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Analytical Calculation for the Fixed Length Lattice *SU*(2)-Higgs Model

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The phase diagram of the lattice system of the *SU*(2) gauge field coupled with the fixed length Higgs field in the fundamental representation has been calculated analytically to the fourth order of the cumulant expansion. The variational parameters have been determined by scanning method. The obtained phase diagram is in fine agreement with the Monte Carlo result. The calculation also has been made for the Polyakov line $\langle L \rangle$ of the *SU*(2) Higgs and the pure *SU*(2) models at finite temperature ($N_\tau = 1$). The best (up to now) approximate analytical result β_c for the pure *SU*(2) model is obtained.

Key words: *SU*(2)-Higgs, cumulant expansion, lattice, phase diagram, fixed length.

1. INTRODUCTION

A series of studies, theoretical analyses [1] and Monte Carlo simulations [2-4], on the lattice system of the *SU*(2) gauge field coupled with the fixed length Higgs field in the fundamental representation has pointed out the existence of the analytical connected confining phase and the Higgs phase. The phase diagram which is in agreement with Monte Carlo simulations has been obtained analytically first by use of the variational cumulant expansion method (VCE) [5]. The free energy is calculated to the third order cumulant expansion, and the variational parameters are determined by the variational minimum condition of the free energy in the second order approximation F_2 w.r.t. the variational parameter. In the variational method with Lagrangian formalism [6], the variational

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parameter is determined by the variational condition of the free energy in the first order approximation F_1. Because of the Jensen inequality, using this condition to get the upper bound of the free energy is justified theoretically. However, using the variational condition for F_2 has not been justified, and it is only used as an effective approximation. While considering higher order cumulant expansions, we have tried to use the variational condition for F_3, the free energy in the third order approximation, but the result is unsatisfactory. Therefore, using the variational conditions for successive orders of F_i to determine parameters cannot be used as a systematic practical approach.

In order to obtain more reliable and quantitative results by VCE, to calculate to higher order expansions is necessary and hopeful. To this end, a systematic approach to determine variational parameters is needed. An "accumulation point" approach suggests to look for the intersecting point of the scanning curves of different orders — the "accumulation point". At the "accumulation point", the results from each orders have the fastest convergent behavior [7]. Nevertheless, the "accumulation points" exist clearly only in the weak coupling region. This leads Kerler and Metz to state that the cumulant expansion method is applicable only in the weak coupling region [7]. However, as has been shown in the study of higher order expansions of $U(1)$ model at $N_\tau = 1$ case [8], it is not applicable just the "accumulation point" approach itself, which is looking for the intersecting point of all orders, but the cumulant expansion method. For some physical quantities, such as the Polyakov line $\langle L \rangle$, the fastest convergent behavior in the weak coupling region reveals in an existence of "accumulation point," while in the intermediate coupling region it reveals in an asymptotical behavior of smaller corrections with higher orders of expansion. The scanning approach should be applied separately to different models and physical quantities. Thus it is worth studying the convergency of the cumulant expansion for the $SU(2)$-Higgs model by scanning approach to determine the variational parameters for higher order expansions.

This paper is the continuation of the work [5]. Now, the free energy F_i is calculated by the cumulant expansion method to one order higher from one side, and of the other side, the scanning approach for determining parameters is studied. The calculated phase transition line is in fine agreement with the Monte Carlo simulations. The phase transition behavior of the model at high temperature ($N_\tau = 1$) is also calculated. Since the fixed length $SU(2)$-Higgs model has a very similar structure with the pure $SU(2)$ gauge model in the cumulant expansion treatment; thus, by the way, we have also studied the temperature deconfining phase transition of the $SU(2)$ gauge model at $N_\tau = 1$, and have obtained the result in fine agreement with the Monte Carlo simulations. It is the best up-to-date approximate analytical result.

2. PHASE DIAGRAM AT ZERO TEMPERATURE

Consider a coupled system of $SU(2)$ gauge field with the fixed length Higgs field in the fundamental representation on a four-dimensional ($d = 4$) hypercubic lattice with the action [2,5]

$$S = S_G + \kappa \sum_l \mathrm{tr} U_l \tag{1}$$

$$S_G = \beta \sum_P \mathrm{tr} U_P \tag{2}$$

where S_G is the Wilson action, $\beta = 2/g^2$, κ is a coupling constant. Notations without explanations are the same as in [5]. According to VCE [9], we introduce a trial action

$$S_0 = J \sum_l \mathrm{tr} U_l \tag{3}$$

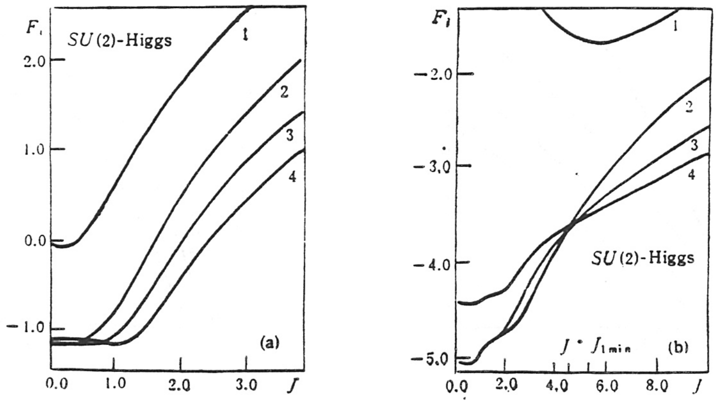

Fig. 1
The scanning pattern of free energies $F_i (i = 1, 2, 3, 4)$ for J.
(a) for $\kappa = 0.4$, $\beta = 1.0$; (b) for $\kappa = 0.4$, $\beta = 2.5$.

The i-th order approximate free energy per site F_i can be expressed as

$$F_i = F_0 - \frac{1}{M}\sum_{n=1}^{i}\frac{1}{n!}K_n \tag{4}$$

$$K_n = \left\langle\left[S_G + (\kappa - J)\sum_l \mathrm{tr}U_l\right]^n\right\rangle_c \tag{5}$$

$$F_0 = -d \cdot \ln\left(\frac{I_1(2J)}{J}\right) \tag{6}$$

where $\langle\cdots\rangle_c$ denotes a cumulant average. We have expanded it to the fourth order, F_4. Although the calculation to the 4-th order is much more complicated, there is no principle difficulty, so the technical details will not be presented here.

Let us consider how to determine the variational parameters. For fixed β, κ, the typical scanning pattern of $F_i(\beta,\kappa,J)$ with respect to J is shown in Fig. 1.

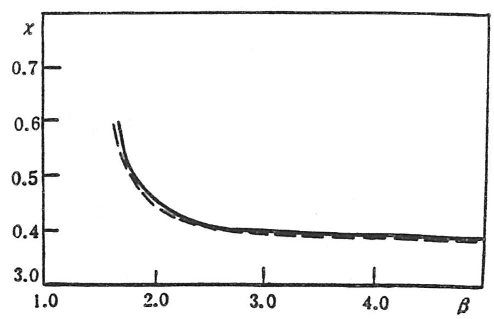

Fig. 2
The phase diagram of the fixed length $SU(2)$-Higgs model at zero temperature. The dashed line is the Monte Carlo result [3].

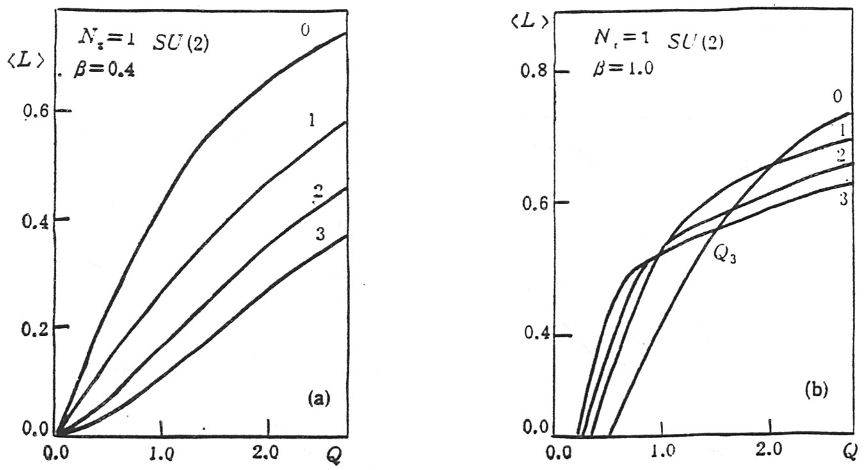

Fig. 3

The scanning pattern of Polyakov line $\langle L\rangle_i (i = 1, 2, 3)$ for Q (a) $N_\tau = 1$, $\beta = 0.4$, for the pure $SU(2)$ model. (b) $N_\tau = 1$, $\beta = 1.0$, for the pure $SU(2)$ model.

For $\beta = 1.0$, the system is in the strong coupling region, F_i converges to the strong coupling value corresponded to $J \approx 0$; For $\beta = 2.5$, F_2, F_3, F_4, intersect in a small region about $J^* \approx 4.6$, which appears approximately as an "accumulation point". The values of F_i show a good convergency. There is a clear minimum for F_1 at $J_{1\min}$, which is not far from J^*. It justifies that by substituting $J_{1\min}$, determined by the variational condition for F_1, into successive higher orders of F_i, we can get improved results. They converge more slowly only. The advantage of the scanning approach is that J^*, corresponding to the fastest convergency of $F_i(\beta,\kappa,J^*)$, can be caught at once. For fixed κ, changing β we will get $F \approx F_4(\beta)$. The discontinuity of the first order derivative of F w.r.t. β gives the first order phase transition point. With increasing κ, this first order phase transition is gradually weakened and disappears finally. The obtained phase diagram shown in Fig. 2 is in good agreement with the Monte Carlo simulation [3] presented by the dashed line.

3. CASE OF FINITE TEMPERATURE (N_τ=1)

The VCE calculation for the varied length $SU(2)$-Higgs model at finite temperature on the lattice has been taken at $N_\tau = 2$ [10]. Here we calculate it when $N_\tau = 1$. As above, we will use the scanning approach to determine the variational parameters. The Polyakov line defined through U_τ on time-like is

$$L = \frac{1}{2}\,\mathrm{tr}U_\tau(x,t) \tag{7}$$

Its statistical average $\langle L\rangle$ may serve as an indicator of deconfining phase transitions [11]. We introduce the corresponding trial action as

$$S_0 = J\sum_\sigma \mathrm{tr}U_\sigma + Q\sum_\tau \mathrm{tr}U_\tau \tag{8}$$

The summations are taken over all space-like and time-like links respectively. To the i-th order approximation

Table 1

	VEC			Mean field	Strong coupling expansion	Monte Carlo
	1st order appr.	2nd order appr.	3rd order appr.			
β_c	0.66666	0.73029	0.75964	0.68 [11] 0.676 [12]	0.96 [13]	0.75 [14]

$$\langle L\rangle \cong \langle L\rangle_i = \langle L\rangle_0 + \sum_{n=1}^{i} \frac{1}{n!} \langle L(S-S_0)^n\rangle_c \tag{9}$$

Similar to the $U(1)$ model [8], to the 3rd order ($i = 3$) all of the cumulant averages $\langle\cdots\rangle_c$ of the connected diagrams with space-like plaquettes or pure space-like links finish by the periodic condition in the time-like direction in $N_\tau = 1$ case. Thus, to the 3rd order approximation we have

$$\langle L\rangle \cong \langle L\rangle_3 = \langle L\rangle_0 + \sum_{n=1}^{3} \frac{1}{n!} \left\langle L\left[S_G + (\kappa - Q)\sum_{\tau} \mathrm{tr}U_\tau\right]^n\right\rangle_c \tag{10}$$

$\langle L\rangle_i (i \le 3)$ will depend only on β, κ and Q. For fixed β, κ, the fastest convergent point for $\langle L\rangle_i$ of each order will be determined by scanning with Q.

First we look at the pure $SU(2)$ gauge model with $\kappa = 0$. Its scanning pattern is similar to that of the $U(1)$ case [7] as shown in Fig. 3. The variational parameters are chosen to be Q_i^* corresponding the intersecting points of $\langle L\rangle_i$ and $\langle L\rangle_0$. In the strong coupling region $Q_i^* = 0$ leads to $\langle L\rangle = 0$ (Fig. 3(a)). Leaving from the strong coupling region, Q_i^* decrease with increasing orders along $\langle L\rangle_0$ (Fig. 3(b)). At points Q_i^*, the i-th order corrections to $\langle L\rangle_0$ are zero.

$$\Delta_i\langle L\rangle = \langle L\rangle_i|_{Q=Q_i^*} - \langle L\rangle_0|_{Q=Q_i^*} = \sum_{n=1}^{i} \frac{1}{n!}\langle L(S-S_0)^n\rangle_c = 0 \tag{11}$$

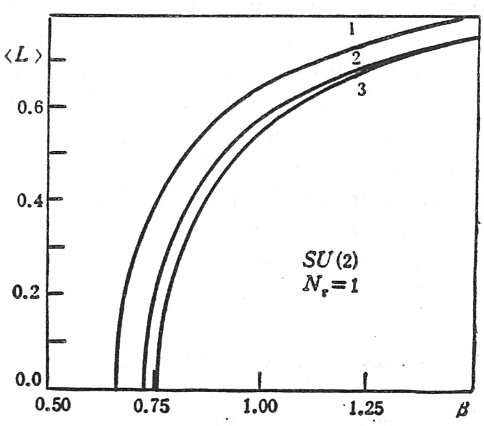

Fig. 4
The Polyakov lines $\langle L\rangle$-β for the first to third order approximations.

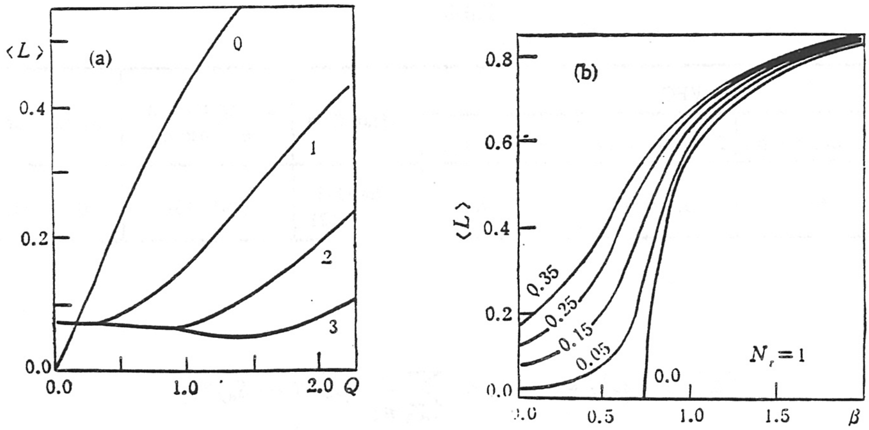

Fig. 5

(a) The scanning pattern of Polyakov line $\langle L\rangle_i (i = 1, 2, 3)$ of the fixed $SU(2)$-Higgs model with Q ($N_\tau = 1$, $\beta = 0.0$, $\kappa = 0.15$). (b) The $\langle L\rangle$-β relations in the third order approximation (κ = 0.0, 0.05, 0.15, 0.25, 0.35, $N_\tau = 1$).

Then

$$\langle L\rangle(\beta) \cong \langle L\rangle_i(\beta)\big|_{Q=Q_i^*} \tag{12}$$

Thus obtained $\langle L\rangle$-β relation is presented in Fig. 4. The corrections which are smaller and smaller with increasing orders show the convergency of the expansion. The first appearances of $\langle L\rangle_i \neq 0$ from $\langle L\rangle = 0$ with increasing β give the deconfining phase transition points β_{ci} in the i-th order approximation. Their values are listed in the Table 1 with results obtained with other methods. Notice that the calculated result of the first order approximation is very close to the mean field result. It is natural, since the early variational approach, which is equivalent to the first order approximation of the cumulant expansion with the parameter determined by the variational condition from F_1, combined with the mean field idea can be used to derive the mean field equation [15]. With higher orders of expansion, β_{ci} nicely converges to the value given by the Monte Carlo simulations. It is the best result obtained by approximate analytical methods up to now. The results also show that the strong coupling result really is rather larger, the reason may lay in the strong coupling method itself [13].

Now let us look at the case with $\kappa \neq 0$. Due to including the Higgs field, in the strong coupling region the scanning curves of various $\langle L\rangle_i$ do not accumulate to the point $Q = 0$ again. But with the variational parameters determined by Eq.(11), the expansion still shows a good convergent behavior in the whole range of coupling constant. For example, Fig. 5(a) demonstrates the scanning result for $\beta = 0$. The parameter value corresponded to the convergency is located at small Q and $\langle L\rangle|_{Q=Q^*} \neq 0$. The results for several values of κ are presented in Fig. 5(b), which are very similar to the corresponding behavior at $N_\tau = 2$ [10]. For $\kappa \neq 0$, no phase transition occurs. There is no Monte Carlo result to be compared with.

The results of this paper show that for the fixed length $SU(2)$-Higgs system in the fundamental representation a good convergent result of $\langle L\rangle$, calculated by the cumulant expansion with the parameters determined by scanning using Eq.(11), can be obtained in the whole range of coupling constant. It is similar to the calculation of $\langle L\rangle$ for $U(1)$ system. It looks as if there will be an universality for different group models. This is under further study.

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