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Letter

H Moment Oscillation and Three Fire Ball Model*

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Abstract The oscillatory behavior of H moments obtained from the experimental data in pp and p$\bar{\text{p}}$ collisions is studied in the Three-Fireball Model (TFM). It is found that similar to that in the negtive binomial distribution (NBD), the H moment in the full TFM distribution is a monotonic function of q, but oscillates when the distribution is truncated by a not too large value of n_{cut}. This shows that H moment oscillation is not a special property of pQCD or NBD, but is common for many truncated multiplicity distributions.

Key words Three-Fireball Model, Negative binomial distribution, H moment oscillation

In recent years the oscillation of H moments as the increasing of moment order q has extracted much attension. It is Dremin[1,2] who first suggested that the ratio H_q of factorial to factorial cumulent moment could be useful in the study of multiplicity distribution (MD). Using pQCD he found that as the moment order q increases H_q oscillates and changes sign, having the first negative minimum located at $q \approx 5$. This prediction has been surprisingly confirmed by experimental data[3]. So, there is an opinion that this oscillation can be regarded as a proof of Local Parton Hadron Duality (LPHD)[4].

On the other hand, it was shown in[5] that the oscillatory behavior of the H moment also appears when it is calculated from the truncated Negative Binomial Distribution. So, many related references discussed H moments in the framework of NBD, either modify NBD to fit the data better[6] or try to explain the oscillation by physical mechanism implied in NBD.

A question then arises: Is the oscillation of H moments really special for pQCD and/or NBD? In this letter we will show that this is not the case. The oscillation of

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H moments can be reproduced by the Three Fire Ball mode[7] as well. This shows that H moment oscillation is in fact a general property common to the model distributions that fit the experimental multiplicity distribution reasonably well.

The generating function $G(z)$ for the multiplicity distribution P_n is defined by

$$G(z) = \sum_{n=0}^{\infty} (1+z)^n P_n . \tag{1}$$

The factorial moments F_q are

$$F_q = \langle n^{|q|} \rangle = \sum_{n=0}^{\infty} P_n \cdot n(n-1)\cdots(n-q+1) \tag{2}$$

$$= \left. \frac{\partial^q G(z)}{\partial Z^q} \right|_{z=0} . \tag{3}$$

and the factorial cumulant moments K_q are defined by

$$G(z) = \exp \sum_q \frac{z^q}{q!} K_q . \tag{4}$$

or

$$K_q = \left. \frac{\partial^q \ln G(z)}{\partial z^q} \right|_{z=0} \tag{5}$$

They are related to F_q by the formula

$$F_q = \sum_{l=0}^{q-1} \binom{q-1}{l} K_{q-l} F_l \qquad (q \geqslant 1) \tag{6}$$

where $F_0 \equiv 1$, $K_0 \equiv 0$, $F_1 = K_1$ The H_q moment is defined by

$$H_q = K_q / F_q . \tag{7}$$

It should be noticed that only even multiplicities are allowed for charged particles, so $P_n = 0$ if n is odd, and that in real experiment only a finite number of particles are produced so the summation in the above equations must be truncated at some value of n.

$$\tilde{P}_n = \begin{cases} P_n & n < n_{\text{cut}} , \\ 0 & n > n_{\text{cut}} . \end{cases}$$

where P_n should be renormalized after truncation.

The higher order moments are affected by the truncation. The factorial moments change only a little, while the factorial cumulant moments in both NBD and TFM oscillate in sign as the order increases, as will be seen in the following.

For the NBD the multiplicity distribution is

$$P_n = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\left[\frac{\langle n\rangle}{k}\right]^n\left[1+\frac{\langle n\rangle}{k}\right]^{-n-k}. \tag{8}$$

For the TFM the multiplicity distribution is

$$\begin{aligned} P_n = {} & \frac{256/\langle n\rangle}{3(1-\alpha)^2(1-3\alpha)^2}\left[(z^3 - 6\beta^{-1}z^2 + 18\beta^{-2}z - 24\beta^{-3})\exp\left(-\frac{4z}{1-\alpha}\right)\right. \\ & \left. + 6(\beta^{-2}z + 4\beta^{-3})\exp(-2z/\alpha)\right] \qquad (|1-3\alpha| > 0.1). \end{aligned} \tag{9}$$

$$P_n = \frac{256/\langle n\rangle}{15\alpha^2(1-\alpha)^4}\left(z^5 - \frac{\beta}{3}z^6 + \frac{\beta^2}{14}z^7\right)\exp(-4z/(1-\alpha)) \qquad (|1-3\alpha| < 0.1) \tag{10}$$

where $z = \frac{n}{\langle n\rangle}$, $\beta = 2(1-3\alpha)/\alpha(1-\alpha)$. Both of them fit the experimental multiplicity distribution well.

The ratio H_q vs. the order q calculated from truncated TFM is shown in Fig. 1 together with the results from NBD and from the experimental data of hadronhadron collisions. The parameters used in the calculation are listed in Tables 1 and 2 respectively.

Table 1 The parameters of the NBD used in the analysis of the *H* moments in pp collisions and in p$\bar{\text{p}}$ collisions

NBD	$\sqrt{s}$/GeV	$\langle n\rangle$	k	n_{cut}
	30.4	10.7	11.0	26
	52.6	12.2	9.4	32
	62.2	13.6	8.2	38
	200	21.2	4.8	58
	546	28.3	3.7	100
	900	35.2	3.7	104

Table 2 The parameters of the TFM used in the analysis of the *H* moments in pp collisions and in p$\bar{\text{p}}$ collisions.

TFM	$\sqrt{s}$/GeV	$\langle n\rangle$	α	n_{cut}
	30.4	10.54	0.520	26
	52.6	12.76	0.540	32
	62.6	13.63	0.502	38
	200	21.6	0.646	58
	540	28.9	0.743	100
	900	34.6	0.770	104

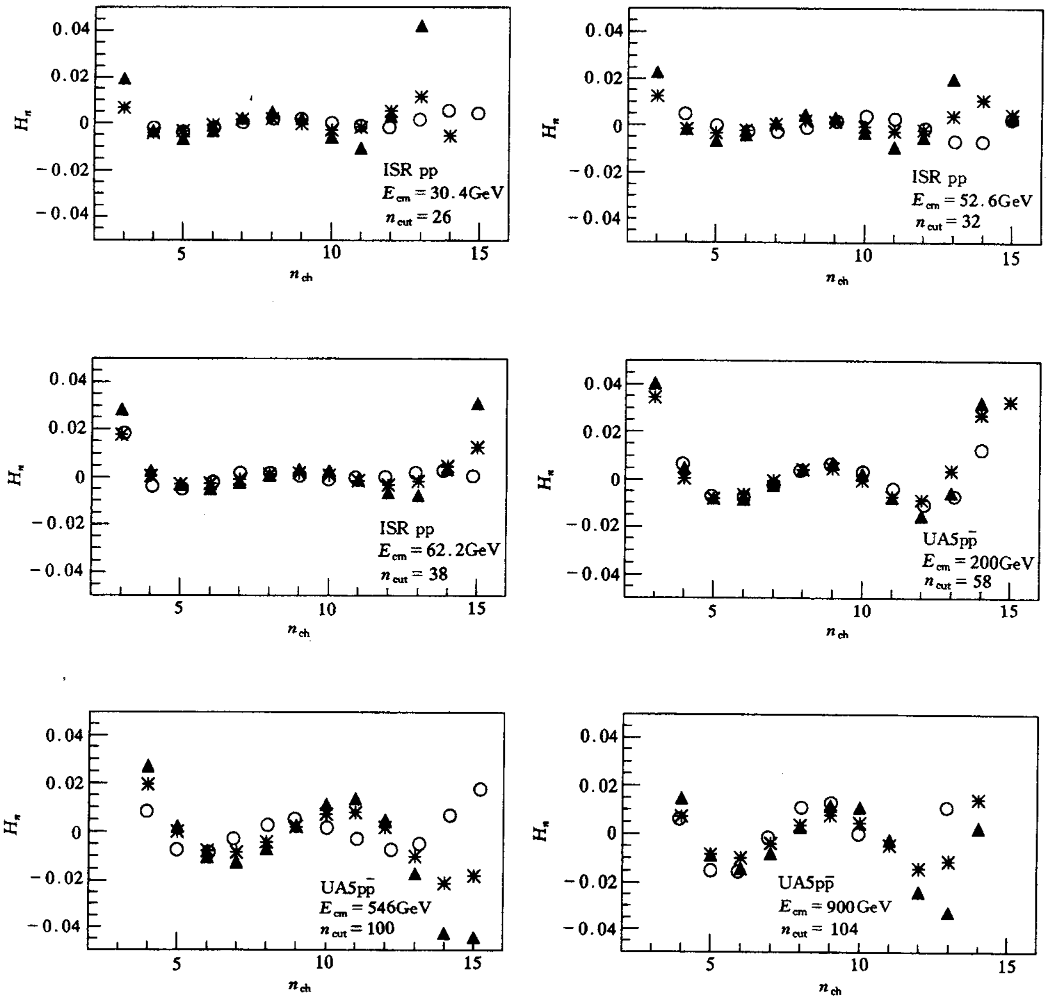

Fig. 1 The ratio H_q vs the order q for a collection of experiments in hadronhadron collisions. [3] The circles are the data points; the full triangles show the prediction of TFM and stars are calculated from NBD. Both fit well the MD' s, after the truncation effect taken into account.

○Exp. Data; ▲TFM; * NBD.

It can be clearly seen from the figures that the results obtained from TFM and NBD after taking into account the effect of truncation both possess the property of oscillation with approximately the same order of magnitude as the experimental data, especially when q is not very large. This shows that the oscillation of H moment is neither a special feature of NBD, nor the dynamical effects of QCD.

Let us remind that the cumulant moment of NBD calculated from the multiplicity generating function is positive decreasing monotonically as the rank increases, and one can get the H moment as a B function ($H_q = kB(q,k)$), which does not oscillate at all.

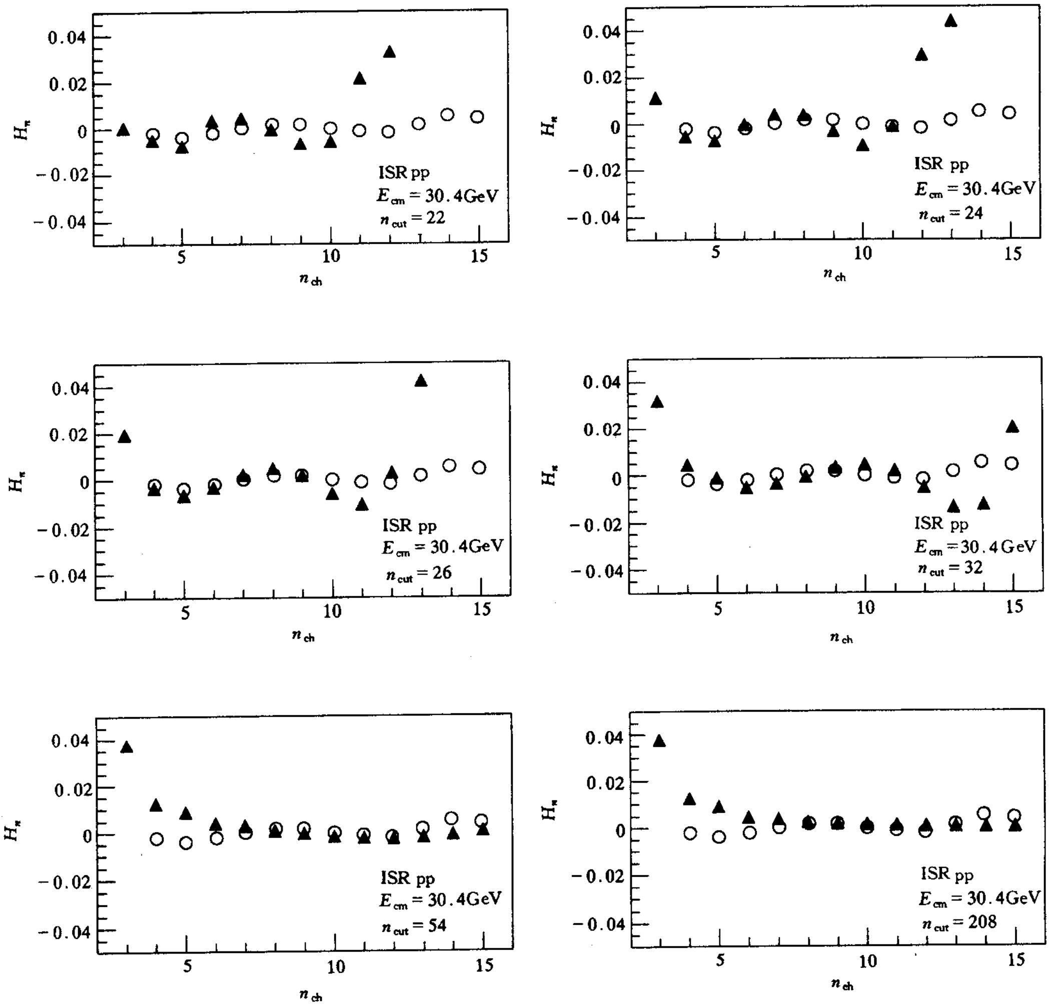

Fig. 2 The results of TFM (full triangles) for $\sqrt{s}$ = 30.4GeV with different truncations are shown together with the experimental data (circles) at the same energy.

○ Exp. Data; ▲TFM.

The oscillation of H moments in NBD is purely and effect of truncation.

TFM has the similiar character. This is shown in Fig. 2, where the results of TFM for $\sqrt{s}$ = 30.4GeV with different n_{cut} are presented together with the ISR pp collision data at the same c. m. energy. It can be seen that different truncation strongly effects the oscillation of H, and when the truncation is large enough oscillation disappears.

It can thus be concluded that when discussing H moment, TFM has similiar feature as NBD, and can also describe the experimental data very well. This means that the oscillation of H moment is not more characteristic for QCD or NBD than for TFM. Therefore, the reason for H moment oscillation can not be found just by studying QCD or NBD. It should be related to some common feature of the truncated multiplicity distributions.

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H矩振荡与三火球模型 *

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摘要 本文用三火球模型(TFM)研究了pp和$p\bar{p}$的实验数据中观察到的H矩振荡行为. 发现,和负二项式分布(NBD)相似,完整的TFM分布的H矩是q的单调函数,而当分布在一个不很大的n_{cut}值被截断时发生振荡. 这表明,H矩振荡不是pQCD或NBD的特殊性质,而是许多被截断的多重数分布的共同性质.

关键词 三火球模型 负二项式分布 H矩振荡

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