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Shell Model Calculation for Light Neutron-Rich Nuclei

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A shell model calculation is carried out for nuclei with $9 \leq A \leq 14$ and $3 \leq Z \leq 5$. In the calculation, ^{8}He is treated as core. The data for 24 experimental energy spectra are selected to determine the parameters of the modified surface δ interaction (MSDI) and the single-particle energies. The energy root-mean square deviation is $RMS = 0.72$ MeV. Then the binding energies, the low-lying excited energy spectra, and the electromagnetic properties are calculated and the results are in good agreement with existing measurements in general. The micromechanism of the halo structure and the parity conversion of the ^{11}Be are discussed in detail. This work is carried out with the shell model code OXBASH.

Key words: shell model, mean-field, unstable nuclei.

1. INTRODUCTION

Experiments have shown that the nuclei ^{11}Li, ^{11}Be, and ^{14}Be, etc., have weakly bound halo structure [1,2]. In addition, the parity of the ground state of ^{11}Be is converted. The naive shell model picture of this state is of spin-parity $1/2^-$, whereas experimentally it is known to be $1/2^+$ [3]. Experimentally the electric-dipole ($E1$) transition from the first $1/2^-$ excited state to the ground state of ^{11}Be shows an anomalous feature. In general, the $E1$ transitions from low-lying excited states to ground state are strongly hindered because they are isoscalar dipole excitations. However, the $B(E1)$ of the $E1$ transition of ^{11}Be from $1/2^-$ to $1/2^+$ is 0.36 W.u. and is on the order of the single-particle transition probability.

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The initial shell model calculation [5] and HF calculation [6] could not give a reasonable explanation for the problems above. The effective interactions (including one- and two-body interactions) used in the initial shell model calculation were determined with the experimental data of stable nuclei. Early in 1960, Talmi and Unna pointed out that the parity conversion of ^{11}Be was caused by the strong proton–neutron monopole interaction in the mean-field [3]. In order to describe these nuclei in terms of the shell model, a great amount of theoretical work has been conducted. N. Fukunishi *et al.* pointed out that the various anomalous features for neutron-rich nuclei in the N = 20 region were caused by the shell gap of $2p1f$ shell and $2s1d$ shell vanishing [7]. We carried out similar work on the neutron-rich nuclei in $N = 8$ region [5]. H. Sagawa *et al.* pointed out pairing block effect was important factor for the conversion of the ^{11}Be parity [8]. Some people introduced the isospin-dependence of the single-particle energies [9].

Now the problem is whether or not the properties of the unstable nuclei can be described by using the effective interaction which determined with the experimental data of unstable nuclei. If this can be carried out, then the shell model can be extended naturally to the range of unstable nuclei and in turn can investigate the properties of the interactions of the unstable nuclei. The difficulty of this method is that the experimental data unstable nuclei are much less then those of the stable nuclei. Therefore, one must choose a suitable potential with small number parameters. It is well known that the modified surface δ interaction (MSDI) [10] has been successfully used to describe the properties of a lot of nuclei in different mass region and it has a few parameters which have specific physical meanings. In this paper, we mainly used this interaction. This work is carried out with shell model code OXBASH [11].

2. MODEL SPACE AND EFFECTIVE INTERACTION-MODIFIED SURFACE δ INTERACTION

To investigate the nuclei many-body system, the main problem is to know the freedoms which determine the property concerned. For the shell model calculation it is how to truncate the model space. For our case the intruder orbits of the *sd* shell are very important for the neutron. It is necessary to extend the neutron model space from $1p$ shell to $2s1d$ shell. In our calculation, the ^{8}He is treated as a core, i.e., the valence protons can only occupy the $1p$ shell, while the valence neutrons can fill $1p_{1/2}$, $1d_{5/2}$, and $2s_{1/2}$ single-particle orbits. The truncation of this model space is based on the following idea: 1) The ^{9}Li and ^{10}Be can be treated as a core when one describes the halo structure of ^{11}Li and ^{11}Be. 2) One can expect that the mean-field for unstable nuclei such as ^{11}Li, ^{11}Be, and ^{14}Be may be greatly different from that of stable nuclei. 3) It has been shown that the $1d_{3/2}$ orbit is not important for the structure of ^{11}Li and ^{11}Be [5,8,9,12]. 4) This model space can greatly simplify the calculation. The model space for neutron and proton are still called $1p - 2s1d$ shell and $1p$ shell, respectively.

Another key problem in the shell-model calculation is the effective interaction. We assume that the single-particle energies (mean-field) and two-body interaction (residual interaction) of unstable nuclei are different from those of stable nuclei. It has been shown the mass-dependence of interaction does exist for the $1p$ shell and $2s1d$ shell nuclei [13]. One can also expect the existence of isospin-dependence of the interaction. Our basic idea is that the properties of unstable nuclei must be described with the interaction determined by experimental data of unstable nuclei. We have pointed out in section 1 that the MSDI is a suitable potential, which can be written as

$$V^{\text{MSDI}} = -4\pi A_T' \, \delta(r_1 - r_2)\delta(r_1 - R_0) + B'(\tau_1 \cdot \tau_2) + C', \tag{1}$$

where r_1 and r_2 are the position vectors of the interacting neutron- and proton-particle, and R is the nuclear radius. A'_T is the strength parameter associated with the isospin T. B' and C' are the correction terms. In our case, there are 9 parameters to be determined.

Table 1
The values of the MSDI parameters and single-particle energies (MeV).

A_0	A_1	B_0	B_1	$\pi 1p_{1/2}$	$\pi 1p_{3/2}$	$\nu 1p_{1/2}$	$\nu 1d_{5/2}$	$\nu 2s_{1/2}$
1.930	0.712	−3.344	1.555	−12.474	−16.661	2.592	6.374	4.894

Table 2
The binding energies of ^{11}Li, $^{11-14}$Be, and $^{12-14}$B (MeV).

Nucleus	^{11}Li	^{11}Be	^{12}Be	^{14}Be	^{12}B	^{13}B	^{14}B
E_B^{exp}	15.131	36.713	39.881	40.081	52.917	57.917	58.766
E_B^{cal}	16.324	36.336	39.894	39.741	52.317	57.275	59.170

In the Fock space, the model Hamiltonian can be written as

$$H = H_{core} + E_c + \sum_{i=1} \varepsilon_i \alpha_i^\dagger \alpha_i + \sum_{\substack{i>j=1 \\ k>l=1}} \langle ij | V | kl \rangle \alpha_i^\dagger \alpha_j^\dagger \alpha_l \alpha_k \ , \tag{2}$$

where H_{core} is the binding energy of the core ^{8}He, ε_i is the i-th single-particle energy, $<ij|V|kl>$ is the two-body interaction and can be obtained by MSDI, E_C is Coulomb interaction and can be obtained by the way as in Ref. 14. The 9 parameters are determined with 24 experimental energies data of nuclei with $9 \le A \le 14$ and $3 \le Z \le 5$ by using the least-squares fit method. Table 1 lists the fitted values of these parameters. The energy root-mean square deviation, defined as

$$RMS^2 = \frac{\sum_{i=1}^{N_d} \{ E_i(\exp) - E_i(\text{th}) \}^2}{(N_d - N_p)} \ . \tag{3}$$

is RMS = 0.72 MeV. Where E_i(exp) and E_i(th) are i-th experimental and calculated energies with respect to core ^{8}He, respectively. N_d and N_p are the number of experimental data and parameters, respectively.

3. BINDING ENERGY AND LOW-LYING EXCITED SPECTRA

Table 2 lists the experimental and calculated binding energies of nuclei ^{11}Li, $^{11-14}$Be, and $^{12-14}$B. It can be seen that all the calculated binding energies are in good agreement with experimental results except the ^{11}Li, for which the difference is about 1.2 MeV. All the experimental data is taken from Ref. 4.

Figure 1 shows the experimental data and the calculated results for low-lying excited states of nuclei ^{11}Be, ^{12}Be, and ^{14}B. The calculated spectra agree with experiments quite well, especially the ground state of ^{11}Be is $1/2^+$, the first excited state is $1/2^-$ with excited energy 0.52 MeV which agrees with experimental data [$E_x(1/2^-)$ = 0.32 MeV]. The low-lying states of ^{14}B are also reproduced in our calculation. In a word, according to our calculated results, the truncated model space and the MSDI are reasonable.

Table 3
The magnetic dipole moments of ^{11}Li, ^{11}Be, and $^{12-14}$B (n.m).

Nucleus	^{11}Li	^{11}Be	^{12}B	^{13}B	^{14}B
μ^{exp}	3.68		1.003	3.178	
μ^{cal}	3.679	−1.876	2.157	2.934	1.288

$g_p^s=5.586$, $g_n^s=-3.826$, $g_p^l=1.000$, $g_n^l=0.0$.

Table 4
The quadruple moments of ^{11}Li and $^{12-14}$B (mb).

Nucleus	^{11}Li	^{12}B	^{13}B	^{14}B
Q^{exp}	−31.000	13.410	47.800	
Q^{cal}	−42.942	24.634	50.964	25.147

$e_p^{eff}=1.5e$, $e_n^{eff}=0.5e$

4. ELECTROMAGNETIC PROPERTIES

As pointed out in section 1, the *E*1 transition from low-lying states to ground state are strongly hindered in general. However, for the light neutron-rich nuclei, this transition is very strong, for example, experimentally, the *E*1 transition probability *B*(*E*1) from $1/2^-$ to $1/2^+$ of ^{11}Be is 0.36 W.u. comparable to the single-particle transitions probability. There may exist low excited electric soft resonance for nuclei ^{11}Li according to its structure.

The magnetic dipole moments and quadrupole moments of nuclei ^{11}Li, ^{11}Be, and $^{12-14}$B are listed in Tables 3 and 4, respectively. The calculated results agree with experimental data.

The calculated *E*1 transition probability of $1/2^-$ to $1/2^+$ of ^{11}Be is 0.097 W.u. with effective charge e_p = 1.5 e and e_n = 0.5 e. This strength does not reach the experimental data, but it is still a very strong transition.

The calculated *B*(*E*1) of ^{11}Li from $1/2^+$, $3/2^+$, and $5/3^+$ to $3/2^-$ are 0.037 e^{2}fm^2, 0.018 e^2, and 0.037 e^{2}fm^2, respectively. These are very strong *E*1 transitions.

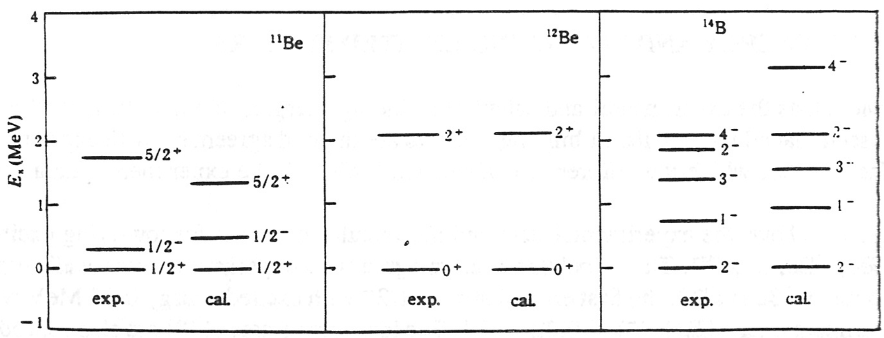

Fig. 1
The spectra of ^{11}Be, ^{12}Be, and ^{14}B.

5. CONCLUSIONS

Our calculation reproduces the ground state property of ^{11}Be. Below we discuss the physical mechanism for the parity conversion and the halo structure of ^{11}Be.

5.1. Energy gap between 1*p* shell and 2*s*1*d* shell for neutron

It can be seen from Table 1 that the energy gap $\Delta\varepsilon$ between $1p$ shell and $2s1d$ shell are

$$\Delta\varepsilon(2s_{1/2}-1p_{1/2})=\varepsilon_{2s_{1/2}}-\varepsilon_{1p_{1/2}}=2.302\text{MeV}, \tag{4}$$

$$\Delta\varepsilon(1d_{5/2}-1p_{1/2})=\varepsilon_{1d_{5/2}}-\varepsilon_{1p_{1/2}}=3.782\text{MeV}. \tag{5}$$

This gap is much smaller than that of stable nuclei which is about 14.5 MeV. It shows that the energy gap between $1p$ shell and $2s1d$ shell decreases as the neutron number increasing. This indicates the strong proton–neutron monopole interaction in the mean-field. This monopole interaction makes, on one hand, the proton potential deeper and more steep and the gap between single-particle state larger, on the other hand, neutron potential shallower and less steep and the gap between $1p$ and $2s1d$ shell smaller. These are the important micromechanism for the formation of halo structure. This result is similar with that in Refs. 7-9. However, it must be pointed out that only the strong monopole interaction is not sufficient to explain the parity conversion and halo structure of ^{11}Be. The valence proton-neutron interaction is equally important.

5.2. Interaction of valence proton and neutron

According the our determined interaction, one can obtain the interaction center

$$E_{center}=\frac{\sum_J(2J+1)E_J}{\sum_J(2J+1)}. \tag{6}$$

where E_j is proton–neutron diagonal two-body matrix elements, J is the coupled momenta of proton and neutron. The results are $E_{center}(\nu 1p_{1/2}-\pi 1p_{3/2}) \approx E_{center}(\nu 2s_{1/2}-\pi 1p_{3/2}) \approx E_{center}(\nu 1d_{5/2}-\pi 1p3) \approx -2.520$ MeV. However, the neutron-proton interaction across the shell is as strong as that within the same shell. This leads to strong configuration mixing. The two $1p_{3/2}$ proton can be polarized and excited to $1p_{1/2}$ orbit due to the strong proton-neutron interaction when the last neutron occupies the *sd* shell. This causes the energy of $1/2^+$ state lower than that of $1/2^-$ state and the parity converted.

The main components of the wave-function of ^{11}Be ground state can be written as

$$|1/2^+> \approx \alpha_1(0_c^+ \times |2s_{1/2}>_\nu)_{1/2^+} + \alpha_2(2_c^+ \times |1d_{5/2}>_\nu)_{1/2^+} . \tag{7}$$

where 0_c^+, 2_c^+ are two configurations of the two $1p_{3/2}$ protons of ^{11}Be. $| >_\nu$ are neutron single-particle states. The resulting values are $\alpha_1 = 0.750$ and $\alpha_2 = 0.534$. These two coupling part components are indispensable to produce the $1/2^+$ ground state. The calculated excited energies are 4.0 MeV and 3.0 MeV, respectively, if one fixes the last neutron in the $1d_{5/2}$ or $2s_{1/2}$ orbits. The ^{11}Be is no longer bounded at these high excited energies. The physical meaning of this coupling is that, as pointed out in Ref. [12], the motion of the core surface (i.e., ^{10}Be) is coupled dynamically with the motion of the single particle.

According to the discussion above, one can expect the configuration mixing is very strong for the neutron-rich nuclei in $N = 8$ region. For example, the calculated transition probability of last two neutrons existed to the $2s1d$ shell for the ground state wave-function of ^{11}Li is more than 22%. The halo structure is related closely to the monopole interaction in the mean-field as well as the valence neutron-proton interaction.

Our calculated results show that the properties of the unstable nuclei can be described in terms of the shell model. The electromagnetic properties are calculated and the resulted $E1$ transition is strong for the nuclei ^{11}Li and ^{11}Be. We have discussed the micromechanism of the parity conversion and halo structure of ^{11}Be. The results show that the micromechansim of the halo structure is caused by the mean-field and the proton–neutron residual interaction; for the parity conversion, the polarization and excitation of the two $1p_{3/2}$ protons are also important.

In order to calculate more precisely the electromagnetic properties and the square-mean radius which are very sensitive to the wave-function, it is necessary to use more realistic interactions and single-particle wave functions. We have not discussed the β decays of these nuclei because it is a very complex problem and more work needs be done on it.

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