High Energy Physics and Nuclear Physics

Volume 20, Number 1

Studies of the Low-Lying States in ^{157,159}Tm

Xing Zheng, Chen Xingqu, and Xu Shuwei[1]

(Department of Modern Physics, Lanzhou University, Lanzhou, China)
[1](Institute of Modern Physics, The Chinese Academy of Sciences, Lanzhou, China)

Properties of the low-lying positive-parity states in ^{157,159}Tm have been investigated by using the triaxial rotor plus particle model with the variable moment of inertia (VMI) of the core. The good agreement between theory and experiment shows that ^{157}Tm and ^{159}Tm may be triaxiality. Furthermore, it is suggested that an excitation rotational band in ^{157}Tm probably already exists in the experimental data.

Key words: triaxial deformation, variable moment of inertia, particle-rotor model.

1. INTRODUCTION

A large number of the experimental data on ^{159}Tm have been accumulated [1]. Recently, using β^+ decay of ^{159}Yb, Tlusty *et al.* [2] studied the properties of low excited states in ^{159}Tm. They believed that two new rotational bands based on [411 1/2$^+$] and [402 5/2$^+$] Nilsson states were identified. On the other hand, the ground-state spin $I = 1/2$ in ^{157}Tm is difficult to explain on the basis of the Nilsson approach if the nuclear shape is axially deformed (as is supposed). It was therefore assumed in Ref. 3 that the nucleus ^{157}Tm had a spherical form. However, from measured isotopic shifts of atomic spectra, the nuclear charge radii of thulium isotopes have been obtained in Ref. 4. There is no significant difference between the charge radii of ^{157}Tm and ^{159}Tm, which means that the nuclear deformations of both nuclei are not appreciably different. The estimate of the effective deformation parameter for ^{157}Tm by the droplet model actually gives $\langle\beta^2\rangle^{1/2} = 0.21$. Very recently, a rotational

Received on February 7, 1995. Supported by the National Natural Science Foundation of China and the China Nuclear Industry Science Foundation.
© 1996 by Allerton Press, Inc. Authorization to photocopy individual items for internal or personal use, or the internal or personal use of specific clients, is granted by Allerton Press, Inc. for libraries and other users registered with the Copyright Clearance Center (CCC) Transactional Reporting Service, provided that the base fee of $50.00 per copy is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923.

band in ^{157}Tm has been assigned by Xu Shuwei *et al.* [5]. Therefore, it is reasonable to assume that the nuclear shape of ^{157}Tm is still to be deformed. Furthermore, the authors of Ref. 4 suggested that the ground-state spin I = 1/2 of ^{157}Tm is the result of [411 1/2] orbit which approaches the Fermi surface at large values of triaxial parameter γ. However, the calculations both in Ref. 6 and in the present paper show that the [411 1/2] orbit goes out from the Fermi surface as the γ deformation increases. Therefore, I = 1/2 is not likely to be the bandhead of the [411 1/2] band. The analyses of the experimental data on the nucleus ^{159}Tm show that there exists some rather large triaxial deformation in the ^{159}Tm [7,8].

The main purpose of the present paper is to study the properties of the low excited positive-parity states of ^{157,159}Tm and to analyze the wavefunction structure of their intrinsic states in terms of the triaxial-particle-rotor model with the variable moment of inertia of the core [6,9,10]. Furthermore, we point out that the ^{157}Tm and ^{159}Tm may be triaxiality. Due to triaxiality, the configurations are strongly mixed. Thus, the rotational bands are labeled by $(\bar{K}\nu)$.

In Section 2, our model is briefly presented. The calculated results compared with the experimental data are given in Section 3. In particular, the properties of the low excited positive-parity states in ^{157}Tm are analyzed. Furthermore, it is pointed out that another excitation rotational band (3/2 18) in ^{157}Tm probably already exists in the observed data [5]. Finally, a summary is given in Section 4.

2. THEORETICAL MODEL

In the assumption that the odd proton in a triaxially deformed potential is coupled with a rotating core, the Hamiltonian of the system is written as [6]

$$H = H_{sp} + H_{rot} + H_{pair}, \tag{1}$$

where H_{sp} is the single-particle Hamiltonian [9,12],

$$H_{sp} = h_{ho}(\varepsilon_2, \gamma) + 2\hbar\omega_0 \rho^2 \varepsilon_4 V_4(\gamma) + V', \tag{2}$$

where $h_{h0}(\varepsilon_2,\gamma)$ is the Hamiltonian of the deformed harmonic oscillator,

$$h_{ho}(\varepsilon_2, \gamma) = \frac{p^2}{2m} + \frac{1}{2} m(\omega_x{}^2 x^2 + \omega_y{}^2 y^2 + \omega_z{}^2 z^2), \tag{3}$$

where ω_x, ω_y and ω_z are the oscillator frequencies. The potential can be expressed by the quadrupole deformation ε_2 and triaxiality γ [13],

$$U(\varepsilon_2 \gamma) = \frac{1}{2}\hbar\omega_0 \rho^2 \left\{1 - \frac{2}{3}\varepsilon_2 \sqrt{\frac{4\pi}{5}}\left[\cos\gamma Y_{20} - \frac{\sin\gamma}{\sqrt{2}}(Y_{22} + Y_{2-2})\right]\right\}, \tag{4}$$

where radius ρ is expressed in the stretched coordinates. The hexadecapole deformation potential $V_4(\gamma)$ is written as [12]

$$V_4 = a_{40} Y_{40} + a_{42}(Y_{42} + Y_{4-2}) + a_{44}(Y_{44} + Y_{4-4}), \tag{5}$$

where parameters a_{4i} are of the form

$$a_{40}=\frac{1}{6}(5\cos^2\gamma+1),$$
$$a_{42}=-\frac{1}{12}\sqrt{30}\sin(2\gamma), \tag{6}$$
$$a_{44}=\frac{1}{12}\sqrt{70}\sin^2\gamma.$$

The term V' of the Nilsson potential is of the form

$$V'=-\kappa\hbar\mathring{\omega}_0\{2\,\boldsymbol{l}_t\cdot\boldsymbol{s}+\mu(\boldsymbol{l}_t^2-\langle\boldsymbol{l}_t^2\rangle_N)\}, \tag{7}$$

where index t in the orbit-angular momentum operator $\boldsymbol{l}_t$ denotes that it is defined in the stretched coordinates. Nilsson parameters κ, μ depend on the main oscillator quantum number N [12,14].

Diagonalizing the Hamiltonian, Eq.(2), we obtain adiabatic single-particle states written as expansions

$$\chi_\nu=\sum_{Nlj\Omega}C^{(\nu)}_{Nlj\Omega}|Nlj\Omega\rangle, \tag{8}$$

With reflection symmetry, Ω are restricted to the values ... −3/2, 1/2, 5/2, ... Their conjugate states are defined as

$$\tilde{\chi}_\nu=\sum_{Nlj\Omega}(-)^{j-\Omega}C^{(\nu)}_{Nlj\Omega}|Nlj-\Omega\rangle, \tag{9}$$

The core Hamiltonian takes

$$H_{\rm rot}=\sum_{k=1}^{3}\frac{\hbar^2}{2J_k}R_k^2=\sum_{k=1}^{3}\frac{\hbar^2}{2J_k}(I_k-j_k)^2, \tag{10}$$

where $\boldsymbol{R}$, $\boldsymbol{I}$, and $\boldsymbol{j}$ are the core angular momentum, the total angular momentum and the single-particle angular momentum, respectively. In the calculations, we take the moments of inertia of the hydrodynamical type

$$J_k=\frac{4}{3}J_0\sin^2\left(\gamma+\frac{2\pi}{3}k\right), \tag{11}$$

The strong coupling basis functions are

$$|IMK\nu\rangle=\left(\frac{2I+1}{16\pi^2}\right)^{1/2}\sum_{Nlj\Omega}C^{(\nu)}_{Nlj\Omega}(D^I_{MK}|Nlj\Omega\rangle+(-)^{I-j}D^I_{M-K}|Nlj-\Omega\rangle). \tag{12}$$

The corresponding total wavefunctions are

$$|IM\rangle=\sum_{K\nu}a_K^{I\nu}|IMK\nu\rangle. \tag{13}$$

where K is the projection of the total angular momentum I on the intrinsic axis 3. The summation above is restricted by the fact that K-Ω must be an even number [6]. Due to triaxiality, $K\neq\Omega$, K is not a good quantum number and the symbol $\bar{K}$ will denote the K value of the largest component in the total wavefunction. Pairing is introduced via a standard BCS procedure, so that the single-particle matrix elements are multiplied by the appropriate pairing factors $uu' + vv'$. Coriolis attenuation factor ξ ($0.5 \leq \xi \leq 1.0$) is used to multiply the off-diagonal single-particle matrix elements.

In order to extend this calculation to include the variable moment of inertia (VMI) of the core, a transformation from the strong coupling basis to the weak coupling basis is used [9, 10],

$$|IMR\alpha N\,lj\rangle = \sum_{M_R m} \langle RM_R jm | IM\rangle |R\alpha M_R\rangle |N\,lj\,m\rangle, \tag{14}$$

where $|R\alpha M_R\rangle$ are the triaxial rotor wavefunctions and $|Nljm\rangle$ are the single-particle wavefunctions, while α labels the different states having the same angular momentum R. It is then straightforward to calculate the core matrix elements as

$$\begin{aligned} &\langle IMK\nu | H_{rot} | IMK'\nu' \rangle \\ &= \sum_{R\alpha Nj} \langle IMK\nu | IMR\alpha N\,lj\rangle E_{R\alpha} \langle IMR\alpha N\,lj | IMK'\nu'\rangle, \end{aligned} \tag{15}$$

where $E_{R\alpha}$ are the eigenenergies of the core.

In the calculations, the Nilsson parameters κ, μ, and the pairing strength G are taken the standard values from Refs. 12 and 14. The Fermi level λ and the pairing gap Δ are derived quantities, not adjustable parameters. 14 single-particle orbits situated near the Fermi level are included in the basis. The VMI parameters have been chosen so that the energy of the core 2^+ state is equal to the experimental value. The deformation parameters ε_2, ε_4 and γ are adopted to give the best agreement between the theoretical and experimental level spectra. The Coriolis attenuation factor ξ is 0.6 for ^{157}Tm and 0.93 for ^{159}Tm.

3. RESULTS AND DISCUSSION

First of all, we elaborate the calculation results of ^{157}Tm, which are compared with the observed data. We give the results and a brief discussion for ^{159}Tm.

3.1. ^{157}Tm

In Fig. 1, the single proton levels as a function of the triaxial parameter γ at $\varepsilon_2 = 0.230$, $\varepsilon_4 = 0.01$ are given. From this figure, we find that the energy of [411 1/2] orbit around $Z = 70$ gradually increases with increasing γ-value so that the [411 1/2] orbit goes far from the Fermi surface. This calculated result coincides with that in Ref. 6. Therefore, it is incorrect that the [411 1/2] orbit approaches the Fermi surface at large values of triaxial deformation as thought by the authors of Ref. 4. The position of [411 1/2] orbit is extremely important for a correct understanding of the ground-state spin $I = 1/2^+$ in ^{157}Tm. In Table 1, the five single-particle wavefunction structure of positive-parity states in the region $Z = 70$ for the deformation parameters $\varepsilon_2 = 0.230$, $\varepsilon_4 = 0.01$, and $\gamma = 35°$ are given. Because of the triaxiality, there is a serious configuration mixing. The main components of 17th, 18th, 19th, 20th, and 21st single-particle orbits are [411 3/2] (which amounts to 52.6% of the total wavefunction), [404 7/2] (63.5%), [402 5/2] (54.5%), [411 1/2] (44.5%), and [402 3/2] (39.7%), respectively. However, other components of these orbits still amount to a rather large ratio; e.g., [402 3/2] amounts to 18.1% in 18th orbit, and [404 7/2] to 19.7% and [400 1/2] to 14.1% in 19th orbit.

In Fig. 2, a comparison between the theoretical and the experimental levels of the low excited positive-parity states for ^{157}Tm is given. The theoretical spectra are labeled by $\bar{K}\nu$, where $\bar{K}$ denotes

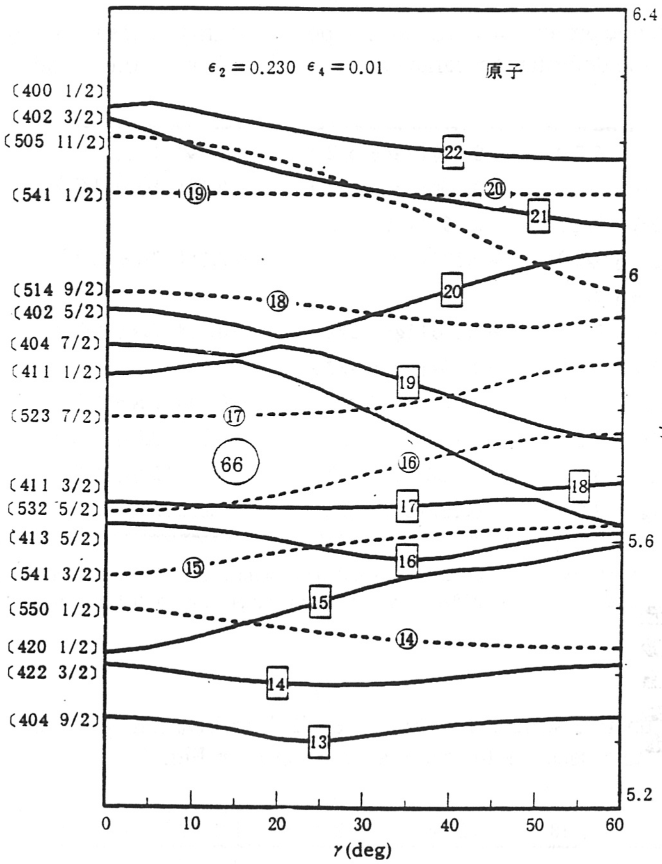

Fig. 1

Single proton levels of the Nilsson potential in the region $Z = 70$ for $\varepsilon_2 = 0.230$, $\varepsilon_4 = 0.01$, and $0° \leq \gamma \leq 60°$.

The positive (full curves) and the negative (dashed curves) parity levels are labeled by their sequence inserted in squares or circles, respectively. On the left of figure, the levels are labeled by their Nilsson quantum number $[Nn_z\Lambda\ \Omega]$ referring to the main component of the wavefunction at $\gamma = 0°$ (axial symmetry).

the K-value of the largest component of the total wavefunction and ν is the level sequence in the single-particle level diagram. From these figure, we can see that (1/2 19) and (3/2 18) bands correspond to the observed ground band and an excitation band, respectively [5]. While both the theoretical (5/2 19) and (7/2 18) bands have not been identified in the experiment yet. It should be pointed out that the Coriolis attenuation factor $\xi = 0.6$ is introduced in the calculations to reproduce the observed level sequence in ^{157}Tm. In Table 2, some wavefunctions of the low excited positive-parity states in ^{157}Tm are given. The base vectors are expressed by $|K\nu\rangle$. Using Tables 1 and 2, one can estimate the contribution from the different intrinsic states. From Table 1 we see that there is a considerable configuration mixing in the single-particle states. In the 19th single-particle orbit, [402 5/2] amounts to 54.5% of the total wavefunction, [404 7/2] to 19.7% and [400 1/2] to 14.1%, while [411 1/2] only amounts to 2.4% of the total wavefunction. From Table 2, for $I = (1/2^+)_1$ state (the bandhead of (1/2 19) band), the $|1/2\ 19\rangle$ state amounts to 87%. Due to triaxiality, $K \neq \Omega$, all intrinsic states in the $\nu = 19$ orbit have made contribution to the $|1/2\ 19\rangle$ state. The

Table 1

Five single proton wavefunctions of positive-parity states in the region $Z = 70$ for the deformation parameters $\varepsilon_2 = 0.230$, $\varepsilon_4 = 0.01$, and $\gamma = 35°$.

$\vert 17\rangle = +0.532\vert 1g_{7/2}\ 5/2\rangle$	$-0.106\vert 2d_{5/2}\ 5/2\rangle$	$-0.140\vert 1g_{7/2}\ 1/2\rangle$	$+0.136\vert 2d_{3/2}\ 1/2\rangle$
$-0.168\vert 3s_{1/2}\ 1/2\rangle$	$-0.174\vert 1g_{9/2}\ 3/2\rangle$	$-0.181\vert 1g_{7/2}\ 3/2\rangle$	$+0.726\vert 2d_{5/2}\ 3/2\rangle$
$+0.116\vert 2d_{3/2}\ 3/2\rangle$	$+0.162\vert 1g_{9/2}\ 7/2\rangle$		
$\vert 18\rangle = -0.286\vert 2d_{5/2}\ 5/2\rangle$	$+0.135\vert 2d_{3/2}\ 1/2\rangle$	$+0.251\vert 3s_{1/2}\ 1/2\rangle$	$+0.425\vert 2d_{3/2}\ 3/2\rangle$
$-0.797\vert 1g_{7/2}\ 7/2\rangle$			
$\vert 19\rangle = -0.204\vert 1g_{9/2}\ 9/2\rangle$	$-0.176\vert 1g_{7/2}\ 5/2\rangle$	$-0.738\vert 2d_{5/2}\ 5/2\rangle$	$+0.156\vert 2d_{3/2}\ 1/2\rangle$
$+0.376\vert 3s_{1/2}\ 1/2\rangle$	$+0.444\vert 1g_{7/2}\ 7/2\rangle$		
$\vert 20\rangle = +0.286\vert 1g_{7/2}\ 5/2\rangle$	$-0.327\vert 1g_{7/2}\ 1/2\rangle$	$+0.340\vert 2d_{5/2}\ 1/2\rangle$	$+0.667\vert 2d_{3/2}\ 1/2\rangle$
$-0.255\vert 3s_{1/2}\ 1/2\rangle$	$+0.151\vert 1g_{7/2}\ 3/2\rangle$	$-0.361\vert 2d_{5/2}\ 3/2\rangle$	
$\vert 21\rangle = -0.430\vert 2d_{5/2}\ 5/2\rangle$	$+0.150\vert 2d_{5/2}\ 1/2\rangle$	$-0.201\vert 2d_{3/2}\ 1/2\rangle$	$-0.427\vert 3s_{1/2}\ 1/2\rangle$
$+0.145\vert 1g_{7/2}\ 3/2\rangle$	$+0.107\vert 2d_{5/2}\ 3/2\rangle$	$-0.630\vert 2d_{3/2}\ 3/2\rangle$	$-0.344\vert 1g_{7/2}\ 7/2\rangle$

The wavefunctions are labeled according to their sequence in the single-particle level diagram and expanded in the basis $|Nlj\Omega\rangle$. (Only the main components > 0.1 are tabulated.)

Table 2

Wavefunction structure of the low excited positive-parity states in ^{157}Tm. Parameters used are the same as those used in Fig. 2.

I	$\vert 1/2\ 18\rangle$	$\vert 3/2\ 18\rangle$	$\vert 5/2\ 18\rangle$	$\vert 7/2\ 18\rangle$	$\vert 1/2\ 19\rangle$	$\vert 3/2\ 19\rangle$	$\vert 5/2\ 19\rangle$	$\vert 7/2\ 19\rangle$
$(1/2)_1$	−0.325				+0.933			
$(5/2)_1$			−0.277		−0.291		+0.880	
$(7/2)_1$		+0.252		−0.886				−0.286
$(3/2)_1$		−0.925			+0.220	−0.274		
$(3/2)_2$	−0.368	+0.252			+0.839			

The base vectors are represented by $|K\nu\rangle$. (Only the main components > 0.2 are tabulated.)

contribution not only comes from [400 1/2] (12.3%) and [411 1/2] (2.1%) but also mainly from [402 5/2] (47.4%) and [404 7/2] (17.1%). For $I = (3/2^+)_1$ state (the bandhead of (3/2 18) band), $|3/2\ 18\rangle$ amounts to 85%. All different intrinsic states in the $\nu = 18$ orbit have a contribution to it, e.g., [404 7/2] amounts to 54.3%, [402 3/2] to 15.4%, and [402 5/2] to 7%. While for both (5/2 19) and (7/2 18) bands, [402 5/2] is predominates in the wavefunction of the 19th orbit while [404 7/2] predominates in 18th orbit. Therefore, the (5/2 19) band is usually called the [402 5/2] band, and the (7/2 18) band is called the [404 7/2] band.

3.2. ^{159}Tm

A comparison between the theoretical and the experimental energy spectra of the low excited positive-parity states in ^{159}Tm is given in Fig. 3. By examining the distribution of the single-particle wavefunctions, we find that in the deformation parameters $\varepsilon_2 = 0.240$, $\varepsilon_4 = 0.01$, and $\gamma = 38°$, the

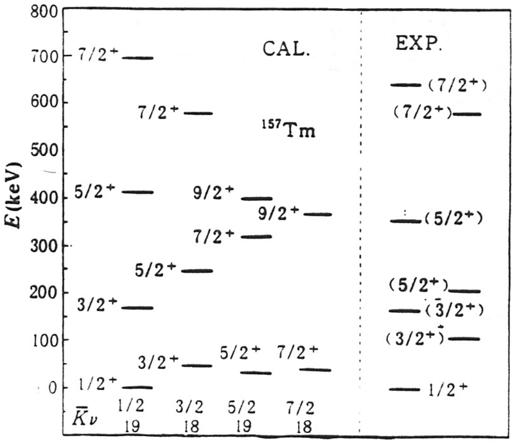

Fig. 2

Comparison between the theoretical and the experimental levels of the low excited positive-parity states for ^{157}Tm.

Parameters of the particle-rotor model with VMI are: deformation parameters $\varepsilon_2 = 0.230$, $\varepsilon_4 = 0.01$, and $\gamma = 35°$; VMI [15] parameters $A_{00} = 0.0549$ MeV, stiff $C = 0.002$ MeV^3, and Coriolis attenuation factor $\xi = 0.6$.

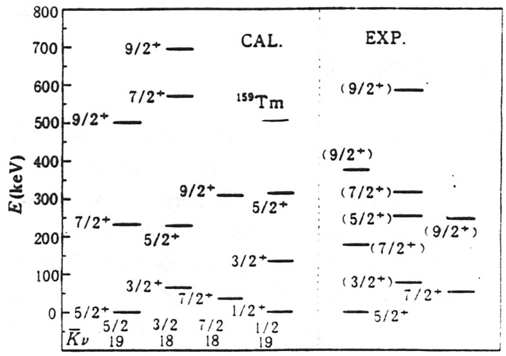

Fig. 3

Comparison between the theoretical and the experimental levels of the low excited positive-parity states for ^{159}Tm.

Parameters of the particle-rotor model with VMI are: deformation parameters $\varepsilon_2 = 0.240$, $\varepsilon_4 = 0.01$, and $\gamma = 38°$; VMI [15] parameters $A_{00} = 0.030$ MeV, stiff $C = 0.001$ MeV^3, and Coriolis attenuation factor $\xi = 0.93$.

[404 7/2] state amounts to 56% of the total wavefunction and [402 3/2] to 21% for $\nu = 18$ orbit. While in the case of $\nu = 19$ orbit, the [402 5/2] state amounts to 48.7% of the total wavefunction, [404 7/2] to 24.8%, [400 1/2] to 14.3%, and [411 1/2] only to 2.7%. Consequently, it is impossible that the observed levels [2] of 77.8, 253.8, 316.9, and 584.3 keV are members of the [411 1/2] band. They are most likely to belong to the (2/3 18) band, the intrinsic states of which originate not only from the [402 2/3] state but also from the others. Since the ground band (5/2 19) is dominated by the [402 5/2] component which may be called the [402 5/2] band. Moreover, the (7/2 18) band corresponds to the [404 7/2] band. The theoretical (1/2 19) band has not been observed yet, but the level $I = 1/2^+$ estimated in Ref. 2 lies very close to the ground state which is most likely the bandhead of the (1/2 19) band.

4. CONCLUSIONS

Using the triaxial-particle-rotor model with VMI and considering the admixture of 14 single-particle orbits around Fermi level, we investigate the properties of low excited positive parity states in ^{157}Tm and ^{159}Tm.

(1) A good agreement between the theoretical and the experimental spectra in ^{157}Tm and ^{159}Tm is obtained. The deformation parameters used are $\varepsilon_2 = 0.230$, $\varepsilon_4 = 0.01$, and $\gamma = 35°$ for ^{157}Tm and $\varepsilon_2 = 0.240$, $\varepsilon_4 = 0.01$, and $\gamma = 38°$ for ^{159}Tm.

(2) Due to triaxiality, $K \neq \Omega$, there exists a serious configuration mixing. It is proposed that a rotational band is labeled by $(\bar{K}\nu)$, where $\bar{K}$ denotes the K value of the largest component of the total wavefunction and ν is the level sequence in the single-particle energy diagram.

(3) The observed 0, 164.7, 353.9, and 640.0 keV levels in ^{157}Tm are members of a rotational band, which corresponds to the theoretical (1/2 19) band, while 105.7, 206.7, and 530.2 keV are most

likely to be included in the (3/2 18) band.

(4) It is impossible that the observed [2] of 77.8, 253.8, 316.9, and 584.3 keV in ^{159}Tm are members of [411 1/2] band. They most likely belong to the (3/2 18) band. The level $I = 1/2^{+}$ estimated in Ref. 2 is most likely the bandhead of the (1/2 19) band.

REFERENCES

[1] M.A. Lee, *Nucl. Data Sheets*, **53**(1988), p. 507.
[2] P. Tlusty *et al.*, *Z. Phys.*, **A341**(1992), p. 435.
[3] C. Ekström *et al.*, *Z. Phys.*, **A316**(1984), p. 239.
[4] G.D. Alkhazov *et al.*, *Nucl. Phys.*, **A477**(1988), p. 37.
[5] X. Shuwei *et al.*, *Phys. Rev.*, **C50**(1994), p. 3147.
[6] S.E. Larsson, G. Leander, and I. Ragnarsson, *Nucl. Phys.*, **A307**(1978), p. 189.
[7] R. Holzmann *et al.*, *Phys. Rev.*, **C31**(1985), p. 421.
[8] I. Hamamoto and H. Sagawa, *Phys. Lett.*, **B201**(1988), p. 415.
[9] I. Ragnarsson and P.B. Semmes, Lund-Mph-88/12.
[10] D. Lieberz *et al.*, *Nucl. Phys.*, **A529**(1991), p. 1.
[11] Ch. Vieu *et al.*, *J. Phys.*, **G4**(1978), p. 531.
[12] T. Bengtsson and I. Ragnarsson, *Nucl. Phys.*, A436(1985), p. 14.
[13] S.E. Larsson, *Physica Scripta*, **8**(1973), p. 17.
[14] S.G. Nilsson *et al.*, *Nucl. Phys.*, **A131**(1969), p. 1.
[15] M.A.J. Mariscotti *et al.*, *Phys. Rev.*, **178**(1969), p. 1864.