# Fission Angular Distributions for the Incomplete Fusion Reactions

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The fragment angular distributions for 35.9, 41.0, 45.6, and 50.0 MeV/u  $^{16}O + ^{197}Au$ , 50 MeV/u  $^{16}O + ^{159}Tb$ , and 50 MeV/u  $^{16}O + ^{209}Bi$  reactions have been measured by the solid state nuclear track detectors. After subtraction of the contribution of intermediate mass fragment (IMF), the fission angular distributions have been fitted by the transition state statistical model (TSM) and the single-spin standard theory to extract the spins of the fissioning nuclei. The relation between spins and bombarding energies is also discussed.

Key words: incomplete fusion fission, sequential fission, fission nuclei spin.

### 1. INTRODUCTION

To measure the fragment angular distributions in the experiments on the nuclear reactions induced by heavy ions is very important for study of nuclear reaction mechanisms. For many years, the theoretical and experimental studies of the fragment angular distributions have progressed extensively. The most experimental results of the fragment angular distributions including in the fission reactions induced by the light particles and the lighter heavy ions show that the measured fission angular distributions can be satisfactorily described by the transition state statistical model (TSM), when the projectile energy is less than 10 MeV/u [1]. However, in near-and sub-coulomb barrier fusion and fission reactions, the anomalous anisotropies of the fission angular distribution were observed [2]. After more heavy projectiles entered the experiments, e.g., in the  $^{32}S + ^{197}Au$ ,  $^{32}S + ^{232}Th$ ,  $^{32}S + ^{238}U$ , and  $^{32}S + ^{248}Cm$  systems, the measured fission angular distributions also exhibit the anomalous anisotropies [3]. These cannot be interpreted by the transition state statistical model. These anomalous anisotropies relate probably to some particular fission processes.

Received on September 17, 1996. Supported by the National Natural Science Foundation of China. 
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In 1980s, the study of the intermediate energy nuclear collisions became very interesting because of the transitional character in the intermediate energy region. It allows one to investigate how nuclear reaction mechanisms change between two extreme types of nuclear behavior. To date, the studies of the formation, the identification, and the decay properties of the highly excited nuclei have attracted much attention in this energy region. The experimental data on the fragment angular distributions produced in such collisions are useful to understand the features of the angular distributions and the reaction mechanisms of the fragments production. However, only few experimental data on the fragment angular distributions obtained in this energy region have been reported so far. The analysis of the fragment angular distributions becomes more difficult because of the coexistence of the complete fusion fission (CF), the incomplete fusion fission (ICF), and the sequence fission (SF) in the transition energy region.

So far, the data on the fragment angular distributions for the 86 MeV/u  $^{12}$ C+ $^{197}$ Au [4], 16 MeV/u  $^{32}$ S+ $^{197}$ Au [5], and 32, 44 MeV/u  $^{40}$ Ar+ $^{197}$ Au [6] reactions have been reported. These angular distributions of given fragment mass number were obtained in the range of  $4\pi$ -solid angle using off-line  $\gamma$ -ray spectroscopy technique. The correlation measurements of the fission partner emitted from the reaction of 5.5-21.7 MeV/u  $^{28}$ Si+ $^{197}$ Au in the larger angular range were measured using six double-grid position sensitive parallel plate avalanche counters (PPAC) with an effective area of 20  $\times$  25 cm² [7]. According to the distribution of linear momentum transfer, the events of the incomplete fusion fission and the sequential fission were distinguished. The angular distributions were fitted by the transition state model (TSM) and spins of fission nuclei corresponding to ICF and SF were extracted.

In this work, fragment angular distributions for the 35.9, 41.0, 45.6, and 50.0 MeV/u  $^{16}O + ^{197}Au$ , 50.0 MeV/u  $^{16}O + ^{159}Tb$ , 50.0 MeV/u  $^{16}O + ^{159}Tb$ , and 50.0 MeV/u  $^{16}O + ^{209}Bi$  reactions have been measured by the solid state nuclear track detectors. After the subtraction of the contribution of intermediate mass fragments (IMF), the fission angular distribution can be fitted by the transition state statistical model (TSM), and the single-spin standard theory to extract the spin of the fission nuclei. The relation between spins and bombarding energies is also discussed.

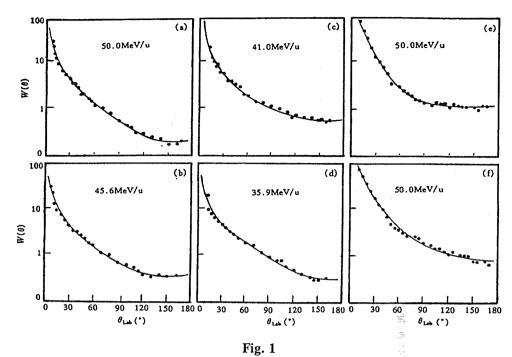
## 2. EXPERIMENTAL PROCEDURE

The experiment was carried out at the Heavy Ion Research Facility, Lanzhou (HIRFL). The energy of  $^{16}$ O ion beam was 50.0 MeV/u. A cylindrical scattering chamber with 20 cm in diameter and 340 cm in length was connected to the tube of the accelerator. The beam passed through a collimator in which were fixed two apertures made of graphite with diameter of 6 mm, and then entered the scattering chamber. The thicknesses of  $^{159}$ Tb,  $^{197}$ Au, and  $^{209}$ Bi were 250.0, 275.3, and 250.0  $\mu$ g/cm², respectively. The target was placed at the center of the scattering chamber and at 45° with respect to the beam direction. A Faraday cup installed at the scattering chamber terminal was connected with a current integrator to measure the beam intensities passing through the target. A cylinder with the diameter of 17.0 cm was fixed at the center of the scattering chamber, and the fragments leaving the target were recorded by the mica track detectors stuck on cylinder's inner wall. The distance from the mica detectors to the center of target was 8.5 cm. After irradiation, the mica detectors were etched with 48% HF at 50°C for 45 minutes. The fragment tracks exhibited the regular diamond shape with the diagonal length of 15  $\mu$ m. The scanning of the fragment tracks was performed with an optical microscope at a total magnification of 40 × 12.5. Thus, the fragment angular distributions for the above three reactions as a function of the laboratory angle in degrees were obtained.

### 3. EXPERIMENTAL RESULTS AND DISCUSSION

### 3.1. Fragment angular distributions

The fragment angular distributions in a laboratory system for 35.9, 41.0, 45.6, and 50.0 MeV/u  $^{16}O + ^{197}Au$ , 50.0 MeV/u  $^{16}O + ^{159}Tb$ , and 50.0 MeV  $^{16}O + ^{209}Bi$  reactions are plotted in Fig. 1. The solid



The fragment angular distributions in the laboratory system for the  $^{16}O + ^{197}Au$  (a-d),  $^{16}O + ^{159}Tb$  (e), and  $^{16}O + ^{209}Bi$  (f) reactions. The solid line connecting the data points show the shape of the distribution.

lines connecting the data points show the shape of the angular distribution. It can be seen from Fig. 1 that the fragment angular distributions in the laboratory system for the above three reactions have similar shape and they are strongly forward peaked. When  $\theta_{Lab} < 30^{\circ}$  the differential cross section is larger, and it decreases rapidly according to the exponent law with increasing angle. The differential cross section decreases slowly when  $\theta_{Lab} > 90^{\circ}$  and it is much less than those in the range of  $\theta_{Lab} < 90^{\circ}$ . The angular distributions in the center of mass system for the  $^{16}O + ^{159}Tb$ ,  $^{16}O + ^{197}Au$ , and  $^{16}O + ^{209}Bi$  reactions are given in Fig. 2. These angular distributions exhibit similar shape and they are asymmetric about  $\theta_{cm} = 90^{\circ}$ . These is a larger differential cross section in  $\theta_{cm} < 30^{\circ}$  region. The anisotropies of angular distributions for the  $^{16}O + ^{197}Au$  reaction increases a little with the bombarding energy increasing.

## 3.2. Theoretical model analysis of fragment angular distribution

It is learned from our previous work [8] that we can observe two types of fragments, i.e., the intermediate mass fragment and the fission fragment. The fission fragments included the contributions from the incomplete fusion fission and the sequential fission reactions, and the angular distributions in the center of mass system for ICF and SF are symmetrical at about  $\theta_{\rm cm}=90^{\circ}$ . Thus, according to the properties of the fission angular distributions, the angular distributions given in Fig. 2 can be analyzed. After subtraction of the contribution of the intermediate mass fragment, the fission angular distributions normalized by the differential cross section at  $\theta=90^{\circ}$  are shown as the experimental points in Fig. 3.

The sequential fission corresponds to the peripheral collision. The spin of fission nuclei is mainly due to the contribution of the partial wave in the grazing angular momentum zone of the entrance

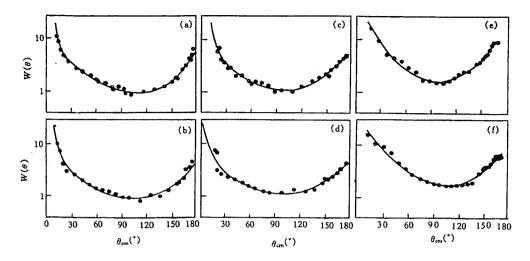


Fig. 2

The fragment angular distributions in the center of mass system for the  $^{16}O + ^{197}Au$  (a-d),  $^{16}O + ^{159}Tb$  (e), and  $^{16}O + ^{209}Bi$  (f) reactions. The solid line connecting the data points show the shape of the distribution.

channel. The angular distribution for SF can be calculated by the single-spin standard theory. We assume the parent nuclei of sequential fission are formed by capturing a partial projectile at the position of the sharp radius of the target nuclei. The spin of nuclei for SF can be written as

$$I_{\rm SF} = 0.2187 A_{\rm t}^{1/3} \langle lmt \rangle_{\rm SF} A_{\rm p} (E_{\rm Lab} / A_{\rm p})^{1/2}. \tag{1}$$

The angular distribution of the sequential fission reaction is given by the following expression,

$$\frac{W(\theta)}{W(90^{\circ})} = \frac{\exp(x\cos^2\theta) J_0(ix\sin^2\theta)}{J_0(ix)},$$
(2)

where  $x = [(I_{SF} + 1/2)/2K_0]^2$ .

The angular distribution of the incomplete fusion fission reaction can be calculated by the transition state statistical model (TSM) [9],

$$W(\theta) = \sum_{I=0}^{I_m} \frac{(2I+1)^2 T_1 \exp[-(I+1/2)^2 \sin^2\theta / 4K_0^2] J_0[i(I+1/2)^2 \sin^2\theta / 4K_0^2]}{\exp[(I+1/2)/(2K_0^2)^{1/2}]},$$
 (3)

where  $T_1$  is the transmission coefficient, we take  $T_1 = 1$  in the intermediate energy region,  $\operatorname{erf}(x)$  is the error function defined by  $\operatorname{erf}(x) = (2/\pi^{1/2}) \int_0^x \exp(-t^2) dt$ , and  $J_0$  is the zero order Bessel function with imaginary argument.

$$K_0^2 = T(I) \mathcal{J}_{\text{eff}} / \hbar^2, \tag{4}$$

where  $\mathcal{J}_{\text{eff}}$  the effective moment of inertia of fission nuclei at the saddle point,  $\mathcal{J}_{\text{eff}}^{-1} = \mathcal{J}_{\parallel}^{-1} - \mathcal{J}_{\perp}^{-1}$ ,  $\mathcal{J}_{\parallel}$  and  $\mathcal{J}_{\perp}$  are the moments of inertia parallel and perpendicular to the nuclear symmetry axis (z-axis).

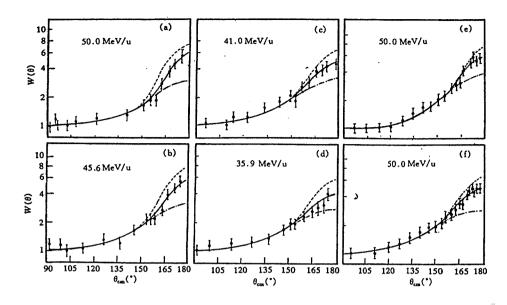


Fig. 3
The fission angular distributions in the center of mass system for the  $^{16}O + ^{197}Au$  (a-d),  $^{16}O + ^{159}Tb$  (e), and  $^{16}O + ^{209}Bi$  (f) ICF and SF reactions. —— ICF+CF; ---- ICF; - · - SF.

They can be calculated by the Cassinian ovaloids [10]. T(I) is the temperature of the fissioning nuclei at the saddle point,

$$T(I) = [(E_{cm} + Q - E_{rot} - B_f) / a]^{1/2},$$
 (5)

For the incomplete fusion reaction,  $E_{\rm cm}=E_{\rm Lab}\langle lmt\rangle A_{\rm p}/A_{\rm CN}$ ,  $A_{\rm CN}=\langle lmt\rangle A_{\rm p}+A_{\rm t}$ ,  $A_{\rm p}$  and  $A_{\rm t}$  are the mass number of the projectile and the target nuclei of the entrance channel, respectively. Q is the thermal energy of fusion reaction,  $B_{\rm f}$  is the effective fission potential barrier height,  $E_{\rm rot}$  is the rotating energy of fissioning nuclei at the saddle point calculated by the formula,  $E_{\rm rot}=\hbar^2 I(I+1)/2\mathcal{J}_1$ , the energy level parameter is taken as  $a=A_{\rm CN}/8.5$ . Thus, the best fit between the sum of angular distribution

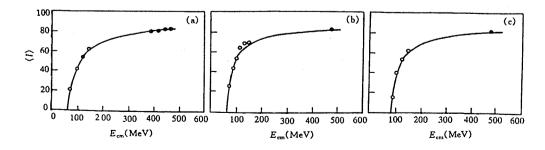


Fig. 4
The average spin of fissioning nuclei  $\langle I \rangle$  for the  $^{16}O + ^{197}Au$  (a-d),  $^{16}O + ^{159}Tb$  (e), and  $^{16}O + ^{209}Bi$  (f) reactions as a function of the center of mass energies.

•: This work;  $\circ$ : data taken from Ref. [11].

Table 1										
The average spin of fissioning nuclei $\langle I \rangle$ and the various										
parameters fitting fission angular distributions.										

Interaction	E <sub>Lab</sub> (MeV/u)		Acn	(lmt)	Q	Jo/Leff	Κ'n	Т	I <sub>m</sub>	$\langle 1 \rangle$
system  16O+197Au					(MeV)			(MeV)	(ħ)	(h)
	ICF	50.0	207	0.622	-1.97	1.267	18.54	4.19	126	84
	ICF	45.6	207	0.651	-1.97	1.436	17.49	4.18	121	80
	ICF	41.0	208	0.684	-5.03	1.399	17.32	4.09	115	76
	ICF	35.9	209	0.722	-18.12	1.388	16.29	3.89	110	74
	SF	50.0	201	0.25	-1.48	1.546	13.12	2.71	36	24
	SF	45.6	201	0.25	-1.48	1.546	13.01	2.64	34	22
	SF	41.0	201	0.25	-1.48	1.546	12.47	2.41	33	22
	SF	35.9	201	0.25	-1.48	1.546	11.99	2.21	30	20
<sup>16</sup> O+ <sup>159</sup> Tb	ICF	50.0	160	0.622	2.89	1.68	14.16	4.39	110	73
	SF	50.0	163	0.25	-0.74	1.761	10.63	2.89	33	22
<sup>16</sup> O+ <sup>209</sup> Bi	ICF	50.0	219	0.25	-15.5	1.289	19.01	4.05	120	80
	SF	50.0	213	0.25	-9.25	1.544	13.89	2.73	37	25

of ICF and SF calculated by Eqs.(2) and (3) and experimental one can be obtained by adjusting parameter  $I_{\rm m}$ . The average spin of the fissioning nuclei for the incomplete fusion reaction  $\langle I \rangle = (2/3)I_{\rm m}$ . The values of various parameters fitting angular distributions, the extracted parameter  $I_{\rm m}$  and the values for the average spins of fissioning nuclei  $\langle I \rangle$  are all given in Table 1.

The values of the average spin of fissioning nuclei for ICF reactions extracted from experimental angular distributions for the  $^{16}O + ^{197}Au$  reaction as a function of the center of mass energies are given in Fig. 4. The values of  $\langle I \rangle$  for CF are also plotted in this figure [11]. It can be seen from Fig. 4 that the complete fusion reaction takes place when  $E_{cm} < 150$  MeV, and the value for the average spin of fission nuclei increases rapidly with the slope of 0.70  $\hbar$ /MeV. The incomplete fusion reaction occurs when  $E_{cm} > 150$  MeV, and its possession proportion becomes larger and larger with increasing bombarding energy. The spin of fission nuclei increases very slowly with the slope of 0.05  $\hbar$ /MeV. The trend of the experimental data for the  $^{16}O + ^{159}$ Tb and  $^{16}O + ^{209}$ Bi reactions is consistent with the result of the  $^{16}O + ^{197}$ Au reaction, though there is only one experimental point for each reaction when  $E_{cm} > 150$  MeV. Our results are also in good agreement with those obtained by Dai *et al.* [7]. It can be also seen from Fig. 4 that the average spin  $\langle I \rangle$  of fissioning nuclei for the incomplete fusion reaction approaches gradually a saturation value when  $E_{cm} > 200$  MeV, and it is about 70-80  $\hbar$  for three reactions studies by the present work. The experimental results show that the incomplete fusion reaction loses not only some linear momentum but also some angular momentum, and therefore it results in the decrease of the values for the spin of fastening nuclei.

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