# Study of the Double $\Lambda$ Hypernuclei ${}^{5}_{\Lambda\Lambda}$ H, ${}^{5}_{\Lambda\Lambda}$ He, ${}^{4}_{\Lambda\Lambda}$ H and ${}^{4}_{\Lambda\Lambda}$ He

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With the aid of the theoretical few-body method, the binding energies of the double  $\Lambda$  hypernuclei  $_{\Lambda}^{5}$ H,  $_{\Lambda}^{5}$ He,  $_{\Lambda}^{4}$ H and  $_{\Lambda}^{4}$ He are calculated. The results show that a stable bound state of  $_{\Lambda}^{5}$ H or  $_{\Lambda}^{5}$ He exists definitely with the double  $\Lambda$  binding energies  $B_{\Lambda\Lambda}(_{\Lambda}^{5}$ H) = 6.75MeV, and  $B_{\Lambda\Lambda}(_{\Lambda}^{5}$ H) = 7.67MeV, respectively. As to  $_{\Lambda}^{4}$ H, the preliminary results manifest that it has a bound state but unstable against the decay to the  $_{\Lambda}^{3}$ H +  $\Lambda$  channel. No bound state exists for  $_{\Lambda}^{4}$ He.

Key words: hypernucleus, few body, double Lamda.

# 1. INTRODUCTION

The study of the double  $\Lambda$  hypernuclei is one topic which has recently generated a great deal of attention. With the discovery of the double  $\Lambda$  hypernuclei  ${}_{\Lambda\Lambda}^6$ He,  ${}_{\Lambda\Lambda}^{10}$ Be [1,2] and  ${}_{\Lambda\Lambda}^{13}$ B [3,4,5], a preliminary understanding of the double  $\Lambda$  hypernuclei has been achieved and efforts to look for more of the double hypernuclei are stimulated. The study of the double  $\Lambda$  hypernuclei requires knowledge of the  $\Lambda$ -N,  $\Lambda$ - $\Lambda$  interactions. Therefore, it plays a special role for testing various models of the two body interactions. Besides, it associates closely with the investigation of the strangeness S=-2 H particle system. The S=-2 system can be produced and observed through the strangeness exchange reaction in the laboratory. The knowledge of the double  $\Lambda$  hypernuclei will be also helpful for the investigation of the strange hadron matter (SHM) produced in relativistic heavy-ion collisions. Baltz et al. [6] calculated the production rate of the strange cluster in  $\Lambda$ u+Au collisions at BNL AGS by

Received on August 2, 1995. Supported by the Science Foundation of the Chinese Academy of Sciences.

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using the composition model. They pointed out that the rich hyperons produced in this collision is the best source of forming the multistrangeness hadron matter.

The double  $\Lambda$  hypernuclei can be produced by the strangeness exchange reaction or the decay of the productions in  $\Xi$ -nucleus interactions and by high-energy heavy-ion collisions.

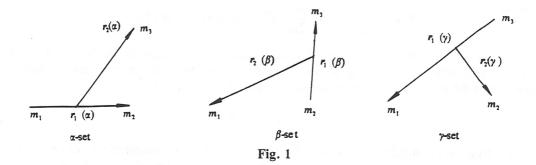
During the study of double hypernuclei, various models for the  $\Lambda$ - $\Lambda$  interaction are proposed. Most of these models are phenomenological, i.e., a form of the  $\Lambda$ - $\Lambda$  interaction which is assumed to be based on some physical considerations, and its parameters are determined by fitting the experimental data of the binding energies of the hypernuclei. However, it is not enough only to study the few double  $\Lambda$  hypernuclei observed. We also want to know whether those phenomenological forces can provide some predictions about the new hypernuclei, particularly the lightest double hypernucleus, because the smaller the particle number in the hypernucleus, the easier the theoretical study will be.

There are a few theoretical calculations on the hypernuclei  $_{\Lambda_{\Lambda}^{5}}^{5}H$  and  $_{\Lambda_{\Lambda}^{5}}^{5}H$ e. The theory predicts a stable bound state but the problem is still open. Tang *et al.* [7] published a theoretical value for the double  $\Lambda$  binding energy  $B_{\Lambda\Lambda}=3.03 \text{MeV}$  and K. S. Myint [8] predicted a value of  $B_{\Lambda\Lambda}=6.3 \text{MeV}$ . The origin of the difference may be due to the fact that the NN,  $\Lambda$ N, and  $\Lambda\Lambda$  interactions used by them are different. Furthermore, the theoretical methods used are also different. Tang *et al.* treated  $_{\Lambda\Lambda}^{5}H$  as a five-body problem and Myint considered it a  $t+\Lambda+\Lambda$  three body problem. Since Tang *et al.* 's research on this problem is from an earlier period (1965), which was a time when our understanding of the  $\Lambda$ N and  $\Lambda\Lambda$  interactions was not so clear, we must consider that their results are more qualitative. More types of forces can be selected by Myint in studying the same problem.

In order to understand the essence of the problem a further investigation is necessary. We have recalculated the binding energy of  $_{\Lambda\Lambda}^{5}H$  by using the few-body theoretical method and the phenomenological force of the  $\Lambda\Lambda$  interaction (which is obtained by fitting the binding energy of  $_{\Lambda\Lambda}^{6}He$ ), so that the results can be compared with the earlier works, and can be used to test the  $\Lambda\Lambda$  interaction used. Meanwhile, the hypernucleus  $_{\Lambda\Lambda}^{5}He$  remains uncalculated, a task that we will accomplish later in the paper.

Tang et al. [7] computed the binding energy of  $_{\Lambda}^{A}H$ . Their conclusion is that there is no bound state for  $_{\Lambda}^{A}H$  or at least no stable state against the decay  $_{\Lambda}^{3}H + \Lambda$ . They concluded that the lightest stable double  $\Lambda$  hypernucleus may be  $_{\Lambda}^{5}H$ . However, Nakaichi-Maeda et al. [9] obtained an entirely different conclusion. They used the central Urbana type of the two-pion-exchange  $\Lambda N$  and  $\Lambda \Lambda$  potentials. The strength of the  $\Lambda N$  interaction is determined by Bodmer et al. from fitting the binding energy  $B_{\Lambda}$  of  $A_{\Lambda}^{5}H$ . Two values of the strength of the  $A\Lambda$  interaction are used, respectively. One is from fitting the binding energy  $A_{\Lambda}^{5}H$ , the other is by fitting the data of  $A_{\Lambda}^{10}B$ . At the same time, a three-body repulsive force is also considered. Their results show that the strengths of the two forces are increased and their difference is decreased if the three-body force is considered. Their conclusion is that the bound state of  $A_{\Lambda}^{4}H$  exists and is furthermore stable against decay  $A_{\Lambda}^{3}H + \Lambda$ . In order to clear if the above-mentioned contradiction, a further investigation is made in this paper.

It is well known that no bound hypernucleus  ${}^3_\Lambda H$  exists. However, the stable hypernucleus  ${}^3_\Lambda H$  does exist but its binding energy is very small. Since the interaction between nucleons in  ${}^4_\Lambda H$  is a weaker spin singlet state force and the strengths of the  $\Lambda\Lambda$  and  $\Lambda N$  interactions are also weaker than that of the NN force, no  ${}^4_\Lambda H$  bound state is reasonable. As to the bound state of  ${}^4_\Lambda H$ , the possibility and stability sensitively depend on the choice the  $\Lambda N$  and  $\Lambda\Lambda$  interactions. The attraction strength of the  $\Lambda N$  singlet interaction is larger than that of the triplet state interaction. As to  ${}^4_\Lambda H$ , the weights of the singlet state and the triplet state compositions of the two-body  $\Lambda N$  interaction are the same as that in  ${}^5_\Lambda H$ e. The calculated result for  ${}^5_\Lambda H$ e directly associates with the results for  ${}^4_\Lambda H$ . The  $\Lambda N$  force used here is different from that in Ref. 9. The  $\Lambda N$  force used in this paper is obtained by fitting the binding energies of the light hypernuclei  ${}^3_\Lambda H$ ,  ${}^4_\Lambda H$ , and the present data of the  $\Lambda P$  scattering, so it is more reasonable to use this interaction in studying  ${}^4_\Lambda H$ . In addition, due to the consideration of the three-body repulsive force in Ref. 9 and a stronger  $\Lambda \Lambda$  force, it is more favorable to obtain the  ${}^4_\Lambda H$  bound state. Another point is that the strength of the  $\Lambda \Lambda$  force used by Nakaichi-Maeda *et al.* is larger



than that of the  $\Lambda N$  force. We therefore recalculate the  $_{\Lambda\Lambda}^4H$  bound state by utilizing the  $\Lambda N$  force provided by Schimert *et al.* [13] as well as the  $\Lambda\Lambda$  force provided by Dalitz *et al.* [13] for fitting the  $_{\Lambda\Lambda}^6H$ e binding energy and we also compare with earlier calculations.

# 2. CALCULATIONS OF AAH AND AAHe

# 2.1. Theoretical method

As to  $^{5}_{\Lambda\Lambda}H$ , we use the three-cluster model

$${}_{\Lambda\Lambda}^{5}H = t + \Lambda + \Lambda \tag{1}$$

Assume the particle masses of the three-body system are  $m_1$ ,  $m_2$ , and  $m_3$ , respectively, and laboratory coordinates are  $R_1$ ,  $R_2$ , and  $R_3$ . There exist three sets of equivalent Jacob coordinates i.e.,  $\alpha$ -set,  $\beta$ -set, and  $\gamma$ -set shown in Fig. 1. The two internal coordinates in  $\alpha$ -set are defined as

$$\mathbf{r}_{1}^{(\alpha)} = \mathbf{R}_{2} - \mathbf{R}_{1}$$
,  
 $\mathbf{r}_{2}^{(\alpha)} = \mathbf{R}_{3} - (m_{1}\mathbf{R}_{1} + m_{2}\mathbf{R}_{2}) / (m_{1} + m_{2})$ .

In the laboratory system, the Hamiltonian of the three-body system is

$$H = -\sum_{i=1}^{3} \frac{\hbar^2}{2m_i} \nabla_{R_i}^2 + \sum_{i \le i}^{3} V_{ij}^i.$$

Removing the motion of the center of mass, the Hamiltonian for the internal motion of the system in the  $\alpha$ -set coordinates is

$$H = -\frac{\hbar^2}{2\mu_1} \nabla_{\eta}^2 - \frac{\hbar^2}{2\mu_2} \nabla_{\eta_2}^2 + \sum_{i < j}^3 V_{ij} , \qquad (2)$$

where  $\mu_1$  and  $\mu_2$  are the reduced masses

$$\mu_1 = \frac{m_1 m_2}{m_1 + m_2}$$
,  $\mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}$ .

and the superscript  $\alpha$  in the above equations is omitted. Equation (2) can be rewritten as

$$H = H_0 + \sum_{i \neq j}^{3} V_{ij} - \frac{1}{2} \omega^2 (\mu_1 r_1^2 + \mu_2 r_2^2), \tag{3}$$

$$H_0 = -\frac{\hbar^2}{2\mu_1} \nabla_{r_1}^2 - \frac{\hbar^2}{2\mu_2} \nabla_{r_2}^2 + \frac{1}{2} \omega^2 (\mu_1 r_1^2 + \mu_2 r_2^2), \tag{4}$$

is the Hamiltonian of two independent harmonic oscillators. Assume that the eigensolutions of  $H_0$  are  $\varphi_{pL}^M$ , where p stands for an aggregation of the harmonic oscillator quantum numbers, the total wave function of the system  $\Psi_L^M$  can be expanded by the harmonic oscillator bases

$$\Psi_L^M = \sum_p C_p \varphi_{pL}^M , \qquad (5)$$

Substituting Eq. (5) into Eq. (3), we can obtain a secular equation. Diagonalizing this equation, we obtain the energy eigenvalue E and the corresponding eigen vectors  $C_p$ . The harmonic oscillator parameter is considered as a variational one and the subspace expanded by the harmonic oscillators is taken to guarantee well-enough the convergency of the variational calculation.

For a three-body system, the matrix elements corresponding to the interaction  $V_{13}$  and  $V_{23}$  can be computed with the aid of the general Talmi-Moshinsky transformation in the  $\beta$ -set and  $\gamma$ -set coordinates, respectively.

# 2.2. Section of interactions

The  $\Lambda\Lambda$  interaction is taken as a single Gaussian potential from Dalitz et al. [13].

$$V_{\Lambda\Lambda}(r) = -52.25 \exp(-r^2/1.034^2),$$
 (6)

and the  $t\Lambda$  interaction is from Ref. 8

$$V_{\rm th}(r) = 359.2 \exp(-r^2/1.25^2) - 324.9 \exp(-r^2/1.41^2),$$
 (7a)

This  $t\Lambda$  potential is obtained by folding the  $\Lambda N$  force with the nuclear density distribution of t. We adjust the strength of the  $V_{t\Lambda}$  potential in order to give the correct binding energy of the hypernucleus  ${}^{\Lambda}_{t\Lambda}$  and find that the  $V_{t\Lambda}$  force is

$$V_{A}(r) = 359.2 \exp(-r^2/1.25^2) - 331.9 \exp(-r^2/1.41^2). \tag{7b}$$

We use the two potentials in our calculation and make a comparison.

# 2.3. Results And discussions

We take the  $\alpha$ -set coordinates which are shown in Fig. 2. Since  $L^{\pi}({}_{\Lambda\Lambda}^5 H) = 0^+, L = l_1 + l_2$ , so  $l_1 = l_2$ . Due to the antisymmetry under the interchange of two  $\Lambda$  particles and the singlet nature of two  $\Lambda$ , the spatial wave function of two  $\Lambda$  is symmetrical.  $L_1$  should be even, i.e.,  $l_1 = l_2 = 0, 2, 4, \ldots$  The cutoff condition of the subspace is  $(2n_1 + l_1 + 2n_2 + l_2) \le N_0$ .  $N_0$  is an integer. We increase the  $N_0$  value sequentially until the variational calculation stable.

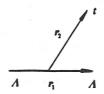


Fig. 2

| tΛ potential | B <sub>M</sub> (MeV) | R <sub>AA</sub> (fm) | $R_{\Lambda\Lambda-\iota}$ (fm) | R <sub>A</sub> (fm) | $R_{A_{1}-A}(fm)$ |
|--------------|----------------------|----------------------|---------------------------------|---------------------|-------------------|
| Eq.(7a)      | 4.60                 | 3.58                 | 2.39                            | 2.98                | 2.86              |
| Eq.(7b)      | 6.75                 | 3.32                 | 2.21                            | 2.76                | 2.65              |

Table 1 Double  $\Lambda$  binding energies of hypernucleus  $^{5}_{\Lambda\Lambda}H$ .

The double  $\Lambda$  binding energy is defined as

$$B_{\Lambda\Lambda} = 2m_{\Lambda} + m_{I} - m({}_{\Lambda\Lambda}^{5}H).$$

The calculations show that if  $N_0$  is increased to 18, the number of bases is 125. If the harmonic oscillator parameter  $\omega = 20 \text{MeV}$  is used, we obtain the stable values of the binding energy of  $^{5}_{\Lambda\Lambda}H$  (as shown in Table 1).

From our calculation, one finds that the binding energy between two  $\Lambda$  hyperons is

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{5}H) - 2B_{\Lambda}({}_{\Lambda}^{4}H) = 2.67 \text{MeV},$$

This value is smaller than that obtained from the double  $\Lambda$  hypernuclei  $_{\Lambda\Lambda}^{6}$ He or  $_{\Lambda\Lambda}^{10}$ Be.

$$\Delta B_{\Lambda\Lambda}^{6} = 4.68 \text{MeV}^{[1]}$$
  
 $\Delta B_{\Lambda\Lambda}^{[0]} = 4.29 \text{MeV}^{[2]}$ 

 $^{5}_{\Lambda\Lambda}$ He is calculated by using the similar method. For the potential  $V_{^{3}\text{He}-\Lambda}$ , we utilize the same form of  $V_{t\Lambda}$  and adjust the strength parameter of  $V_{^{3}\text{He}-\Lambda}$  so that it can fit the experimental  $B_{\Lambda}$  of  $^{4}_{\Lambda}$ He.

$$V_{3_{H_2-A}}(r) = 359.2 \exp(-r^2/1.25^2) - 334.6 \exp(-r^2/1.41^2).$$
 (8)

The computed results are as follows

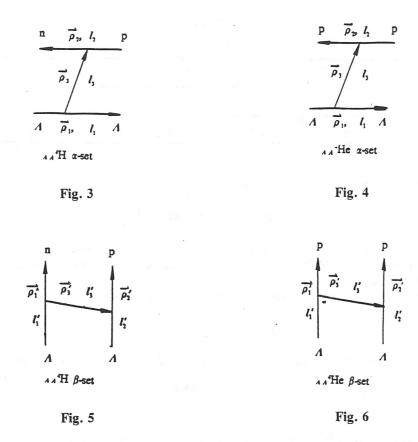
$$R_{\Lambda\Lambda} = 3.22 \text{fm}, R_{\Lambda\Lambda - 14e} = 2.14 \text{fm},$$
  
 $R_{\Lambda - 14e} = 2.68 \text{fm}, R_{\Lambda 14e - \Lambda} = 2.57 \text{fm};$   
 $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^5\text{He}) = 7.67 \text{MeV};$   
 $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^5\text{He}) = 2.89 \text{MeV}.$ 

we can see that  $B_{\Lambda\Lambda}(\Lambda_{\Lambda}^5He)$  is 1MeV larger than  $B_{\Lambda\Lambda}(\Lambda_{\Lambda}^5H)$ . But the  $B_{\Lambda\Lambda}(\Lambda_{\Lambda}^5He)$  and  $B_{\Lambda\Lambda}(\Lambda_{\Lambda}^5H)$  are almost the same. Our results show that the double  $\Lambda$  binding energy by using the folding potential Eq. (7a) is closer to Tang *et al.*'s result, while the results by fitting  ${}^{4}_{\Lambda}H$  data is closer to Myint's result. We consider the latter more reasonable.

# 3. CALCULATIONS OF AH AND AH

# 3.1. Theoretical method

 $^{4}_{\Lambda\Lambda}$ H and  $^{4}_{\Lambda\Lambda}$ He are the systems consisting of four particles. We treat them by using the few-body variational method. The Jacobi coordinates shown in Figs. 3 and 4 are used for  $^{4}_{\Lambda\Lambda}$ H and  $^{4}_{\Lambda\Lambda}$ He, respectively.



The expanding method for the four-body system based on the harmonic oscillator bases is similar to that of the three-body system, except the bases functions are now the coupled states of the three harmonic oscillator products. It is important to analyze the symmetry of Figs. 3 and 4. It is well known that the energy of the state having a fully spatial symmetry is always the lowest. So the spin and isospin in the lowest state possess the quantum numbers

$$S_{\Lambda\Lambda} = 0, \begin{cases} S_{pn} = 1 \\ T_{pn} = 0 \end{cases}, \begin{cases} S_{pp} = 0 \\ T_{pp} = 1 \end{cases}.$$

Then the bases function can be chosen as

$$\begin{split} \psi_{k}^{(1)} = & \left\{ \varphi_{n_{1} l_{1}}(\vec{\rho_{1}}) [\varphi_{n_{2} l_{2}}(\vec{\rho_{2}}) \varphi_{n_{3} l_{3}}(\vec{\rho_{3}})]_{l_{23}} \right\}_{L^{2}} \chi_{S_{\Lambda \Lambda = 0}}(\Lambda \Lambda) \chi_{S_{pn = 1}}(p_{n}), & \text{for } {}_{\Lambda \Lambda}^{4} H; \\ \psi_{k}^{(2)} = & \left\{ \varphi_{n_{1} l_{1}}(\vec{\rho_{1}}) [\varphi_{n_{2} l_{2}}(\vec{\rho_{2}}) \varphi_{n_{3} l_{3}}(\vec{\rho_{3}})]_{l_{23}} \right\}_{L^{2}} \chi_{S_{\Lambda \Lambda = 0}}(\Lambda \Lambda) \chi_{S_{pp = 0}}(p_{p}), & \text{for } {}_{\Lambda \Lambda}^{4} He. \end{split}$$

Here the quantum numbers are selected as follows:  $l_1$  = even,  $l_2$  = even, antisymmetry requirement;  $l_2+l_3=l_{23}$ ,  $l_{23}+l_1=L$ , so  $l_{23}=l_1$ , angular momentum coupling rule;  $(-1)^{l_1+l_2+l_3}=+1$ , parity conservation. Besides  $N=2(n_1+n_2+n_3)+l_1+l_2+l_3\leq N_0$ .

The  $\beta$ -set coordinates related to Figs. 3 and 4 are shown in Figs. 5 and 6, respectively. When the bases functions are changed from the  $\alpha$ -set to the  $\beta$ -set with the aid of the Talmi-Moshinsky

transformation, the quantum numbers in the  $\beta$ -set must satisfy the following conditions:

- (1) Energy conservation  $N' = 2(n'_1 + n'_2 + n'_3) + l'_1 + l'_2 + l'_3 = N$ .
- (2) Angular momentum coupling rule  $l'_2 + l'_3 = l'_{23}$ ,  $l_{23} + l'_1 = L$ , and  $l'_{23} = l'_1$ .
- (3) Parity conservation  $(-1)^{l'_1+l'_2+l'_3} = +1$ .

If the interaction is spin dependent, calculate the interaction matrix elements between the particles 1 and 4 or 2 and 3; for the spatial wave functions, a Talmi-Moshinsky transformation will be necessary, and the spin wave function simultaneously needs to transformed.

$$[\chi_{s_{AA}=0} (12)\chi_{s_{pa}=1}(34)]_{S=1}$$

$$= \frac{1}{2} [\chi_{S_{Aa}=0}(14)\chi_{S_{Ap}=1}(23)]_{S=1} - \frac{1}{2} [\chi_{S_{Aa}=1}(14)\chi_{S_{Ap}=0}(23)]_{S=1}$$

$$+ \frac{1}{\sqrt{2}} [\chi_{S_{Aa}=1}(14)\chi_{S_{Ap}=1}(23)]_{S=1} ;$$
(9)

$$[\chi_{s_{AA}=0} (12)\chi_{s_{AB}=0}(34)]_{S=0}$$

$$= \frac{1}{2} [\chi_{S_{A_0}=0}(14)\chi_{S_{A_0}=0}(23)]_{S=0} + \frac{\sqrt{3}}{2} [\chi_{S_{A_0}=1}(14)\chi_{S_{A_0}=1}(23)]_{S=0}.$$
(10)

# 3.2. Choice of interactions

From Ref. 14, the  $\Lambda N$  interaction is taken as the following

$$V_{\Lambda N} = \frac{1}{2} \left( 1 + P_{\Lambda N}^{\sigma} \right) V_{t}(r) + \frac{1}{2} \left( 1 - P_{\Lambda N}^{\sigma} \right) V_{s}(r) + V_{csb}(r). \tag{11}$$

where  $V_{csb}(r) = -\tau_3^j \vec{\sigma}_{\Lambda} \cdot \vec{\sigma}_{N} w_0 \exp[-\lambda (r - d_{\Lambda N})]$  is the charge asymmetry term;  $V_t$  is the triplet force, and  $V_s$  is the singlet one.  $V_t$  and  $V_s$  are taken, respectively, as

$$V_{s}(r) = -59.7 \exp \left[ -\frac{r^{2}}{1.045^{2}} \right],$$

$$V_{t}(r) = -41.79 \exp \left[ -\frac{r^{2}}{1.045^{2}} \right].$$

By evaluating the matrix elements of Eq. (11) between the spin wave functions, we obtain a spin-averaged potential,

$$V_{\Lambda N}(r) = \frac{1}{4} V_{s}(r) + \frac{3}{4} V_{t}(r).$$

The NN force is taken from the paper of I. Reichstein [15]

$$V_{ij} = \left[ \frac{1 + P_{ij}^{\sigma}}{2} V_{t} + \frac{1 - P_{ij}^{\sigma}}{2} V_{s} \right] + \frac{e^{2}}{4r_{ij}} (1 + \tau_{iz})(1 + \tau_{jz}) . \tag{12}$$

| $N_{0}$ | Bases number M | B(MeV) |
|---------|----------------|--------|
| 6 200   | 33             | 0.942  |
| 8       | 75             | 2.02   |
| 10      | 153            | 2.25   |

Table 2 The calculation values of the binding energy B of  $_{\Lambda\Lambda}^{4}H$ .

where

$$V_t = -66.92 \exp(-0.415r^2),$$
  
 $V_r = -29.05 \exp(-0.292r^2).$ 

The  $\Lambda\Lambda$  force is taken to be the same as Eq. (6).

# 3.3. Calculation results and discussions

The binding energies B of  $_{\Lambda}^{\Lambda}H$  computed by using different  $N_0$  values are given in Table 2. We can see that the convergency of the B values is rather slow with increasing  $N_0$ . In order to save the computing time, we utilize the following formula to evaluate the binding energy as  $N_0$  tends to be infinite

$$B(M) = B(\infty) + C \cdot M^{-\alpha}.$$

If the data in Table 2 are put into the above equation, we will obtain

$$\alpha = 1.802$$
,  $C = 760.39$  (MeV),  $B(\infty) = 2.34$  MeV.

and then we obtain

$$B_{\Lambda\Lambda} = 0.13 \text{MeV}.$$

This value is close to the experimental value  $B_{\Lambda}(^{\lambda}_{\Lambda}H)$ . Our results approach the calculated value derived by Tang *et al.* and do not support the calculation obtained by Nakaichi-Maeda *et al.* The latter explicitly used the larger  $\Lambda\Lambda$  interaction which is even larger than the  $\Lambda N$  interaction and leads to a deeper bound state. We think that this result is unreasonable. Present understanding of the  $\Lambda N$  and  $\Lambda\Lambda$  interactions remains unclear, and differences between various models are often great. Furthermore, the stability of  $^{4}_{\Lambda\Lambda}H$  depends sensitively on the choice of the  $\Lambda N$  and  $\Lambda\Lambda$  forces, so a more intensive and systematic study is necessary.

### 4. SUMMARY

In this paper, the binding energies of the double  $\Lambda$  hypernuclei  $_{\Lambda\Lambda}^{5}H$ ,  $_{\Lambda\Lambda}^{5}He$ ,  $_{\Lambda\Lambda}^{4}H$  and  $_{\Lambda\Lambda}^{4}He$  are calculated. We arrive at the following conclusions:

- (1) There exist stable bound hypernuclei  ${}_{\Lambda\Lambda}^5H$  and  ${}_{\Lambda\Lambda}^5He$ . The binding energies of the double hyperons are  $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^5H)=6.75 \,\mathrm{MeV}$ ,  $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^5H)=7.67 \,\mathrm{MeV}$ , and  $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^5H)=2.67 \,\mathrm{MeV}$ ,  $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^5H)=2.89 \,\mathrm{MeV}$ , respectively.
- (2) No bound  $_{\Lambda\Lambda}^{4}$ He exists. As to  $_{\Lambda\Lambda}^{4}$ H, we find that it has a bound state but is unstable against the decay  $_{\Lambda}^{3}$ H +  $\Lambda$ . Since the stability of  $_{\Lambda\Lambda}^{4}$ H sensitively depends on the choice of the interactions, a further study is necessary.

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