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用玻色化顶点算子实现与 Z_n 对称的 Belavin R 矩阵相关的 Z–F代数

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摘 要

根据 Asai, Jimbo, Miwa 和 Pugai 给出的玻色化的 $A_{n-1}^{(1)}$ 面模型的顶点算子，用面顶点对应关系，得到了与 Z_n 对称的 Belavin R矩阵相关的玻色化顶点算子，它们实现了与这种 R矩阵对应的 Zamolodchikov – Fadeev 代数.

关键词 玻色化，顶点算子，Z–F代数，Belavin R矩阵.

1 引 言

计算统计模型的多点函数的一种方便的办法是利用与它的玻尔兹曼权对应的玻色化顶点算子[1—3]，这种顶点算子满足以这种模型的R矩阵为结构常数的 Zamolodchkov、Fadeev 代数[4—7]. 用这种方法，可以算出重新规一化的多点函数的积分表达式.

继三角型顶点算子[1—3]之后，由 Lukyanov 和 Pugai[8]给出了与 ABF[9]模型 R矩阵相关的玻色化顶点算子，Miwa 等还给出了与之对应的反射边玻色算子[10]. 最近 Asai, Jimbo, Miwa 和 Pugai 给出了 $A_{n-1}^{(1)}$面模型[11]顶点算子的玻色化方法[12]，这些漂亮的工作将使椭圆型面统计模型热力学函数的计算大大地推进.

在他们工作的基础上，我们用面顶点对应关系[13—15]得到了与 Z_n Belavin 模型 R矩阵相关的顶点算子——这些算子的交换关系组成的 Z–F代数以这种 R矩阵为结构常数.

虽然在一般意义上，$A_{n-1}^{(1)}$面模型与 Z_n Belavin 模型是等价的，然而直接给出与后者相关的顶点算子仍然有其作用，首先是不必用面权，省去了面模型中的一些复杂问题，其次是，Z_n Belavin 模型本身与几种物理上有实际应用的模型有更密切的关系，比如 T–J 模型等. 因此，我们推导这种顶点算子可能对这方面有益.

在下文，首先找出一种 intertwiner, 它能建立顶点 R矩阵与文献 [12] 中的玻尔兹曼权 W之间的面顶对应. 然后，介绍面模型的顶点算子和交换关系，最后，把 intertwiner 和他们的顶点算子结合起来得到我们要求的顶点算子.

2　顶面对应关系

给定整数$n>0$, $\mathrm{Im}\tau>0$, 任意$w\in C$, 可以构造Z_n对称的R矩阵[16,17]，定义$n\times n$矩阵，g, h, I_α:

$$g_{jk}=\omega^j\delta_{jk},\ h_{jk}=\delta_{j+1,k},\ \omega=\exp\left(\frac{2\pi\mathrm{i}}{n}\right),\ I_\alpha=I_{(\alpha_1,\alpha_2)}=g^{\alpha_2}h^{\alpha_1},\ (\alpha_1,\alpha_2)\in Z_n^2.$$

定义$I_\alpha^{(j)}=I\otimes I\otimes\cdots\otimes I_\alpha\otimes I\otimes\cdots\otimes I$, I_α在第j个空间，

$$W_\alpha(z)=\frac{1}{n}\theta\begin{bmatrix}\frac{1}{2}+\frac{\alpha_1}{n}\\ \frac{1}{2}+\frac{\alpha_2}{n}\end{bmatrix}\left(z+\frac{w}{n},\tau\right)\Bigg/\theta\begin{bmatrix}\frac{1}{2}+\frac{\alpha_1}{n}\\ \frac{1}{2}+\frac{\alpha_2}{n}\end{bmatrix}\left(\frac{w}{n},\tau\right)=\frac{\sigma_\alpha\left(z+\frac{w}{n}\right)}{\sigma_\alpha\left(\frac{w}{n}\right)},$$

$$\theta\begin{bmatrix}a\\ b\end{bmatrix}(z,\ \tau)=\sum_{m\in z}\exp\{\mathrm{i}\pi(m+a)[(m+a)\tau+2(z+b)]\},$$

$$\sigma_\alpha(z)=\theta\begin{bmatrix}\frac{1}{2}+\frac{\alpha_1}{n}\\ \frac{1}{2}+\frac{\alpha_2}{n}\end{bmatrix}(z,\tau)\ ,$$

Z_n对称的 Belavin R 矩阵就是

$$R_{jk}(z)=\sum_{\alpha\in Z_n^2}W_\alpha(z)I_\alpha^{(j)}(I_\alpha^{-1})^{(k)}\ . \tag{1}$$

它满足 YBE:

$$R_{12}(z_1-z_2)R_{13}(z_1)R_{23}(z_2)=R_{23}(z_2)R_{13}(z_1)R_{13}(z_1-z_2)\ .$$

给定矢量$a\in Z^n$，约定基矢为 $e_\mu=\underbrace{(0,\ 0,\ \cdots,\ 1,\ 0,\ \cdots,\ 0)}_{\text{第}\mu\text{个位置}}$可以定义$A_{n-1}^{(1)}$的玻尔兹曼权[11]

$$W(a/z)_{\mu\mu}^{\mu\mu}=\frac{\sigma_0(z+w)}{\sigma_0(w)}\ ,$$

$$\left.\begin{aligned}W(a/z)_{\mu\nu}^{\nu\mu}&=\frac{\sigma_0(z+a_{\mu\nu}w)}{\sigma_0(a_{\mu\nu}w)}\\ W_z(a/z)_{\mu\nu}^{\mu\nu}&=\frac{\sigma_0(z)\,\sigma_0(a_{\mu\nu}w-w)}{\sigma_0(w)\,\sigma_0(a_{\mu\nu}w)}\end{aligned}\right\}\ \mu\neq\nu\ , \tag{2}$$

其余情形下，$W(a|z)_{\mu\nu}^{\mu'\nu'}=0$. 其中

$$a=(a_1,\ a_2,\ \cdots,\ a_n),\ \bar{a}_\mu=a_\mu-\frac{1}{n}\sum_{l=0}^{n-1}a_l+w_j,$$

w_j是n个一般的复数，$a_{\mu\nu}=\bar{a}_\mu-\bar{a}_\nu$.

再定义顶面对应的 intertwiner

$$\varphi^{(k)}_{\mu,a}(z) = \theta^{(k)}(z + nw\bar{a}_\mu) , \tag{3}$$

$$\theta^{(j)}(u) = \theta\begin{bmatrix}\frac{1}{2}-\frac{j}{n}\\ \frac{1}{2}\end{bmatrix}(u, n\tau) ,$$

这样，顶面对应关系就是

$$R(z_1 - z_2)\varphi_{\mu, a+e_\nu}(z_1)\otimes\varphi_{\nu, a}(z_2) = \sum_{\mu'\nu'} W(a|z_1 - z_2)^{\mu\nu}_{\mu'\nu'}\varphi_{\mu', a}(z_1)\otimes\varphi_{\nu', a+e_{\mu'}}(z_2). \tag{4}$$

还可以定义行矢量$\tilde{\varphi}_{\mu,a}(z)$，使下式成立:

$$\sum_k \tilde{\varphi}^{(k)}_{\mu,a}(z) \ \ \varphi^{(k)}_{\nu,a}(z) = \delta_{\mu\nu} , \tag{5}$$

这样，就有

$$\sum_\mu \varphi_{\mu, a}(z) \ \ \tilde{\varphi}_{\mu,a}(z) = I , \tag{6}$$

I是 $n\times n$ 单位矩阵.

将(4)式右乘$\tilde{\varphi}_{\mu,a+e_\nu}(z_1)\otimes\tilde{\varphi}_{\nu,a}(z_2)$，左乘$\tilde{\varphi}_{\mu'',a}(z_1)\otimes\tilde{\varphi}_{\nu'', a+e_{\mu''}}(z_2)$，对$\mu$，$\nu$求和，(4)式左边变为

$$\tilde{\varphi}_{\dots}\otimes\tilde{\varphi}_{\dots}R(z_1 - z_2) \ \ I\otimes I = \tilde{\varphi}_{\dots}\otimes\tilde{\varphi}_{\dots}R(z_1 - z_2) ,$$

右边变为

$$\sum_{\substack{\mu'\nu'\\ \mu\nu}} W(a|z_1 - z_2)^{\mu\nu}_{\mu'\nu'} \ \ \delta_{\mu'\mu''}\tilde{\varphi}_{\nu'', a+e_{\mu''}}(z_2)\varphi_{\nu', a+e_{\mu'}}(z_2)\times\tilde{\varphi}_{\dots}\tilde{\otimes}\varphi_{\dots}$$

$$= \sum_{\substack{\nu'\\ \mu\nu}} W(a|z_1 - z_2)^{\mu\nu}_{\mu''\nu'} \underbrace{\tilde{\varphi}_{\nu'',a+e_{\mu''}}(z_2)\varphi_{\nu',a+e_{\mu''}}(z_2)}_{\delta_{\nu'\nu''}}\times\cdots$$

$$= \sum_{\mu\nu} W(a|z_1 - z_2)^{\mu\nu}_{\mu''\nu''}\times\tilde{\varphi}_{\mu,a+e_\nu}(z_1)\otimes\tilde{\varphi}_{\nu,a}(z_2)$$

$$= 左边 = \tilde{\varphi}_{\mu'',a}(z_1)\otimes\tilde{\varphi}_{\nu'',a+e_{\mu''}}(z_2)R(z_1 - z_2). \tag{7}$$

3 修改的顶面对应关系

(7)式中的面玻尔兹曼权并不是 Asai 等人在玻色化顶点算子的交换关系中的那一个，为了能应用他们的结果[12]，需要修改 R矩阵和$\tilde{\varphi}$.

首先令

$$R'(z) = f(z)R(z) ,$$

$$f(z) = r_1\left(-\frac{z}{w}\right)\frac{\sigma_0(w)}{\sigma_0(w+z)} ,$$

其中

$$r_1(v) = y^{\frac{r-1}{r}\frac{n-1}{n}}\frac{g_1(y^{-1})}{g_1(y)}, \quad y = x^{2v}, \quad x = \mathrm{e}^{\pi i w},$$

$$g_1(y) = \{x^2 y\}\{x^{2r+2n-2}y\} / \{x^{2r}y\}\{x^{2n}y\},$$

$$r = \frac{\tau}{w},$$

$$\{u\} = \prod_{k,j=0}^{\infty}(1 - x^{2rk+2nj}u),$$

然后，修改$\tilde{\varphi}$，令

$$\tilde{\varphi}'_{\mu,a}(z) = \lambda(\mu, a, z)\tilde{\varphi}_{\mu,a}(z),$$

$$\lambda(\mu, a, z) = \mathrm{e}^{\pi i \frac{\omega}{r}\left[n a_\mu^{-2} + 2\bar{a}_\mu \frac{z}{\omega} - 2\left(1 - \frac{1}{n}\right)\sum_l a_l \frac{z}{\omega}\right]},$$

这样，就有修改了的顶面对应关系

$$\tilde{\varphi}'_{\mu',a}(z_1)\otimes\tilde{\varphi}'_{\nu', a+e_{\mu'}}(z_2)R'(z_1 - z_2) = \sum_{\mu\nu} W'(a|z_1 - z_2)^{\mu\nu}_{\mu'\nu'}\tilde{\varphi}'_{\mu, a+e_\nu}(z_1)\otimes\tilde{\varphi}'_{\nu, a}(z_2), \tag{8}$$

其中

$$W'(a|z)^{\mu\mu}_{\mu\mu} = 1,$$

$$\left.\begin{aligned} W'(a|z)^{\mu\nu}_{\mu\nu} &= r_1\left(-\frac{z}{\omega}\right)\mathrm{e}^{\pi i \frac{w}{r}\left(-2a_{\mu\nu} - 2\frac{z}{w}\right)} \times \frac{\sigma_0(-z)\sigma_0(a_{\mu\nu}w - w)}{\sigma_0(-z-w)\sigma_0(a_{\mu\nu}w)} \\ W'(a|z)^{\mu\nu}_{\nu\mu} &= r_1\left(-\frac{z}{\omega}\right)\mathrm{e}^{\pi i \frac{w}{r}(a_{\mu\nu} - 1)\left(-\frac{z}{w}\right)} \times \frac{\sigma_0(-z-a_{\mu\nu}w)\sigma_0(w)}{\sigma_0(-z-w)\sigma_0(a_{\mu\nu}w)} \end{aligned}\right\} \mu \neq \nu. \tag{9}$$

令 $[v] = \mathrm{e}^{\pi i \frac{w}{r} v^2}\sigma_0(wv)$，就有

$$W'(a|-wv)^{\mu\mu}_{\mu\mu} = 1,$$

$$W'(a|-wv)^{\mu\nu}_{\mu\nu} = r_1(v)\frac{[v][a_{\mu\nu} - 1]}{[v-1][a_{\mu\nu}]},$$

$$W'(a|-wv)^{\mu\nu}_{\nu\mu} = r_1(v)\frac{[v - a_{\mu\nu}][1]}{[v-1][a_{\mu\nu}]}. \tag{10}$$

与文献 [12] 一致.

4　面模型中的顶点算子

这节介绍文献 [12] 中的顶点算子，根据文献 [12]，规定玻色子$\beta^j_m(1\leqslant j\leqslant n-1,\ m\in z\setminus\{0\})$，要求 $r = \frac{\tau}{w}$ 为大于 $n+1$ 的整数，

$$[\beta^j_m, \beta^k_{m'}] = m\frac{[(n-1)m]_x[(r-1)m]_x}{[nm]_x\ [rm]_x}\delta_{m+m',0}\ (j = k),$$

$$= -mx^{sgn(j-k)nm}\frac{[m]_x[(r-1)m]_x}{[nm]_x\ \ [rm]_x}\ \delta_{m+m',0}\ (j \neq k)\ , \tag{11}$$

$$[a]_x = (x^a - x^{-a})/(x - x^{-1}),\ x = \mathrm{e}^{\pi i w}\ ,$$

β^n_m由$\sum_{j=1}^{n} x^{-2jm}\beta^j_m = 0$ 确定．再考虑p_μ, q_μ满足$\mu = 1, \cdots, n$,

$$[\mathrm{i}p_\mu,\ q_\nu] = \delta_{\mu\nu}\ . \tag{12}$$

考虑正交归一基矢$\{e_\mu\}$，满足$<e_\mu, e_\nu> = \delta_{\mu\nu}$, $\bar{e}_\mu = e_\mu - \frac{1}{n}\sum_l e_l$, $P_\mu = p_\mu - \frac{1}{n}\sum_l p_l + \frac{1}{\sqrt{r(r-1)}}$ w'_μ ,再令真空为$|0\rangle$, $p_\mu|0\rangle = 0$, 当$m > 0$时，$\beta^j_m|0\rangle = 0$,

$$|l,\ k\rangle = \mathrm{e}^{\mathrm{i}\sqrt{\frac{r}{r-1}}Q_l - \mathrm{i}\sqrt{\frac{r-1}{r}}Q_k}|0\rangle,$$

$l = \sum l_\mu e_\mu$, $Q_l = \sum l_\mu q_\mu$, $k = \sum k_\mu e_\mu$, $Q_k = \sum k_\mu q_\mu$, $F_{lk} = C[\{\beta^j_{-1}, \beta^j_{-2}, \cdots\}_{1\leqslant j\leqslant n}]\ |l,\ k\rangle$.

规定$\hat{a}_\mu = -\sqrt{\frac{r}{r-1}}\ p_\mu$, $\hat{\bar{a}}_\mu = \hat{a}_\mu - \frac{1}{n}\sum_l \hat{a}_l + w''_\mu$, 则

$$\hat{a}_\mu \mathrm{e}^{\mp\mathrm{i}\sqrt{\frac{r-1}{r}}q_\nu} = \mathrm{e}^{\mp\mathrm{i}\sqrt{\frac{r-1}{r}}q_\nu}(\hat{a}_\mu \pm \delta_{\mu\nu}), \tag{13}$$

$$\hat{a}_\mu|lk\rangle = [\langle e_\mu,\ \left(-\frac{r}{r-1}\right)l\rangle + k_\mu]|l,\ k\rangle.$$

在P_μ和$\hat{\bar{a}}_\mu$定义式中，$\{w'_\mu\}$和$\{w''_\mu\}$是两组一般的复数，需满足．

$$w''_\mu - w'_\mu = l_\mu\ r^2/(r-1) + c\ , \tag{14}$$

其中c是与μ无关的任意常数．

按文献[12]定义，对$j = 1, \cdots, n-1$,

$$\xi_j(v) = \mathrm{e}^{\mathrm{i}\sqrt{\frac{r-1}{r}}[(q_j - q_{j+1}) - \mathrm{i}(p_j - p_{j+1})\log y]}\ \vdots\mathrm{e}^{\sum_{m\neq 0}\frac{1}{m}(j^j_m - j^{j+1}_m)(x^j y)^{-m}}\ ,$$

$$\eta_j(v) = \mathrm{e}^{-\mathrm{i}\sqrt{\frac{r-1}{r}}\left(\sum_{v=1}^{j} q_v - \mathrm{i}\left(\sum_{v=l}^{j} P_v\right)\log y\right)}\vdots\mathrm{e}^{-\sum_{m\neq 0}\frac{1}{m}\sum_{k=1}^{j} x^{(j-2h+1)m}\beta^k_m y^{-m}}\ ,$$

$$x = \mathrm{e}^{\pi i w}\ ,$$

$$y = x^{2v} = \mathrm{e}^{-2\pi i z},$$

按照文献[12]，引入$\pi_\mu = \sqrt{r(r-1)}\ P_\mu$, $\pi_{\mu\nu} = \pi_\mu - \pi_\nu$, 引入顶点算子：

$$\Phi_\mu(v) = \oint \prod_{j=1}^{\mu-1}\frac{\mathrm{d}y_j}{2\pi \mathrm{i} y_j}\ \ \eta_1(v)\xi_1(v_1)\cdots\xi_{\mu-1}\ (v_{\mu-1}) \times \prod_{j=1}^{\mu-1} f(v_j - v_{j-1},\ \pi_{j\mu})\ , \tag{15}$$

$v_0 = v$, $y_j = x^{2v_j}$, 积分路线是满足$x|y_{j-1}| < |y_j| < x^{-1}|y_{j-1}|$的围线原点的简单闭曲线，就会有[12]

$$\Phi_\mu(v_1)\ \Phi_\nu(v_2) = \sum_{\mu'\nu'} \Phi_{\nu'}(v_2)\Phi_{\mu'}\ (v_1) W''(a|v_1 - v_2)^{\ \mu\nu}_{\ \mu'\nu'}\ , \tag{16}$$

其中

$$W''(a|v)_{\mu\nu}^{\mu'\nu'} = W'(a|z)_{\mu\nu}^{\mu'\nu'} .$$

也就是

$$\Phi_\mu\left(-\frac{z_1}{w}\right)\Phi_\nu\left(-\frac{z_2}{w}\right) = \sum_{\mu'\nu'}\Phi_{\nu'}\left(-\frac{z_2}{w}\right)\Phi_{\mu'}\left(-\frac{z_1}{w}\right)W'(a|z_1-z_2)_{\mu'\nu'}^{\mu\nu} 。\tag{17}$$

在(17)式中，玻尔兹曼权 W''中是用算符$\pi_{\mu\nu}$代表(10)式中的 $a_{\mu\nu}$的，由于$\pi_\mu F_{l,k} = [\langle \bar{e}_\mu, rl-(r-1)k\rangle, + w_\mu']F_{lk}$, $\pi_{\mu\nu} F_{l,k} = [\langle e_\mu - e_\nu, rl-(r-1)k\rangle + w_\mu' - w_\nu']F_{l,k}$，令 $\hat{a}_{\mu\nu} = \hat{a}_\mu - \hat{a}_\nu$，

$$\hat{a}_{\mu\nu} F_{lk} = [\langle e_\mu - e_\nu , \frac{-r}{r-1}\ l + k\rangle + w_\mu'' - w_\nu'']F_{lk} ,$$

由(14)式，有

$$\begin{aligned}&\langle e_\mu - e_\nu , rl\rangle + w_\mu' - w_\nu' \\ &= \langle e_\mu - e_\nu , \frac{-r}{r-1}\ l\rangle + w_\mu'' - w_\nu'' ,\end{aligned}\tag{18}$$

又由于 $[v+r] = -[v]$，而作用在 F_{lk}上 $(\pi_{\mu\nu} - a_{\mu\nu}) F_{lk} = \underbrace{\langle e_\mu - e_\nu , rk\rangle F_{lk}}_{\text{整数}\times r}$．于是用$\pi_{\mu\nu}$和算符 $a_{\mu\nu}$构成的(见(10)式)玻尔兹曼权 $W'(a|-wv)$就相同.注意，由(13)式，(15)式，就会有

$$\hat{a}_\mu \Phi_\nu(v) = \Phi_\nu(\hat{a}_\mu + \delta_{\mu\nu}) .\tag{19}$$

5　对应于Z_n对称R矩阵的顶点算子

由(8)和(17)，可以得到对应于R矩阵的顶点算子.

将(17)式右乘$\tilde{\Phi}'_{\mu,a+e_\nu}(z_1+\delta)\otimes\tilde{\Phi}'_{\nu,a}(z_2+\delta)$对$\mu\nu$求和，(其中，$a_\mu, \bar{a}_\mu$是(13)式中的算子，但在 F_{lk}左边它们都是普通数，所以运算有意义)这样得到:

$$\text{方程左边} = \sum_{\mu\nu}\Phi_\mu(v_1)\Phi_\nu(v_2)\ \tilde{\Phi}'_{\mu,a+e_\nu}\ (z_1+\delta)\otimes\tilde{\Phi}'_{\nu,a}(z_2+\delta)$$

$$= \sum_\mu \Phi_\mu(v_1)\tilde{\Phi}'_{\mu,a}(z_1+\delta)\otimes\sum_\nu\Phi_\nu\tilde{\Phi}'_{\nu,a}(z_2+\delta) ,$$

$$\text{方程右边} = \sum_{\mu'\nu'}\Phi_{\nu'}(v_2)\Phi_{\mu'}(v_1)\tilde{\Phi}'_{\mu,a}(z_1+\delta)\otimes\tilde{\Phi}'_{\nu',a+e_{\mu'}}\ (z_2+\delta)\ R'(z_1-z_2)$$

$$= P_{12}\left[\sum_{\nu'}\tilde{\Phi}_{\nu'}(v_2)\tilde{\Phi}'_{\nu',a}(z_2+\delta)\otimes\sum_{\mu'}\Phi_{\mu'}(v_1)\,\tilde{\Phi}_{\mu',a}(z_1+\delta)\right]R'(z_1-z_2),$$

其中 P_{12}交换附加空间的指标，所以对应于顶点模型的顶点算子是:

$$\sum_\mu \Phi_\mu\left(-\frac{z}{w}\right)\tilde{\Phi}'_{\mu,a,}(z+\delta) = \psi(z),$$

其中δ是任意复数，它们满足$\psi^i(z_1)\psi^j(z_2)F_{l,k} = \sum_{i'j'}R'(z_1-z_2)_{i,j'}^{i,j}\psi^{j'}(z_2)\psi^{i'}(z_1)\ F_{l,k}$.

利用模变换$\theta\begin{bmatrix} a \\ b \end{bmatrix}\left(z/\tau, -\frac{1}{\tau}\right) = \text{const}\ e^{\frac{\pi i z^2}{\tau}}\theta\begin{bmatrix} b \\ -a \end{bmatrix}(z, \tau)$可以直接将文献[12]中的玻尔兹曼权(10)化为与(2)式成比例的形式，这样就不需要修改$\tilde{\Phi}$而得到改变τ，w之后的R矩阵的顶点算子.

还可以进一步研究顶点算子，使它们满足如:

$$\psi_j^*(z_2)\psi_{i'}(z_1) = \sum_{ij'} R'(z_1 - z_2)_{i,j}^{i',j'}\psi_i(z_1)\psi_{j'}^*(z_2)$$

等关系的玻色化顶点算子.

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Bosonized Vertex Operators Realize Z–F Algebra for Z_n Symmetric Belavin R Matrix

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Abstract

From the bosonization of vertex operators for $A^{(1)}_{n-1}$ face model given by Asai, Jimbo, Miwa and Pugai, using vertex–face correspondence, we obtain bosonized vertex operators for Z_n symmetric Belavin R matrix, which realize the Zamolodchikov–Fadeev algebra for that R matrix.

Key words bosonization, vertex operators, Z–F algebra, Belavin R matrix.