模型的 R 矩阵[13]，其定义为

$$R_{12}(z)=\frac{1}{n}\sum_{\alpha\in Z_n^2} W_\alpha(z) I_\alpha\otimes I_\alpha^{-1}. \tag{8}$$

$\alpha=(\alpha_1,\alpha_2),\alpha_1,\alpha_2\in Z_n$,$I_\alpha$是 $n\times n$ 矩阵，$I_\alpha=g^{\alpha_2}h^{\alpha_1}$,$g_{ij}=\omega^i\delta_{ij}$,$h_{ij}=\delta_{i+1,j}$,$\omega=\mathrm{e}^{\frac{2\pi\sqrt{-1}}{n}}$, g,h 均为 $n\times n$ 矩阵，$i\in Z_n$,

$$W_\alpha(z)=\frac{\sigma_\alpha\left(z+\frac{w}{n}\right)}{\sigma_\alpha\left(\frac{w}{n}\right)}\ ,\quad \sigma_\alpha(z)\equiv\theta\begin{bmatrix} \frac{1}{2}+\frac{\alpha_1}{n} \\ \frac{1}{2}+\frac{\alpha_2}{n} \end{bmatrix}(z,\tau)\ . \tag{9}$$

还可构造满足下列关系的行矢量$\tilde{\phi}$和$\bar{\phi}$,

$$\sum_K \tilde{\phi}_{a-\hat{\mu},\,a}^{(k)}(z)\ \phi_{a-\hat{\nu},\,a}^{(k)}(z)=\delta_{\mu\nu}\ ,\qquad \sum_k \bar{\phi}_{a,a+\hat{\mu}}^{(k)}(z)\ \phi_{a,\,a+\hat{\nu}}^{(k)}(z)=\delta_{\mu\nu}\ . \tag{10}$$

上式关系又可写成

$$\sum_\mu \phi_{a-\hat{\mu},\,a}(z)\tilde{\phi}_{a-\hat{\mu},\,a}(z)=I,\qquad \sum_\mu \phi_{a,\,a+\hat{\mu}}(z)\bar{\phi}_{a,\,a+\hat{\mu}}(z)=I. \tag{11}$$

$K\in Z_n$. 对于其它情况，$\tilde{\phi}$,ϕ定义为

$$\tilde{\phi}_{b,a}(z)=0\qquad (b\neq a-\hat{\mu}),\qquad \bar{\phi}_{a,c}(z)=0\qquad (c\neq a+\hat{\mu}). \tag{12}$$

利用$\tilde{\phi}$,$\bar{\phi}$,ϕ之间的关系，面–顶角对应可写成以下几种形式，

$$1\otimes\bar{\phi}_{a,d}(z_2)R_{12}(z_1-z_2)\phi_{a,b}(z_1)\otimes 1=\sum_c W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z_1-z_2)\phi_{d,\,c}(z_1)\otimes\bar{\phi}_{b,\,c}(z_2)\ , \tag{13}$$

$$\tilde{\phi}_{c,b}(z_1)\otimes 1R_{12}(z_1-z_2)1\otimes\phi_{a,\,b}(z_2)=\sum_d W\begin{bmatrix} d & c \\ a & b \end{bmatrix}(z_1-z_2)\tilde{\phi}_{d,\,a}(z_1)\ \otimes\phi_{d,\,c}(z_2), \tag{14}$$

$$\bar{\phi}_{d,\,c}(z_1)\otimes\bar{\phi}_{a,\,d}(z_2)\ R_{12}(z_1-z_2)=\sum_b W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z_1-z_2)\bar{\phi}_{a,\,b}(z_1)\ \otimes\bar{\phi}_{b,\,c}(z_2), \tag{15}$$

$$\tilde{\phi}_{d,\,c}(z_1)\otimes\tilde{\phi}_{a,\,d}(z_2)\ R_{12}(z_1-z_2)=\sum_b W\begin{bmatrix} a & d \\ b & c \end{bmatrix}(z_1-z_2)\tilde{\phi}_{a,\,b}(z_1)\ \otimes\tilde{\phi}_{b,\,c}(z_2)\,. \tag{16}$$

现在,我们来构造面型因式化 L 算子。定义面型因式化 L 算子为

$$L\begin{bmatrix} a & d' \\ b & c' \end{bmatrix} = \tilde{\phi}_{d',c'}(z)\phi_{a,b}(z) . \tag{17}$$

其中 $c' = (c_0', c_1', \cdots, c_{n-1}') \in Z^n$, $\bar{c}_\mu' = c_\mu' - \frac{1}{n}\sum_\nu c_\nu' + \delta_\mu'$, δ_μ'是一些一般的复数. 由

$$\sum_a \bar{\phi}_{d',c'}(z)\phi_{a,b}(z)\tilde{\phi}_{a,b}(z)\phi_{d',c'}(z) = 1,$$

可知

$$L^{-1}\begin{bmatrix} a & d' \\ b & c' \end{bmatrix}(z) = \bar{\phi}_{a,b}(z)\phi_{d',c'}(z). \tag{18}$$

利用面-顶角对应关系(7),(13)—(16)式,可以证明

$$\sum_{s'} W\begin{bmatrix} d' & c' \\ s' & b' \end{bmatrix}(z_1 - z_2) L\begin{bmatrix} e & d' \\ f & s' \end{bmatrix}(z_1) L\begin{bmatrix} f & s' \\ a & b' \end{bmatrix}(z_2) =$$

$$\sum_s L\begin{bmatrix} e & d' \\ s & c' \end{bmatrix}(z_2) L\begin{bmatrix} s & c' \\ a & b' \end{bmatrix}(z_1) W\begin{bmatrix} e & s \\ f & a \end{bmatrix}(z_1 - z_2) , \tag{19}$$

$$\sum_{s'} L^{-1}\begin{bmatrix} f & s' \\ e & d' \end{bmatrix}(z_1) L^{-1}\begin{bmatrix} a & b' \\ f & s' \end{bmatrix}(z_2) W\begin{bmatrix} b' & s' \\ c' & b' \end{bmatrix}(z_1 - z_2) =$$

$$\sum_s W\begin{bmatrix} a & f \\ s & e \end{bmatrix}(z_1 - z_2) L^{-1}\begin{bmatrix} s & c' \\ e & d' \end{bmatrix}(z_2) L^{-1}\begin{bmatrix} a & b' \\ s & c' \end{bmatrix}(z_1) , \tag{20}$$

$$\sum_{s'} L\begin{bmatrix} f & s' \\ a & b' \end{bmatrix}(z_1) W\begin{bmatrix} s' & d' \\ b' & c' \end{bmatrix}(z_1 - z_2) L^{-1}\begin{bmatrix} f & s' \\ e & d' \end{bmatrix}(z_2) =$$

$$\sum_s L^{-1}\begin{bmatrix} a & b' \\ s & c' \end{bmatrix}(z_2) W\begin{bmatrix} f & e \\ a & s \end{bmatrix}(z_1 - z_2) L\begin{bmatrix} e & d' \\ s & c' \end{bmatrix}(z_1) . \tag{21}$$

利用这些关系,可以从面型反射方程(4)的已知解来构造一个新解.

3 新解的构造

现在来构造面型反射方程(4)的解. 将面型反射方程(4)式的等式两边同时左乘 $L\begin{bmatrix} d & d' \\ c & c' \end{bmatrix}(z_1) L\begin{bmatrix} e & e' \\ d & d' \end{bmatrix}(z_2)$ 和右乘 $L^{-1}\begin{bmatrix} b & b' \\ c & c' \end{bmatrix}(-z_1) L^{-1}\begin{bmatrix} a & a' \\ b & b' \end{bmatrix}(-z_2)$ 并对 b, c, d 求和得

$$LHS=\sum_{fs}\sum_{bcd}L\begin{bmatrix}d & d'\\ c & c'\end{bmatrix}(z_1)\,L\begin{bmatrix}e & e'\\ d & d'\end{bmatrix}(z_2)\,W\begin{bmatrix}e & d\\ s & c\end{bmatrix}(z_1-z_2)K\begin{pmatrix}s & f\\ & e\end{pmatrix}(z_1)\times$$
$$W\begin{bmatrix}f & s\\ b & c\end{bmatrix}(z_1+z_2)\,K\begin{pmatrix}b & a\\ & f\end{pmatrix}(z_2)L^{-1}\begin{bmatrix}b & b'\\ c & c'\end{bmatrix}(-z_1)\,L^{-1}\begin{bmatrix}a & a'\\ b & b'\end{bmatrix}(-z_2).$$

利用(20)式得

$$LHS=\sum_{fs}\sum_{bcs'}W\begin{bmatrix}e' & d'\\ s' & c'\end{bmatrix}(z_1-z_2)\,L\begin{bmatrix}e & e'\\ s & s'\end{bmatrix}(z_1)\,K\begin{pmatrix}s & f\\ & e\end{pmatrix}(z_1)\,L\begin{bmatrix}s & s'\\ c & c'\end{bmatrix}(z_2)\times$$
$$W\begin{bmatrix}f & s\\ b & c\end{bmatrix}(z_1+z_2)\,L^{-1}\begin{bmatrix}b & b'\\ c & c'\end{bmatrix}(-z_1)\,K\begin{pmatrix}b & a\\ & f\end{pmatrix}(z_2)\,L^{-1}\begin{bmatrix}a & a'\\ b & b'\end{bmatrix}(-z_2).$$

利用(21)式得

$$LHS=\sum_{s'f'}\sum_{f}W\begin{bmatrix}e' & d'\\ s' & c'\end{bmatrix}(z_1-z_2)\sum_{s}L\begin{bmatrix}e & e'\\ s & s'\end{bmatrix}(z_1)\,K\begin{pmatrix}s & f\\ & e\end{pmatrix}(z_1)\,L^{-1}\begin{bmatrix}f & f'\\ s & s'\end{bmatrix}(-z_1)\times$$
$$W\begin{bmatrix}f' & s'\\ b' & c'\end{bmatrix}(z_1+z_2)\sum_{b}L\begin{bmatrix}f & f'\\ b & b'\end{bmatrix}(z_2)\,K\begin{pmatrix}b & a\\ & f\end{pmatrix}(z_2)\,L^{-1}\begin{bmatrix}a & a'\\ b & b'\end{bmatrix}(-z_2). \tag{22}$$

同理，由(19)式和(21)式得

$$RHS=\sum_{s'f'}\sum_{f}\sum_{d}L\begin{bmatrix}e & e'\\ d & d'\end{bmatrix}(z_2)\,K\begin{pmatrix}d & f\\ & e\end{pmatrix}(z_2)\,L^{-1}\begin{bmatrix}f & c'\\ d & d'\end{bmatrix}\,W\begin{bmatrix}f' & d'\\ b' & c'\end{bmatrix}(z_1+z_2)\times$$
$$\sum_{s}L\begin{bmatrix}f & f'\\ s & s'\end{bmatrix}(z_1)\,K\begin{pmatrix}s & a\\ & f\end{pmatrix}(z_1)\,L^{-1}\begin{bmatrix}a & a'\\ s & s'\end{bmatrix}(-z_1)\,W\begin{bmatrix}a' & s'\\ b' & c'\end{bmatrix}(z_1-z_2). \tag{23}$$

令

$$K'\begin{pmatrix}s' & f'\\ & e'\end{pmatrix}(z)=\sum_{s}L\begin{bmatrix}e & e'\\ s & s'\end{bmatrix}(z)\,K\begin{pmatrix}s & f\\ & e\end{pmatrix}(z)\,L^{-1}\begin{bmatrix}f & f'\\ s & s'\end{bmatrix}(-z), \tag{24}$$

并取$K\begin{pmatrix}s & f\\ & e\end{pmatrix}(z)=K\begin{pmatrix}s & f\\ & e\end{pmatrix}(z)\,\delta_{ef}$，有

$$\sum_{f's'}W\begin{bmatrix}e' & d'\\ s' & c'\end{bmatrix}(z_1-z_2)\,K'\begin{pmatrix}s' & f'\\ & e'\end{pmatrix}(z_1)W\begin{bmatrix}f' & s'\\ b' & c'\end{bmatrix}(z_1+z_2)\,K'\begin{pmatrix}b' & a'\\ & f'\end{pmatrix}(z_2)\,\delta_{ae}=$$
$$\sum_{f's'}K'\begin{pmatrix}d' & f'\\ & e'\end{pmatrix}(z_2)W\begin{bmatrix}f' & d'\\ s' & c'\end{bmatrix}(z_1+z_2)\,K'\begin{pmatrix}s' & a'\\ & f'\end{pmatrix}(z_1)\,W\begin{bmatrix}a' & s'\\ b' & c'\end{bmatrix}(z_1-z_2)\,\delta_{ae}. \tag{25}$$

即$K'\left(\begin{matrix} s' & f' \\ & e' \end{matrix}\right)(z)$满足面型反射方程(4)，因此利用原来面型反射方程(4)的一个对角解，借助面型因式化L算子，得到了面型反射方程(4)的一个新解.

M. T. Batchelor 等人[6]给出了 $A_{n-1}^{(1)}$面模型反射方程的一个对角解

$$K\left(\begin{matrix} a+\hat{\mu} & b \\ & a \end{matrix}\right)(z)=\frac{\sigma_0(\bar{a}_\mu w-\xi+z)}{\sigma_0(\bar{a}_\mu w-\xi-z)}f_a(z)\delta_{ab}. \tag{26}$$

ξ为任意参数，$f_a(z)$为任意解析函数. 由此可得

$$K'\left(\begin{matrix} s' & f' \\ & e' \end{matrix}\right)(z)=\sum_s\tilde{\phi}_{e',s'}(z)\phi_{a,s}(z)K\left(\begin{matrix} s & a \\ & a \end{matrix}\right)(z)\bar{\phi}_{a,s}(-z)\phi_{f',s'}(-z). \tag{27}$$

令$f'=e'+\hat{\mu}'$，$S'=e'+\hat{\mu}'-\hat{\nu}'$，$s=a+\hat{\mu}$，得

$$K'\left(\begin{matrix} e'+\hat{\mu}' & e'+\hat{\mu}'-\hat{\nu}' \\ & e' \end{matrix}\right)(z)=\sum_\mu\tilde{\phi}_{e',e'+\hat{\mu}'}(z)\phi_{a,a+\hat{\mu}'}(z)\frac{\sigma_0(\bar{a}_\mu w-\xi+z)}{\sigma_0(\bar{a}_\mu w-\xi-z)}f_a(z)\times$$

$$\bar{\phi}_{a,a+\hat{\mu}}(-z)\phi_{e'+\hat{\mu}'-\hat{\nu}',e'+\hat{\mu}'}(-z). \tag{28}$$

由(11)式可以知道$\tilde{\phi}_{e',e'+\hat{\mu}'}(z)$可从矩阵$\tilde{M}(z)$的逆矩阵获得，其中

$$\tilde{M}(z)_{i\lambda}=\phi^{(i)}_{e'+\hat{\mu}'-\hat{\lambda},e'+\hat{\mu}'}(z)=\theta^{(i)}(z-nw(\bar{e}'_\lambda+\delta_{\mu'\lambda}-1),\ n\tau)。 \tag{29}$$

定义$\tilde{M}'(z)$为把矩阵$\tilde{M}(z)$第μ'列元素用列矢量$\phi_{a,a+\hat{\mu}}(z)$相应分量所替换而得到的矩阵，则有

$$\tilde{\phi}_{e',e'+\hat{\mu}'}(z)\phi_{a,a+\hat{\mu}}(z)=\frac{\det\tilde{M}'(z)}{\det\tilde{M}(z)}. \tag{30}$$

如有一$n\times n$矩阵$A_{ij}=\theta^{(i)}(nz_j)$，则可证明[14]

$$\det A=C(\tau)\sigma_0\left(\sum_i z_i-\frac{n-1}{2}\right)\prod_{j<k}(z_j-z_k). \tag{31}$$

利用(31)式，得

$$\tilde{\phi}_{e',e'+\hat{\mu}'}(z)\phi_{a,a+\hat{\mu}}(z)=\frac{\sigma_0\left(z+w\delta'+(n-1)w-\frac{n-1}{2}+w(\bar{e}'_{\mu'}-\bar{a}_\mu)\right)}{\sigma_0\left(z+w\delta'+(n-1)w-\frac{n-1}{2}\right)}\times$$

$$\prod_{j\neq\mu'}\frac{\sigma_0(w(\bar{e}'_j-\bar{a}_\mu-1))}{\sigma_0(w(\bar{e}'_j-\bar{e}'_{\mu'}-1))}. \tag{32}$$

$\delta' = \sum_{v}\delta'_{v}$. 从(11)式可知$\bar{\phi}_{a,\ a+\hat{\mu}}(-z)$可从矩阵 $\bar{M}(-z)$的逆矩阵获得,其中

$$\bar{M}(-z)_{i\lambda} = \phi^{(i)}_{a,\ a+\hat{\lambda}}(-z) = \theta^{(i)}(-z-nw\bar{a}_{\lambda},\ n\tau). \tag{33}$$

定义 $\bar{M}'(-z)$为把矩阵 $\bar{M}(-z)$第μ列元素用列矢量$\phi_{e'+\hat{\mu}'-\hat{v}',\ e'+\hat{\mu}'}(-z)$相应的分量替换而得到的矩阵,则有

$$\bar{\phi}_{a,\ a+\hat{\mu}}(-z)\phi_{e'+\hat{\mu}'-\hat{v}',\ e'+\hat{\mu}'}(-z) = \frac{\det\bar{M}'(-z)}{\det\bar{M}(-z)}. \tag{34}$$

利用(31)式得

$$\bar{\phi}_{a,\ a+\hat{\mu}'}(-z)\phi_{e'+\hat{\mu}'-\hat{v}',\ e'+\hat{\mu}'}(-z) = \frac{\sigma_0\left(-z+w\delta-\frac{n-1}{2}+w(\bar{a}_{\mu}-\bar{e}'_{v'}-\delta_{\mu'v'}+1)\right)}{\sigma_0\left(-z+w\delta-\frac{n-1}{2}\right)}\times \prod_{j\neq\mu}\frac{\sigma_0(w(\bar{a}_j-\bar{e}'_{v'}-\delta_{\mu'v'}+1))}{\sigma_0(w(\bar{a}_j-\bar{a}_{\mu}))}. \tag{35}$$

$\delta = \sum_{v}\delta_{v}$. 最后得到面型反射方程(4)的一个新解

$$K'\begin{pmatrix} e'+\hat{\mu}' & e'+\hat{\mu}'-\hat{v}' \\ & e' \end{pmatrix}(z) = \sum_{\mu}\frac{\sigma_0\left(z+w\delta'+(n-1)w-\frac{n-1}{2}+w(\bar{e}'_{\mu'}-\bar{a}_{\mu})\right)}{\sigma_0\left(z+w\delta'+(n-1)w-\frac{n-1}{2}\right)}\times \frac{\sigma_0\left(-z+w\delta-\frac{n-1}{2}+w(\bar{a}_{\mu}-\bar{e}'_{v'}-\delta_{\mu'v'}+1)\right)}{\sigma_0\left(-z+w\delta-\frac{n-1}{2}\right)}\times\prod_{j\neq\mu'}\frac{\sigma_0(w(\bar{e}'_j-\bar{a}_{\mu}-1))}{\sigma_0(w(\bar{e}'_j-\bar{e}'_{\mu'}-1))} \prod_{k\neq\mu}\frac{\sigma_0(w(\bar{a}_k-\bar{e}'_{v'}-\delta_{\mu'v'}+1))}{\sigma_0(w(\bar{a}_j-\bar{a}_{\mu}))}\times\frac{\sigma_0(\bar{a}_{\mu}w-\xi+z)}{\sigma_0(\bar{a}_{\mu}w-\xi-z)}f_a(z). \tag{36}$$

4 结论

从(36)式来看,首先$K'\begin{pmatrix} e'+\hat{\mu}' & e'+\hat{\mu}'-\hat{v}' \\ & e' \end{pmatrix}(z)$是一个非对角解. 其次,在$A_{n-1}^{(1)}$面模型中,$a$与$e'$的取值是相互独立的,而(36)式的存在条件只要求$\bar{a}_{\mu}$中的$\delta_{\mu}$是一些一般的复数,因此$\bar{a}_{\mu}$是可以任意选取的. 故可将$\bar{a}_0$, $\bar{a}_1$, $\cdots$, $\bar{a}_{n-1}$视为相互独立的参数,加上任意参数ξ,在(36)式中就含有$n+1$个任意的参数.

参 考 文 献

1 Sklyanin E K. J. Phys., 1988, **A21**:2735

2 Behrend R E, Pearce P A, O'Brien D L. J. Stat. Phys., 1996, **84**:1—48

3 Ahn C, Koo W M. Nucl. Phys., 1996, **B468**:461—486

4 Batchelor M T, Fridkin V, Zhou Y K. J. Phys., 1996, **A29**:L61—L67

5 Fan H, Hou B Y, Shi K J. J. Phys., 1995, **A28**:4743—4749

6 Batchelor M T, Fridkin V, Kuniba A et al. Phys. Lett., 1996, **B376**:266—274

7 Zhou Y K, Batchelor M T. J. Phys., 1996, **A29**: 1987—1996

8 Chen min, Hou Boyu, Shi Kangjie. High Energy Phys. and Nucl. Phys. (in Chinese), 1996, **20**:794
(陈敏，侯伯宇，石康杰. 高能物理与核物理，1996，**20**:794)

9 Shi kangjie, Li GuangLiang, Fan Heng et al. High Energy Phys. and Nucl. Phys. (in Chinese), 1999, **23**(5):425
(石康杰，李广良，范桁等. 高能物理与核物理，1999，**23**(5)：425)

10 Shi Kangjie, Li GuangLiang, Fan Heng et al. High Energy Phys. and Nucl. Phys. (in Chinese), 1998, **22**:1100
(石康杰，李广良，范桁等. 高能物理与核物理，1998，**22**:1100)

11 Behrend R E, Pearce P A. J. Phys., 1996, **A29**: 7827—7835

12 Jimbo M, Miwa T, Okado M. Nucl. Phys., 1988, **B300**: 74

13 Belavin A A. Nucl. Phys., 1980, **B180**:109

14 Quano Y H, INS-T-513, Proc. of the Workshop on New Aspects of Quantum Field Theories, Nojiri S. ed. 29—45

Polyparametioner Solution to the Reflection Equation in the $A_{n-1}^{(1)}$ Face Model

Shi Kangjie Li Guangliang Fan Heng Hou Boyu

(Institute of Modern Physics, Northwest University, Xi'an 710069)

Abstract By making urs of the diagonal solution of the reflection equation of the $A_{n-1}^{(1)}$ IRF model and the face factorized L operator, we obtain a nondiagonal solution with $n+1$ parameters to the reflection equation for the $A_{n-1}^{(1)}$ face model.

Key words $A_{n-1}^{(1)}$ IRF model, face factorized L operator, reflection equation, nondiagonal solution

Received 13 October 1998