第 17 卷 第 3 期　　高能物理与核物理　　Vol. 17, No. 3
1993 年 3 月　　HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS　　Mar., 1993

阿贝尔手征群陪集纯规范场的恒等式和重整化*

邱　荣
(福州大学物理系，350002)

摘　要

导出了与“阿贝尔手征群陪集空间纯规范场的生成泛函路径积分测度和有效作用量联合起来在手征群变换下具有不变性”相应的恒等式. 利用此恒等式建造了重整化方程，并由方程的解将阿贝尔手征群陪集纯规范场理论重整化.

通常，导出与 BRS 不变性相应的 W-T 恒等式时，生成泛函积分测度与有效作用量各自具有 BRS 不变性[1]. 但当群 G 是手征群时，由于生成泛函中费米场的路径积分测度在手征变换下的改变[2]，使得阿贝尔手征群陪集纯规范场理论的生成泛函作用量中增加了附加项，破坏了通常规范场理论的 BRS 不变性. 幸而，在任意规范下，费米场路径积分测度的改变和有效作用量的改变抵消，两者联合起来仍具有手征群变换下的不变性，因而我们可在文献[3]完成了这种场的量子化问题的基础上进一步来导出与“阿贝尔手征群陪集空间纯规范场生成泛函路径积分测度和有效作用量联合起来在手征群变换下具有不变性“相应的恒等式. 再由这恒等式和鬼方程出发建造重整化方程，并用方程的解将阿贝尔手征群陪集空间纯规范场理论重整化.

一

考虑最简单的手征群 $G = U(1) \times U(1)_5$. 在子群 $H = U(1)$ 上定域不变的拉氏函数密度可写成:

$$\mathscr{L}^{(A)} = -\frac{1}{4} F_{\mu\nu}^2 - \bar{\psi}\gamma_\mu(\partial_\mu - ieA_\mu)\psi - m\bar{\psi}\psi, \tag{1}$$

式中 A_μ 是子群 $H = U(1)$ 的矢量规范场，$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. 引入陪集空间纯规范场 $\phi_0(x) \in G/H$，它可以参数化为 $\phi_0(x) = e^{i\theta(x)\gamma_5}$，对费米场 $\psi(x)$、$\bar{\psi}(x)$ 作手征变换:

$$\psi''(x) = e^{i\theta(x)\gamma_5}\psi(x), \quad \bar{\psi}''(x) = \bar{\psi}(x)e^{i\theta(x)\gamma_5}, \tag{2}$$

并利用 $\gamma_\mu\gamma_5 = -\gamma_5\gamma_\mu$，可将 $\mathscr{L}^{(A)}$ 写成:

本文 1991 年 11 月 5 日收到.
* 福建省自然科学基金资助.

$$\mathscr{L}^{(B)}=-\frac{1}{4}F^{a}_{\mu\nu}-\bar{\phi}\gamma_{\mu}(\partial_{\mu}-ieA_{\mu}-i\gamma_5\partial_{\mu}\theta)\phi-m\bar{\phi}e^{-2i\theta(x)\gamma_5}\phi(x), \tag{3}$$

已略去 $\bar{\phi}$ 和 ϕ 上的“″”号。与(3)式相应，任意规范 $F^{a}_{B}[\phi、\bar{\phi}、A、\theta]=0$ 下的格林函数生成泛函路径积分，在引入鬼场 C^{+a}、C^{a} 并完成量子化后可写成：($\alpha=1,2$)

$$\begin{aligned}Z^{[B]}=&\int[\mathrm{d}\phi\mathrm{d}\bar{\phi}\mathrm{d}A_{\mu}\mathrm{d}\theta\mathrm{d}C^{+a}\mathrm{d}C]\exp i\left\{\int\mathrm{d}^4x\left[\mathscr{L}^{(B)}+\theta G\right.\right.\\&\left.\left.-\frac{1}{2\alpha}(F^{a}_{B}[\phi、\bar{\phi}、A_{\mu}、\theta])^2\right]+\int\mathrm{d}^4x\mathrm{d}^4yC^{+a}(x)M_{\alpha\beta}(x,y)C^{\beta}(y)\right\}\\=&\int[\mathrm{d}\phi]e^{iS_{\rm eff}},\end{aligned} \tag{4}$$

式中 $[\mathrm{d}\phi]=[\mathrm{d}\phi\mathrm{d}\bar{\phi}\mathrm{d}A_{\mu}\mathrm{d}\theta\mathrm{d}C^{+a}\mathrm{d}C]$,

$$S_{\rm eff}=\int\mathrm{d}^4x\left[\mathscr{L}^{(B)}+\theta G-\frac{1}{2\alpha}(F^{a}_{B})^2\right]+\int\mathrm{d}^4x\mathrm{d}^4yC^{+a}(x)M_{\alpha\beta}(x,y)C^{\beta}(y), \tag{5}$$

$$M=(M_{\alpha\beta}(x,y))=\left(\frac{\delta F^{a\,g}_{B}(x)}{\delta u_{\beta}(y)}\right)_{g=1}=\begin{pmatrix}\dfrac{\delta F^{1g}_{B}(x)}{\delta u_1(y)} & \dfrac{\delta F^{1g}_{B}(x)}{\delta u_2(y)}\\ \dfrac{\delta F^{2g}_{B}(x)}{\delta u_1(y)} & \dfrac{\delta F^{2g}_{B}(x)}{\delta u_2(y)}\end{pmatrix}_{g=1}, \tag{6}$$

$\theta G=-\theta\dfrac{ie^2}{8\pi^2}F_{\mu\nu}\tilde{F}_{\mu\nu}$，这是由于费米场在手征变换下引起生成泛函积分测度变化而引入的[2]，即

$$\begin{aligned}[\mathrm{d}\phi''\mathrm{d}\bar{\phi}'']&=[\mathrm{d}\phi\mathrm{d}\bar{\phi}]e^{-2i\mathrm{tr}\theta(x)\gamma_5}=[\mathrm{d}\phi\mathrm{d}\bar{\phi}]e^{i\int\mathrm{d}^4x\frac{ie^2}{8\pi^2}\theta F_{\mu\nu}\tilde{F}_{\mu\nu}},\\ [\mathrm{d}\phi\mathrm{d}\bar{\phi}]&=[\mathrm{d}\phi''\mathrm{d}\bar{\phi}'']e^{-i\int\mathrm{d}^4x\theta\frac{ie^2}{8\pi^2}F_{\mu\nu}\tilde{F}_{\mu\nu}}.\end{aligned} \tag{7}$$

可以证明，$\mathscr{L}^{(B)}$ 在下列变换

$$\begin{cases}\delta\phi(x)=i[C_1(x)+\gamma_5C_2(x)]\xi\phi(x),\ \delta\bar{\phi}(x)=i\bar{\phi}(x)[-C_1(x)+\gamma_5C_2(x)]\xi\\ \delta A_{\mu}(x)=\dfrac{1}{e}\partial_{\mu}(C_1(x)\xi),\qquad \delta\theta(x)=C_2(x)\xi\\ \delta C^{+a}(x)=-\dfrac{1}{\alpha}F^{a}_{B}[\phi,\bar{\phi},A,\theta]\xi,\qquad \delta C^{a}(x)=0\quad(\alpha=1,2)\end{cases} \tag{8}$$

下不变，式中 ξ 是 Grassmann 常数。实际上

$$\begin{aligned}\delta F^2_{\mu\nu}&=2F_{\mu\nu}\delta F_{\mu\nu}=2F_{\mu\nu}\delta(\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu})=0,\\ \delta[\bar{\phi}\gamma_{\mu}(\partial_{\mu}-ieA_{\mu}-i\gamma_5\partial_{\mu}\theta)\phi]&=(\delta\bar{\phi})\gamma_{\mu}(\partial_{\mu}-ieA_{\mu}-i\gamma_5\partial_{\mu}\theta)\phi\\&\quad+\bar{\phi}\gamma_{\mu}(-ie\delta A_{\mu}-i\gamma_5\partial_{\mu}\delta\theta)\phi+\bar{\phi}\gamma_{\mu}(\partial_{\mu}-ieA_{\mu}-i\gamma_5\partial_{\mu}\theta)\delta\phi=0,\\ \delta(\bar{\phi}e^{-2i\theta(x)\gamma_5}\phi)&=[(\delta\bar{\phi})\phi+\bar{\phi}\delta\phi-2i\bar{\phi}\gamma_5(\delta\theta)\phi]e^{-i2\theta\gamma_5}=0,\end{aligned}$$

所以 $\delta\mathscr{L}^{(B)}=0$。

又可以证明： $\delta\delta\bar{\phi}=\delta\delta\phi=\delta\delta A_{\mu}=\delta\delta\theta=\delta\delta C_{\alpha}=0$[4], (9)

实际上 $\delta\delta\bar{\phi}=\delta[i\bar{\phi}(x)(-C_1+\gamma_5C_2)\xi]=i\bar{\phi}(x)[-C_1+\gamma_5C_2]\xi i[-C_1+\gamma_5C_2]\xi=0$，因 $\xi^2=0$ 或 $\because C_1^2=C_2^2=0$ 和 $C_1C_2+C_2C_1=0$，同理 $\delta\delta\phi=0$ 又

$$\delta\delta A_{\mu}=\frac{1}{e}\partial_{\mu}\delta C_1(x)\xi=0,\quad \delta\delta\theta=\delta C_2(x)\xi=0.$$

引入和 ψ、$\bar\psi$、A_μ、θ、C_α、C^+_α、$\delta\psi'$、$\delta\bar\psi'$、$\delta A'_\mu$、$\delta\theta'$、$\delta C'$ 相应的外源 $\bar\eta$、η、J_μ、Ω、ζ^+_α、ζ_α、$\bar K$、K、U_μ、ω、V_α。对应量的对易性质相同，而 $\delta\psi'$、$\delta\bar\psi'$、$\delta A'_\mu$、$\delta\theta'$、$\delta C'$ 中各已从 $\delta\psi$、$\delta\bar\psi$、δA_μ、$\delta\theta$、δC 中除去 ξ 因子，因而和 ψ、$\bar\psi$、A_μ、θ、C 的对易性质相反。这样可把格林函数生成泛函 $Z^{(B)}(0)$ 扩充成：

$$Z[\bar\eta,\eta,J_\mu,\Omega,\zeta^+_\alpha,\zeta_\alpha,\bar K,K,U_\mu,\omega,V_\alpha] = \int[d\phi]e^{iS_{\mathrm{eff}}}\exp i\int d^4x[\bar\eta\psi + \bar\psi\eta + J_\mu A_\mu + \Omega\theta + \zeta^+_\alpha C^\alpha + C^+_\alpha\zeta^\alpha + \bar K\delta\psi' - \delta\bar\psi' K - U_\mu\delta A'_\mu - \omega\delta\theta' + V^\alpha\delta C'_\alpha], \tag{10}$$

积分是不会由于变换积分变量而改变积分值的。因而格林函数生成泛函(10)式，在(8)式变换下，应具有不变性。已知 $\mathscr{L}^{(B)} = \mathscr{L}'^{(B)}$ 和(9)式，又由(8)式知，$A_\mu(x)$、$\theta(x)$、$C^+_\alpha(x)$ 和 $C^\alpha(x)$ 的积分测度是不变的，而 $\psi(x)$、$\bar\psi(x)$ 的变换也可表示为：

$$\psi''(x) = e^{i[C_1(x)+\gamma_5 C_2(x)]\xi}\psi(x),\ \bar\psi''(x) = \bar\psi(x)e^{i[-C_1(x)+\gamma_5 C_2(x)]\xi}.$$

于是费米场积分测度的变化为：

$$[d\psi d\bar\psi] = [d\psi'' d\bar\psi'']e^{2i\,\mathrm{tr}C_2(x)\xi\gamma_5}.$$

利用[3]

$$2\mathrm{tr}C_2(x)\xi\gamma_5 = \int d^4x\,\mathrm{tr}C_2(x)\xi\gamma_5(e^{i\not D_L(x)\not D_R(x)/M^2} + e^{i\not D_R\not D_L/M^2})\delta(x-x')_{\substack{x'=x\\ M\to\infty}}$$

来计算得

$$2\mathrm{tr}C_2(x)\xi\gamma_5 = -\frac{ie^2}{8\pi^2}\int d^4x C_2(x)F_{\mu\nu}\tilde F_{\mu\nu}\xi,$$

其中 $\not D_L(x) = \gamma_\mu(\partial_\mu - ieA_\mu(x) - i\partial_\mu\theta(x))$，$\not D_R(x) = \gamma_\mu(\partial_\mu - ieA_\mu + i\partial_\mu\theta(x))$ 是拉格朗日函数密度所决定的左右手 Dirac 算子。同时，

$$\delta\int d^4x\theta(x)G(x) = -\frac{ie^2}{8\pi^2}\int d^4x C_2(x)F_{\mu\nu}\tilde F_{\mu\nu}\xi$$

即

$$\int d^4x\theta(x)G(x) = \int d^4x\theta'(x)G'(x) + \frac{ie^2}{8\pi^2}\int d^4x C_2(x)F_{\mu\nu}\tilde F_{\mu\nu}\xi,$$

所以 $\theta(x)G(x)$ 的改变与费米场泛函积分测度的改变相消。

又有

$$\delta\left[C^{+\alpha}M_{\alpha\beta}C^\beta - \frac{1}{2\alpha}(F^\alpha_B[\psi,\bar\psi,A,\theta])^2\right]$$

$$= (\delta C^{+\alpha})M_{\alpha\beta}C^\beta + C^{+\alpha}\delta(M_{\alpha\beta}C^\beta) - \frac{1}{\alpha}F^\alpha_B\delta F^\alpha_B[\psi,\bar\psi,A,\theta]$$

$$= -\frac{1}{\alpha}F^\alpha_B[\psi,\bar\psi,A,\theta]\xi M_{\alpha\beta}C^\beta - \frac{1}{\alpha}F^\alpha_B\delta F^\alpha_B = 0.$$

因为例如在线性规范

$$F^\alpha_B[\psi,\bar\psi,A,\theta] = f^\alpha_i\phi_i \tag{$*$}$$

条件下($\phi_i=\psi$、$\bar\psi$、A、θ)，$\delta F^\alpha_B = f^\alpha_i\delta\phi_i$，$\delta\delta F^\alpha_B = f^\alpha_i\delta\delta\phi_i = 0$。根据(*)式和 $M_{\alpha\beta}$ 的定义[5]

$$M_{\alpha\beta}=\frac{\delta F_B^a}{\delta u_\beta}=\frac{\delta F_B^a}{C_\beta\xi}$$

得

$$\delta F_B^a=M_{\alpha\beta}C_\beta\xi=-\xi M_{\alpha\beta}C_\beta,$$

$$\delta(M_{\alpha\beta}C^\beta)\xi=\delta\delta F_B^a=0\to\delta(M_{\alpha\beta}C^\beta)=0.$$

这样,(10)式的不变性就可以写成:

$$\int[\mathrm{d}\phi]\mathrm{e}^{iS_{\mathrm{eff}}}\Big\{\exp i\int\mathrm{d}^4x[\bar\eta(\phi+\delta\phi)+(\bar\phi+\delta\bar\phi)\eta+J_\mu(A^\mu+\delta A^\mu)+\Omega(\theta+\delta\theta)$$
$$+\zeta_a^+(C^a+\delta C^a)+(C_a^++\delta C_a^+)\zeta^a+\bar K(\delta\phi'+\delta\delta\phi')-(\delta\bar\phi'+\delta\delta\bar\phi')K$$
$$-U_\mu(\delta A'^\mu+\delta\delta A'^\mu)-\omega(\delta\theta'+\delta\delta\theta')+V^a(\delta C'_a+\delta\delta C'_a)]$$
$$-\exp i\int\mathrm{d}^4x[\bar\eta\phi+\bar\phi\eta+J_\mu A^\mu+\Omega\theta+\zeta_a^+C^a+C_a^+\zeta^a+\bar K\delta\phi'-\delta\phi'K$$
$$-U_\mu\delta A'_\mu-\omega\delta\theta'+V^a\delta C'_a]\Big\}=0,$$

即

$$\int[\mathrm{d}\phi]\Big\{\Big[\exp i\int\mathrm{d}^4x(\bar\eta\delta\phi+\delta\bar\phi\eta+J_\mu\delta A^\mu+\Omega\delta\theta+\zeta_a^+\delta C^a+\delta C_a^+\zeta^a)\Big]$$
$$-1\Big\}\mathrm{e}^{iS_{\mathrm{eff}}}\exp i\int\mathrm{d}^4x[\bar\eta\phi+\bar\phi\eta+J_\mu A^\mu+\Omega\theta+\zeta_a^+C^a+C_a^+\zeta^a$$
$$+\bar K\delta\phi'-\delta\bar\phi'K-U_\mu\delta A'^\mu-\omega\delta\theta'+V^a\delta C'_a]=0.$$

将{ }中的量展开并取一级小量,即得:

$$\int[\mathrm{d}\phi]\{\bar\eta\delta\phi+\delta\bar\phi\eta+J_\mu\delta A^\mu+\Omega\delta\theta+\zeta_a^+\delta C^a+\delta C_a^+\zeta^a\}\mathrm{e}^{iS_{\mathrm{eff}}+\text{外源项}}=0.$$

把 $\delta\phi$、$\delta\bar\phi$、δA_μ、$\delta\theta$、δC_a、δC_a^+ 换成各自的外源导数:

$$\frac{\delta\phi}{\xi}=\delta\phi'\to\frac{\delta}{i\delta\bar K},\frac{\delta\bar\phi}{\xi}=\delta\bar\phi'\to-\frac{\delta}{i\delta K},\ \frac{\delta A_\mu}{\xi}=\delta A'_\mu\to-\frac{\delta}{i\delta U_\mu},$$

$$\frac{\delta\theta}{\xi}=\delta\theta'\to-\frac{\delta}{i\delta\omega},\ \frac{\delta C_a}{\xi}=\delta C'_a\to\frac{\delta}{i\delta V_a},$$

$$\delta C_a^+=-\frac{1}{\alpha}F_B^a[\phi,\bar\phi、A、\theta]\zeta\to-\frac{1}{\alpha}F_B^a\left[\frac{\delta}{i\delta\bar\eta},-\frac{\delta}{i\delta\eta},\frac{\delta}{i\delta J_\mu},\frac{\delta}{i\delta\Omega}\right]\xi$$

代入前式并移到积分号外,消去 ζ 便得到以格林函数生成泛函 Z 表示的恒等式:

$$\Big\{\bar\eta\frac{\delta}{\delta\bar K}-\frac{\delta}{\delta K}\eta-J_\mu\frac{\delta}{\delta U_\mu}-\Omega\frac{\delta}{\delta\omega}+\zeta_a^+\frac{\delta}{\delta V^a}$$
$$-\frac{1}{\alpha}F_B^a\left[\frac{\delta}{\delta\bar\eta},-\frac{\delta}{\delta\eta},\frac{\delta}{\delta J_\mu},\frac{\delta}{\delta\Omega}\right]\zeta_a\Big\}$$
$$\cdot Z[\bar\eta、\eta、J_\mu、\Omega、\zeta_a^+、\zeta_a、\bar K、K、U_\mu、\omega、V_a]=0. \tag{11}$$

用 $Z=\mathrm{e}^{iW}$ 代入上式,便得以连通格林函数生成泛函表示的恒等式:

$$\Big\{\bar\eta\frac{\delta}{\delta\bar K}-\frac{\delta}{\delta K}\eta-J_\mu\frac{\delta}{\delta U_\mu}-\Omega\frac{\delta}{\delta\omega}+\zeta_a^+\frac{\delta}{\delta V^a}-\frac{1}{\alpha}F_B^a\left[\frac{\delta}{\delta\bar\eta},-\frac{\delta}{\delta\eta},\frac{\delta}{\delta J_\mu},\frac{\delta}{\delta\Omega}\right]\zeta_a\Big\}$$
$$\cdot W[\bar\eta、\eta、J_\mu、\Omega、\zeta_a^+、\zeta_a、\bar K、K、U_\mu、\omega、V_a]=0. \tag{12}$$

将外源 $\bar{\eta}(x)$、$\eta(x)$、$J_\mu(x)$、$\Omega(x)$、$\zeta_\alpha^+(x)$、$\zeta_\alpha(x)$ 等按照定义

$$\phi(x)=\frac{\delta W[J]}{\delta J(x)}=\frac{1}{iZ[J]}\frac{\delta Z[J]}{\delta J(x)}=\langle 0|\hat{\phi}(x)|0\rangle^J$$

即

$$\phi(x)=\frac{\delta W}{\delta\bar{\eta}(x)},\ \bar{\Phi}(x)=-\frac{\delta W}{\delta\eta(x)},\ A_\mu(x)=\frac{\delta W}{\delta J_\mu(x)},\theta(x)=\frac{\delta W}{\delta\Omega(x)},$$

$$C_\alpha(x)=\frac{\delta W}{\delta\zeta^+(x)},\ C_\alpha^+(x)=-\frac{\delta W}{\delta\zeta_\alpha(x)}. \tag{13}$$

换成新的变量 $\phi(x)$、$\bar{\Phi}(x)$、$A_\mu(x)$、$\theta(x)$、$C_\alpha(x)$、$C_\alpha^+(x)$，又将W按照

$$\Gamma[\phi、\bar{\Phi}、A_\mu、\theta、C_\alpha、C_\alpha^+、\bar{K}、K、U_\mu\omega、V_\alpha]=W[\bar{\eta}、\eta、J_\mu、\Omega、\zeta_\alpha^+、\zeta_\alpha、\bar{K}、K、U_\mu、\omega、V_\alpha]$$
$$-\int d^4x[\bar{\eta}\phi+\bar{\Phi}\eta+J_\mu A^\mu+\Omega\theta+\zeta_\alpha^+C^\alpha+C_\alpha^+\zeta^\alpha], \tag{14}$$

换成正规顶角生成泛函 $\Gamma[\phi、\bar{\Phi}、A_\mu、\theta、C_\alpha、C_\alpha^+、\bar{K}、K、U_\mu、\omega、V_\alpha]$，并将

$$\frac{\delta\Gamma}{\delta\phi(x)}=\int d^4y\frac{\delta W[\bar{\eta}]}{\delta\bar{\eta}}\frac{\delta\bar{\eta}(y)}{\delta\phi(x)}-\int d^4y\frac{\delta\bar{\eta}}{\delta\phi(x)}\phi(y)+\int d^4y\frac{\delta\phi(y)}{\delta\phi(x)}\bar{\eta}(y)=\bar{\eta}(x),$$

$$\eta(x)=-\frac{\delta\Gamma}{\delta\bar{\Phi}(x)},J_\mu=-\frac{\delta\Gamma}{\delta A_\mu},\ \Omega=-\frac{\delta\Gamma}{\delta\theta},\zeta_\alpha^+=\frac{\delta\Gamma}{\delta C^\alpha},\zeta^\alpha=-\frac{\delta\Gamma}{\delta C_\alpha^+} \tag{15}$$

及

$$\frac{\delta W}{\delta\bar{K}}=\frac{\delta\Gamma}{\delta\bar{K}},\ \frac{\delta W}{\delta K}=\frac{\delta\Gamma}{\delta K},\ \frac{\delta W}{\delta U_\mu}=\frac{\delta\Gamma}{\delta U_\mu},\ \frac{\delta W}{\delta\omega}=\frac{\delta\Gamma}{\delta\omega},\ \frac{\delta W}{\delta V_\alpha}=\frac{\delta\Gamma}{\delta V_\alpha},$$

$$\frac{\delta\Gamma}{\delta\bar{\eta}}=\frac{\delta W}{\delta\bar{\eta}}-\phi(x)=0\rightarrow\frac{\delta W}{\delta\bar{\eta}}=\phi,\cdots\cdots, \tag{16}$$

代入(12)式，就得到用正规顶角生成泛函表示的恒等式：

$$\frac{\delta\Gamma}{\delta\phi}\frac{\delta\Gamma}{\delta\bar{K}}+\frac{\delta\Gamma}{\delta K}\frac{\delta\Gamma}{\delta\bar{\Phi}}+\frac{\delta\Gamma}{\delta A_\mu}\frac{\delta\Gamma}{\delta U_\mu}+\frac{\delta\Gamma}{\delta\theta}\frac{\delta\Gamma}{\delta\omega}+\frac{\delta\Gamma}{\delta C^\alpha}\frac{\delta\Gamma}{\delta V_\alpha}$$
$$+\frac{1}{\alpha}F_B^\alpha[\phi、\bar{\Phi}、A、\theta]\frac{\delta\Gamma}{\delta C_\alpha^+}=0. \tag{17}$$

为了简化(17)式，我们来求鬼的运动方程．将(10)式中 $C_\alpha^+(x)$ 作平移变换：

$$C_\alpha^+(x)\rightarrow C_\alpha^+(x)+\lambda(x),\lambda(x)$$

为无穷小量，

$$\phi、\bar{\Phi}、A_\mu、C_\alpha、\theta\ 不变.$$

由Z的不变性得：

$$Z=\int[d\phi]\left\{1+i\int d^4x\lambda(x)\left[\int d^4yM_{\alpha\beta}(x,y)C^\beta(y)+\zeta_\alpha\right]\right\}e^{iS_{eff}+i\int d^4x[\cdots\cdots]}.$$

对 $\lambda(x)$ 求微商得：

$$\int[d\phi]\left\{\int d^4yM_{\alpha\beta}(x,y)C^\beta(y)+\zeta_\alpha\right\}e^{iS_{eff}+i\int d^4x[\cdots\cdots]}.$$

由于

$$\int d^4yM_{\alpha\beta}(x,y)C^\beta(y)=\int d^4y\frac{\delta F_B^\alpha[\phi、\bar{\Phi}、A、\theta]}{C^\beta(y)\xi}C^\beta(y)$$

$$= -\int d^4y \frac{\delta F^a_B[\phi、\bar{\phi}、A、\theta]}{\xi C^\beta} C^\beta = -\frac{1}{\xi}\delta F^a_B[\phi、\bar{\phi}、A、\theta]$$

$$\to -\frac{1}{\xi}\delta F^a_B\left[\frac{\delta}{i\delta\bar{\eta}}, -\frac{\delta}{i\delta\eta}, \frac{\delta}{i\delta J_\mu}, \frac{\delta}{i\delta\Omega}\right],$$

代入前式并移到积分号外,得:

$$\left\{-\frac{1}{\xi}\delta F^a_B\left[\frac{\delta}{i\delta\bar{\eta}}, -\frac{\delta}{i\delta\eta}, \frac{\delta}{i\delta J_\mu}, \frac{\delta}{i\delta\Omega}\right] + \zeta_\alpha\right\} Z = 0. \tag{18}$$

将 $Z = e^{iW}$ 代入,则得:

$$\left\{-\frac{1}{\xi}\delta F^a_B\left[\frac{\delta}{\delta\bar{\eta}}, -\frac{\delta}{\delta\eta}, \frac{\delta}{\delta J_\mu}, \frac{\delta}{\delta\Omega}\right] + \zeta_\alpha\right\} W = 0. \tag{19}$$

因

$$\zeta_\alpha = -\frac{\delta\Gamma}{\delta C^+_\alpha}, \quad \frac{\delta\Gamma}{\delta\bar{\eta}} = \frac{\delta W}{\delta\bar{\eta}} - \phi(x) = 0 \to \frac{\delta W}{\delta\bar{\eta}} = \phi(x),$$

同理得:

$$-\frac{\delta W}{\delta\eta} = \bar{\phi}(x), \quad \frac{\delta W}{\delta J_\mu} = A_\mu, \quad \frac{\delta W}{\delta\Omega} = \theta(x),$$

将以上关系代入(19)式,得以正规顶角生成泛函表示的鬼方程:

$$\left\{\frac{1}{\xi}\delta F^a_B[\phi、\bar{\phi}、A_\mu、\theta] + \frac{\delta\Gamma}{\delta C^+_\alpha}\right\} = 0. \tag{20}$$

定义不包含规范固定项的正规顶角生成泛函 $\hat{\Gamma}$:

$$\Gamma = \hat{\Gamma} - \frac{1}{2\alpha}\int (F^a_B[\phi、\bar{\phi}、A、\theta])^2 d^4x, \tag{21}$$

将(21)式对场量求泛函导数再乘以对相应外源的泛函导数,并注意 ϕ、$\bar{\phi}$ 的反对易关系及下列关系:

$$\begin{cases} \dfrac{\delta\phi}{\xi} = \delta\phi' = \dfrac{\delta Z}{iZ\delta\bar{K}} = \dfrac{\delta W}{\delta\bar{K}} = \dfrac{\delta\Gamma}{\delta\bar{K}} = \dfrac{\delta\hat{\Gamma}}{\delta\bar{K}}, \\ -\dfrac{\delta\bar{\phi}}{\xi} = -\delta\bar{\phi}' = \dfrac{\delta Z}{iZ\delta K} = \dfrac{\delta W}{\delta K} = \dfrac{\delta\Gamma}{\delta K} = \dfrac{\delta\hat{\Gamma}}{\delta K}, \\ -\dfrac{\delta A_\mu}{\xi} = -\delta A'_\mu = \dfrac{\delta Z}{iZ\delta U_\mu} = \dfrac{\delta W}{\delta U_\mu} = \dfrac{\delta\Gamma}{\delta U_\mu} = \dfrac{\delta\hat{\Gamma}}{\delta U_\mu}, \\ -\dfrac{\delta\theta}{\xi} = -\delta\theta' = \dfrac{\delta Z}{iZ\delta\omega} = \dfrac{\delta W}{\delta\omega} = \dfrac{\delta\Gamma}{\delta\omega} = \dfrac{\delta\hat{\Gamma}}{\delta\omega}, \end{cases} \tag{22}$$

便得:

$$\begin{aligned} \frac{\delta\Gamma}{\delta\phi}\frac{\delta\Gamma}{\delta\bar{K}} &= \left(\frac{\delta\hat{\Gamma}}{\delta\phi} + \frac{1}{\alpha}F^a_B[\phi、\bar{\phi}、A、\theta]\frac{\delta F^a_B}{\delta\phi}\right)\frac{\delta\hat{\Gamma}}{\delta\bar{K}} \\ &= \frac{\delta\hat{\Gamma}}{\delta\phi}\frac{\delta\hat{\Gamma}}{\delta\bar{K}} + \frac{1}{\alpha}F^a_B[\phi、\bar{\phi}、A、\theta]\frac{\delta F^a_B}{\delta\phi}\frac{\delta\phi}{\xi}, \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{\delta\Gamma}{\delta K}\frac{\delta\Gamma}{\delta\bar{\phi}} &= \frac{\delta\Gamma}{\delta K}\left(\frac{\delta\hat{\Gamma}}{\delta\bar{\phi}} + \frac{1}{\alpha}F^a_B[\phi、\bar{\phi}、A、\theta]\frac{\delta F^a_B}{\delta\bar{\phi}}\right) \\ &= \frac{\delta\hat{\Gamma}}{\delta K}\frac{\delta\hat{\Gamma}}{\delta\bar{\phi}} - \frac{1}{\alpha}F^a_B[\phi、\bar{\phi}、A、\theta]\frac{\delta F^a_B}{\delta\bar{\phi}}\frac{\delta\bar{\phi}}{\xi}, \end{aligned} \tag{24}$$

$$\frac{\delta\Gamma}{\delta A_\mu}\frac{\delta\Gamma}{\delta U_\mu}=\left(\frac{\delta\hat{\Gamma}}{\delta A_\mu}-\frac{1}{\alpha}F_B^a[\phi、\bar{\Phi}、A、\theta]\frac{\delta F_B^a}{\delta A_\mu}\right)\frac{\delta\hat{\Gamma}}{\delta U_\mu}$$
$$=\frac{\delta\hat{\Gamma}}{\delta A_\mu}\frac{\delta\hat{\Gamma}}{\delta U_\mu}+\frac{1}{\alpha}F_B^a[\phi、\bar{\Phi}、A、\theta]\frac{\delta F_B^a}{\delta A_\mu}\frac{\delta A_\mu}{\xi},\qquad(25)$$

$$\frac{\delta\Gamma}{\delta\theta}\frac{\delta\Gamma}{\delta\omega}=\left(\frac{\delta\hat{\Gamma}}{\delta\theta}-\frac{1}{\alpha}F_B^a[\phi、\bar{\Phi}、A、\theta]\frac{\delta F_B^a}{\delta\theta}\right)\frac{\delta\hat{\Gamma}}{\delta\omega}$$
$$=\frac{\delta\hat{\Gamma}}{\delta\theta}\frac{\delta\hat{\Gamma}}{\delta\omega}+\frac{1}{\alpha}F_B^a[\phi、\bar{\Phi}、A、\theta]\left.\frac{\delta F_B^a}{\delta\theta}\right)\frac{\delta\theta}{\xi}.\qquad(26)$$

考虑到

$$\delta F_B^a[\phi、\bar{\Phi}、A、\theta]=\frac{\delta F_B^a}{\delta\phi}\delta\phi+\frac{\delta F_B^a}{\delta\bar{\Phi}}\delta\bar{\Phi}+\frac{\delta F_B^a}{\delta A_\mu}\delta A_\mu+\frac{\delta F_B^a}{\delta\theta}\delta\vartheta,\qquad(27)$$

将(23)—(26)式相加代入(17)式，便得：

$$\frac{\delta\hat{\Gamma}}{\delta\phi}\frac{\delta\hat{\Gamma}}{\delta\bar{K}}+\frac{\delta\hat{\Gamma}}{\delta K}\frac{\delta\hat{\Gamma}}{\delta\bar{\Phi}}+\frac{\delta\hat{\Gamma}}{\delta A_\mu}\frac{\delta\hat{\Gamma}}{\delta U_\mu}+\frac{\delta\hat{\Gamma}}{\delta\theta}\frac{\delta\hat{\Gamma}}{\delta\omega}+\frac{\delta\hat{\Gamma}}{\delta C_\alpha}\frac{\delta\hat{\Gamma}}{\delta V_\alpha}$$
$$+\frac{1}{\alpha}F_B^a[\phi,\bar{\Phi},A,\theta]\frac{1}{\xi}\delta F_B^a[\phi,\bar{\Phi},A,\theta]$$
$$+\frac{1}{\alpha}F_B^a[\phi、\bar{\Phi}、A、\theta]\frac{\delta\hat{\Gamma}}{\delta C_\alpha^+}=0.\qquad(28)$$

利用(21)式，(20)式成为

$$\frac{1}{\xi}\delta F_B^a[\phi、\bar{\Phi}、A、\theta]+\frac{\delta\hat{\Gamma}}{\delta C_\alpha^+}=0.\qquad(29)$$

将(29)式代入(28)式得：

$$\frac{\delta\hat{\Gamma}}{\delta\phi}\frac{\delta\hat{\Gamma}}{\delta\bar{K}}+\frac{\delta\hat{\Gamma}}{\delta K}\frac{\delta\hat{\Gamma}}{\delta\bar{\Phi}}+\frac{\delta\hat{\Gamma}}{\delta A_\mu}\frac{\delta\hat{\Gamma}}{\delta U_\mu}+\frac{\delta\hat{\Gamma}}{\delta\theta}\frac{\delta\hat{\Gamma}}{\delta\omega}+\frac{\delta\hat{\Gamma}}{\delta C_\alpha}\frac{\delta\hat{\Gamma}}{\delta V_\alpha}=0.\qquad(30)$$

引入定义

$$A*B=\sum_i\left(\frac{\delta A}{\delta U_i}\frac{\delta B}{\delta\phi_i}+\frac{\delta A}{\delta\phi_i}\frac{\delta B}{\delta U_i}\right),$$

(30)式也可写为

$$\hat{\Gamma}*\hat{\Gamma}=0,\qquad(30')$$

式中 ϕ_i 代表 $\phi、\bar{\Phi}、A_\mu、\theta,C_\alpha,U_i$ 代表 $\bar{K}、K、U_\mu、\omega$ 和 V_α。(30)式就是阿贝尔手征群陪集纯规范场 θ 所满足的恒等式。它是泛函积分测度和有效作用量在(8)式变换下都改变，只有两者联合起来才不变的情况下导出的。鬼方程(29)也和 θ 有关。

利用(22)和(27)式，(29)式还可写成：

$$\frac{\delta\hat{\Gamma}}{\delta C_\alpha^+}+\left[\frac{\delta F_B^a}{\delta\phi}\frac{\delta\hat{\Gamma}}{\delta\bar{K}}-\frac{\delta F_B^a}{\delta\bar{\Phi}}\frac{\delta\hat{\Gamma}}{\delta K}-\frac{\delta F_B^a}{\delta A_\mu}\frac{\delta\hat{\Gamma}}{\delta U_\mu}-\frac{\delta F_B^a}{\delta\theta}\frac{\delta\hat{\Gamma}}{\delta\omega}\right]=0.\qquad(29a)$$

若取线性规范 $F_B^a[\phi、\bar{\Phi}、A、\theta]=f_i^a\phi_i$，则

$$\delta F_B^a=\frac{\delta F_B^a}{\delta\phi}\delta\phi+\frac{\delta F_B^a}{\delta\bar{\Phi}}\delta\bar{\Phi}+\frac{\delta F_B^a}{\delta A_\mu}\delta A_\mu+\frac{\delta F_B^a}{\delta\theta}\delta\theta$$
$$=\xi\left[f_\phi^a\frac{\delta\hat{\Gamma}}{\delta\bar{K}}-f_{\bar{\Phi}}^a\frac{\delta\hat{\Gamma}}{\delta K}-f_{A_\mu}^a\frac{\delta\hat{\Gamma}}{\delta U_\mu}-f_\theta^a\frac{\delta\hat{\Gamma}}{\delta\omega}\right]$$

$$= \xi\left[f'^{\alpha}_{\psi}\frac{\delta\hat{\Gamma}}{\delta\bar{K}} + f'^{\alpha}_{\bar{\psi}}\frac{\delta\hat{\Gamma}}{\delta K} + f'^{\alpha}_{A\mu}\frac{\delta\hat{\Gamma}}{\delta U_\mu} + f'^{\alpha}_{\theta}\frac{\delta\hat{\Gamma}}{\delta\omega}\right] = \xi f'^{\alpha}_{i}\frac{\delta\hat{\Gamma}}{\delta U_i},$$

式中 $U_i = \bar{K}、K、U_\mu、\omega$，则(29)式又可写成：

$$\frac{\delta\hat{\Gamma}}{\delta C^+_\alpha} + f'^{\alpha}_{i}\frac{\delta\hat{\Gamma}}{\delta U_i} = 0. \tag{29b}$$

二

现在利用(29)和(30)式来建立重整化方程. 既然(29)和(30)或 (29b) 和 (30′) 式是 $\hat{\Gamma}$ 必需满足的方程式，则各级近似下的 $\hat{\Gamma}$ 就都必须满足它们. 在树图近似下，正规顶角的生成泛函 $\Gamma_0[S_0]$ 等于系统的作用量 S_0[1]：

$$\Gamma_0[S_0] = S_0,$$

$$\hat{\Gamma}_0[S_0] = \hat{S}_0 = S_0 + \frac{1}{2\alpha}\int d^4x(F^\alpha_B[\psi、\bar{\psi}、A、\theta])^2,$$

所以由作用量 S_0 建造的正规顶角生成泛函 $\Gamma_0[S_0]$ 当然亦满足(29b)和(30′)式，即有

$$\hat{S}_0 * \hat{S}_0 = 0 \tag{31}$$

和

$$\frac{\delta\hat{S}_0}{\delta C^+_\alpha} + f'^{\alpha}_{i}\frac{\delta\hat{S}_0}{\delta U_i} = 0. \tag{32}$$

用 $\hat{\Gamma}_0$ 生成的正规顶角，即由它导出的费曼规则都有限.

但如用 $\hat{S}_0$ 来建造单圈近似的正规顶角生成泛函 $\hat{\Gamma}[S_0]$

$$\hat{\Gamma}[S_0] = \hat{\Gamma}_0[\hat{S}_0] + \hat{\Gamma}_1[\hat{S}_0] = \hat{S}_0 + \hat{\Gamma}_1[\hat{S}_0],$$

则由 $\hat{S}_0$ 生成的正规顶角仍然是有限的；而 $\hat{\Gamma}_1[\hat{S}_0]$ 生成的正规顶角有有限的部分 $\hat{\Gamma}^f_1[\hat{S}_0]$，也有发散的部分 $\hat{\Gamma}^d_1[\hat{S}_0]$，即

$$\hat{\Gamma}_1[\hat{S}_0] = \hat{\Gamma}^f_1[\hat{S}_0] + \hat{\Gamma}^d_1[\hat{S}_0].$$

我们在作用量 $\hat{S}_0$ 上加上抵消项 $\Delta\hat{S}_0$，即令

$$\hat{S}_1 = \hat{S}_0 + \Delta\hat{S}_0.$$

用此来建造正规顶角生成泛函 $\hat{\Gamma}[\hat{S}_0 + \Delta\hat{S}_0]$，并取单圈近似

$$\hat{\Gamma}[\hat{S}_0 + \Delta\hat{S}_0] \simeq \hat{\Gamma}_0[\hat{S}_0 + \Delta\hat{S}_0] + \hat{\Gamma}_1[\hat{S}_0 + \Delta\hat{S}_0]$$

$$\simeq \hat{S}_0 + \Delta\hat{S}_0 + \hat{\Gamma}_1[\hat{S}_0] = \hat{S}_0 + \Delta\hat{S}_0 + \hat{\Gamma}^f_1[\hat{S}_0] + \hat{\Gamma}^d_1[\hat{S}_0],$$

如选抵消项 $\Delta\hat{S}_0 = -\hat{\Gamma}^d_1[\hat{S}_0]$，

则在单圈近似下，

$$\hat{\Gamma}[\hat{S}_0 + \Delta\hat{S}_0] = \hat{S}_0 + \hat{\Gamma}^f_1[\hat{S}_0]$$

生成的正规顶角就都是有限的. 这时，由 $\hat{S}_0$ 构造的 $\hat{\Gamma}[\hat{S}_0]$

$$\hat{\Gamma}[\hat{S}_0] = \hat{S}_0 + \hat{\Gamma}^f_1[\hat{S}_0] + \hat{\Gamma}^d_1[\hat{S}_0]$$

虽是发散的，但仍要满足(29b)和(30′)式，即

$$(\hat{S}_0 + \hat{\Gamma}^f_1[\hat{S}_0] + \hat{\Gamma}^d_1[\hat{S}_0]) * (\hat{S}_0 + \hat{\Gamma}^f_1[\hat{S}_0] + \hat{\Gamma}^d_1[\hat{S}_0]) = 0,$$

$$\frac{\delta}{\delta C_a^+}(\hat{S}_0 + \hat{\Gamma}_1^f[\hat{S}_0] + \hat{\Gamma}_1^d[\hat{S}_0]) + f_i'^a \frac{\delta}{\delta U_i}(\hat{S}_0 + \hat{\Gamma}_1^f[\hat{S}_0] + \hat{\Gamma}_1^d[\hat{S}_0]) = 0.$$

因在零级(树图)近似下，$\hat{S}_0$ 满足(31)和(32)式，所以在一级(单圈)近似下，由上式可得

$$\hat{S}_0 * (\hat{\Gamma}_1^f[\hat{S}_0] + \hat{\Gamma}_1^d[\hat{S}_0]) = 0,$$

$$\frac{\delta}{\delta C_a^+}(\hat{\Gamma}_1^f[\hat{S}_0] + \hat{\Gamma}_1^d[\hat{S}_0]) + f_i'^a \frac{\delta}{\delta U_i}(\hat{\Gamma}_1^f[\hat{S}_0] + \hat{\Gamma}_1^d[\hat{S}_0]) = 0.$$

其中有限部分 $\hat{\Gamma}_1^f[\hat{S}_0]$ 和发散部分 $\hat{\Gamma}_1^d[\hat{S}_0]$ 应各自满足上述方程，因而

$$\hat{S}_0 * \hat{\Gamma}_1^d[\hat{S}_0] = 0, \tag{33}$$

$$\frac{\delta \hat{\Gamma}_1^d[\hat{S}_0]}{\delta C_a^+} + f_i'^a \frac{\delta \hat{\Gamma}_1^d[\hat{S}_0]}{\delta U_i} = 0, \tag{34}$$

就是用来确定在单圈近似下的抵消项 $\Delta\hat{S}_0 = -\hat{\Gamma}_1^d[\hat{S}_0]$ 的方程. 类似地可得确定双圈近似下的抵消项 $\Delta\hat{S}_1 = -\hat{\Gamma}_2^d[\hat{S}_1]$ 的方程. 类推，便可得逐级以致无穷级近似下抵消项应满足的方程. 由此可知，(33)和(34)式的形式就是确定抵消项的一般方程，即重整化方程.

可以证明，(33)和(34)式的一般解是：

$$\Delta S_0 = -\Gamma_1^d[\hat{S}_0] = \sum_\tau a_\tau G_\tau + \hat{S}_0 * R, \tag{35}$$

$$k = \int d^4x \left\{ \sum_i b_i (U_i - f_i'^a C_a^+)\phi_i + b_c C_a V_a \right\}, \tag{36}$$

式中 G_τ 是规范不变的泛函，a_τ 是相应的系数，b_i 和 b_c 是取小量的常系数.

把抵消项 ΔS_0 加到 S_0，就得到使正规顶角有限且有意义的作用量：

$$\begin{aligned} S_0 + \Delta S_0 &= S_0 + \sum_\tau a_\tau G_\tau + \hat{S}_0 * k \\ &= \hat{S}_0 - \frac{1}{2\alpha}\int d^4x\,(F_B^a[\phi, \bar{\Phi}、A、\theta])^2 + \sum_\tau a_\tau G_\tau + \hat{S}_0 * k \\ &= \hat{S}_0 + \sum_j \left(\frac{\delta \hat{S}_0}{\delta \phi_j}\frac{\delta k}{\delta U_j} + \frac{\delta \hat{S}_0}{\delta U_j}\frac{\delta k}{\delta \phi_j} \right) + \sum_\tau a_\tau G_\tau \\ &\quad - \frac{1}{2\alpha}\int d^4x (F_B^a[\phi、\bar{\Phi}、A、\theta])^2. \end{aligned}$$

令

$$\phi_i \to \phi_i' = \phi_i + \frac{\delta k}{\delta U_i} = (1 + b_i)\phi_i = Y_i \phi_i,$$

$$\begin{aligned} U_i - f_i'^a C_a^+ \to U_i' - f_i'^a C_a^{+1} &= U_i - f_i'^a C_a^+ - \frac{\delta k}{\delta \phi_i} = (1 - b_i)(U_i - f_i'^a C_a^+) \\ &= Y_i^{-1}(U_i - f_i'^a C_a^+), \end{aligned}$$

$$U_i' = Y_i^{-1} U_i, \quad C_a'^+ = Y_i^{-1} C_a^+,$$

$$C_a \to C_a' = C_a + \frac{\delta k}{\delta V_a} = C_a + b_c C_a = (1 + b_c)C_a = Y_c C_a = Y_V^{-1} C_a,$$

$$V_a \to V_a' = V_a - \frac{\delta k}{\delta C_a} = V_a - b_c V_a = (1 - b_c)V_a = Y_c^{-1} V_a = Y_V V_a,$$

则

$$\hat{S}_0[\phi_i,U_i]\to\hat{S}_0[\phi'_i,U'_i]=\hat{S}_0[\phi_i,U_i]+\frac{\delta\hat{S}_0}{\delta\phi_i}(\phi'_i-\phi_i)-\frac{\delta\hat{S}_0}{\delta U_i}(U'_i-U_i).$$

$$S_0+\Delta S_0=\hat{S}_0[\phi'_i,U'_i]-\frac{\delta\hat{S}_0}{\delta\phi_i}(\phi'_i-\phi_i)+\frac{\delta\hat{S}_0}{\delta U_i}(U'_i-U_i)+\sum_i\left(\frac{\delta\hat{S}_0}{\delta\phi_i}\frac{\delta k}{\delta U_i}+\frac{\delta\hat{S}_0}{\delta U_i}\frac{\delta k}{\delta\phi_i}\right)+\sum_\tau a_\tau G_\tau-\frac{1}{2\alpha}\int d^4x(F^\alpha_B[\phi、\bar{\phi}、A、\theta])^2$$

$$=\hat{S}_0[\phi'_i,U'_i]+\sum_\tau a_\tau G_\tau[\phi'_i]-\frac{1}{2\alpha}\int d^4x(F^\alpha_B[\phi、\bar{\phi}、A、\theta])^2. \tag{37}$$

这表明，抵消项 $\hat{S}_0*k$ 的作用是将变量作如下的变换:

$$\phi'_i=Y_i\phi_i,U'_i=Y_i^{-1}U_i,C^{+\prime}_\alpha=Y_i^{-1}C^+_\alpha,V'_\alpha=Y_VV_\alpha,\ C'_\alpha=Y_V^{-1}C_\alpha. \tag{38}$$

$\sum a_\tau G_\tau$ 才是通常所谓的抵消项，规范固定项是与重整化无关的.

三、

下面利用上述结果将阿贝尔手征群陪集空间纯规范场理论重整化. 在手征变换下，与路径积分测度联合起来不变的阿贝尔手征群陪集纯规范场的有效作用量，在任意规范下为(5)式，对于 $U(1)$ 场，当 $F^\alpha_B=F^\alpha_B[\phi、\bar{\phi}、A、\theta]$ 时，有 $M_{\alpha\beta}=0$. 例如，F^α_B 只是 A 的函数时，规范变换为

$$A'_\mu=A_\mu+\frac{1}{e}\partial_\mu(C_1(x)\xi),$$

取朗道规范 $\partial_\mu A_\mu=0,\partial^\mu A'_\mu=\partial^\mu\left[A_\mu+\frac{1}{e}\partial_\mu(C_1(x)\xi)\right]=\frac{1}{e}\partial^2C_1\xi$,

$$M_{\alpha\beta}=\frac{\delta F^\alpha_B}{\delta u}=\frac{1}{e}\frac{(\partial^2\delta C_1)\xi}{\delta u}=0,\because\delta C_1=0.$$

当 F^α_B 是 $\phi、\bar{\phi}、\theta$ 的函数时也一样，因而

$$\int d^4x\int d^4y C^+_\alpha(x)M_{\alpha\beta}(x,y)C_\alpha(y) \tag{39}$$

可以略去.

除去场量变换式(8)中的 ξ 再乘以 e 得:

$$\delta\phi'=-ie[C_1(x)+\gamma_5C_2(x)]\phi(x),\delta\bar{\phi}'=ie\bar{\phi}[-C_1(x)+\gamma_5C_2(x)],$$
$$\delta A'_\mu=\partial_\mu C_1(x),\ \delta\theta'=eC_2(x),\ \delta C'_\alpha=0,$$

这些场量都是二次变换不变量，它们与各自相应的外源 $\bar{K}、K、U_\mu、\omega、V_\alpha$ 构成新的作用量为:

$$\int d^4x\{-U_\mu\partial_\mu C_1(x)+ie\bar{K}[C_1(x)+\gamma_5C_2(x)]\phi(x)-ie\bar{\phi}(x)[-C_1(x)+\gamma_5C_2(x)]-e\omega C_2(x)\}, \tag{40}$$

(5)式加(40)式，并考虑(39)式，得

$$S_0 = \int d^4x \left\{ -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \bar{\Phi}\gamma_\mu(\partial_\mu - ieA_\mu - i\gamma_5\partial_\mu\theta)\psi - m\bar{\Phi}e^{-2i\theta(x)\gamma_5}\psi \right.$$
$$- \theta\frac{ie^2}{8\pi^2}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\nu A_\mu - \partial_\mu A_\nu) - \frac{1}{2\alpha}\int d^4x (F_B^a)^2 - U_\mu\partial_\mu C_1(x)$$
$$\left. + ie\bar{K}[C_1(x) + \gamma_5 C_2(x)]\psi(x) - ie\bar{\Phi}(x)[-C_1(x) + \gamma_5 C_2(x)] - \omega e C_2(x) \right\}. \tag{41}$$

现在来求抵消项 $\Delta S_0 = -\Gamma^d[S_0] = \sum_\tau a_\tau G_\tau + \hat{S}_0 * k$.

由(36)式知

$$k = \int d^4x \{ b_A(U_\mu - f'^{\ a}_A C^+_a)A_\mu + b_\psi(\bar{K} - f'^{\ a}_\psi C^+_a)\psi + b_{\bar{\psi}}(K - f'^{\ a}_{\bar{\psi}} C^+_a)\bar{\Phi}$$
$$+ b_\theta(\omega - f'^{\ a}_\theta C^+_a)\theta + b_e V_a C_a \},$$

根据(37)和(38)式知，抵消项 $\hat{S}_0 * k$ 附加于 $S_0 + \sum_\tau a_\tau G_\tau$ 的作用就是用

$$\left.\begin{array}{ll} A'_\mu = Y_A A_\mu, U'_\mu = Y_A^{-1}U_\mu, C^{+\prime}_a = Y_i^{-1}C^+_a, & Y_A = 1 + b_A \\ \psi' = Y_\psi \psi, \quad \bar{K}' = Y_\psi^{-1}\bar{K}, & Y_\psi = 1 + b_\psi \\ \bar{\Phi}' = Y_{\bar{\psi}}\bar{\Phi}, \quad K' = Y_{\bar{\psi}}^{-1}K, & Y_{\bar{\psi}} = 1 + b_{\bar{\psi}} \\ \theta' = Y_\theta\theta, \quad \omega' = Y_\theta^{-1}\omega, & Y_\theta = 1 + b_\theta \\ C'_a = Y_V^{-1}C_a, V'_a = Y_V V_a, & Y_V = 1 + b_V \end{array}\right\} \tag{42}$$

代替 $\hat{S}_0$ 中的 A_μ、U_μ、C^+_a、ψ、$\bar{K}$、$\bar{\Phi}$、K、C_a、θ、ω.

规范不变的项有：

$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2, -\bar{\Phi}\gamma_\mu(\partial_\mu - ieA_\mu - i\gamma_5\partial_\mu\theta)\psi, -m\bar{\Phi}e^{-i2\theta(x)\gamma_5}\psi,$$

所以规范不变的抵消项为：

$$\sum_\tau a_\tau G_\tau = -\int d^4x \left[a_e \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + a_K \bar{\Phi}\gamma_\mu(\partial_\mu - ieA_\mu \right.$$
$$\left. - i\gamma_5\partial_\mu\theta)\psi + a_m m\bar{\Phi}e^{-2i\theta(x)\gamma_5}\psi \right]. \tag{43}$$

由于 G_τ 的规范不变性，所以(43)式中的场量一开始就可用(42)式中带撇的量来表，令 $Y_e = 1 + a_e, Y_m = 1 + a_m, Y_K = 1 + a_K$，则(43)式应写成：

$$\sum_\tau a_\tau G_\tau = -\int d^4x \left\{ \frac{1}{4}(Y_e - 1)Y_A^2(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (Y_K - 1)Y_\psi Y_{\bar{\psi}}\bar{\Phi}\gamma_\mu(\partial_\mu \right.$$
$$\left. - ieY_A A_\mu - i\gamma_5 Y_\theta\partial_\mu\theta)\psi + (Y_m - 1)Y_\psi Y_{\bar{\psi}} m\bar{\Phi}e^{-i2Y_\theta\theta(x)\gamma_5}\psi \right\}. \tag{44}$$

由(37)、(41)、(42)和(44)式，得重整化后的作用量为：

$$S = S_0 + \Delta S_0 = \int d^4x \left\{ -\frac{1}{4}Y_e Y_A^2(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - Y_K Y_\psi Y_{\bar{\psi}}\bar{\Phi}\gamma_\mu(\partial_\mu - ieY_A A_\mu \right.$$
$$- iY_\theta\gamma_5\partial_\mu\theta)\psi - Y_m Y_\psi Y_{\bar{\psi}} m\bar{\Phi}e^{-2iY_\theta\theta\gamma_5}\psi$$

$$- Y_\theta Y_A^2 \frac{ie^2}{8\pi^2}\theta(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\nu A_\mu - \partial_\mu A_\nu)$$
$$- (Y_A Y_V)^{-1} U_\mu \partial_\mu C_1(x) + ieY_V^{-1}\bar{K}[C_1(x) + \gamma_5 C_2(x)]\psi(x)$$
$$- ieY_V^{-1}\bar{\psi}[-C_1(x) + \gamma_5 C_2(x)]K - eY_\theta^{-1}Y_V^{-1}\omega C_2(x)$$
$$- \frac{1}{2\alpha}(F_B^\alpha[\psi, \bar{\psi}, A, \theta])^2\Big\}, \quad (45)$$

由这样的重整化作用量建立的生成泛函可以生成高一级近似为有限的正规顶角，逐级使用这种方法，就可得重整化的阿贝尔手征群陪集纯规范场理论.

下面将重整化常数用 Y_i 表示出来.

设(45)式中的场量、质量、作用常数和规范参数都是重整化了的量，并且用裸量表示的作用量和(45)式有同样的形式且相等即

$$S = \int d^4x \Big\{ -\frac{1}{4}(\partial_\mu A_{0\nu} - \partial_\nu A_{0\mu})^2 - \bar{\psi}_0 \gamma_\mu(\partial_\mu - ie_0 A_{0\mu} - i\gamma_5 \partial_\mu \theta_0)\psi_0$$
$$- m_0 \bar{\psi}_0 e^{-2i\theta_0 \gamma_5}\psi_0 - \frac{ie_0^2}{8\pi^2}\theta_0(\partial_\mu A_{0\nu} - \partial_\nu A_{0\mu})(\partial_\nu A_{0\mu} - \partial_\mu A_{0\nu})$$
$$- U_{0\mu}\partial_\mu C_1^0(x) + ie_0 \bar{K}_0[C_1^0(x) + \gamma_5 C_2^0(x)]\psi_0$$
$$- ie_0 \bar{\psi}_0[-C_1^0(x) + \gamma_5 C_2^0(x)]K_0 - e_0 \omega_0 C_2^0(x)$$
$$- \frac{1}{2\alpha_0}(F_B^\alpha[\psi_0, \bar{\psi}_0, A_0, \theta_0])^2\Big\}. \quad (46)$$

裸量和重整化量间由重整化常数联系起来的关系为:

$$\begin{cases} \psi_0 = \sqrt{Z_2}\psi,\ \bar{\psi}_0 = \sqrt{Z_2}\bar{\psi},\ A_{0\mu} = \sqrt{Z_3}A_\mu, \theta_0 = \sqrt{Z_\theta}\theta, C_\alpha^0 = \sqrt{\tilde{Z}_3}C_\alpha, \\ \bar{K}_0 = \sqrt{Z_K}\bar{K}, K_0 = \sqrt{Z_K}K, U_{0\mu} = \sqrt{Z_U}U_\mu,\ \omega_0 = \sqrt{Z_\omega}\omega, \\ e_0 = Z_e e,\ m_0 = Z_m m,\ \alpha_0 = Z_\alpha \alpha. \end{cases} \quad (47)$$

比较(45)、(46)和(47)式，由(45)和(46)式第一、第二、第三行得:

$$Z_3 = Y_e Y_A^2,\ Z_2 = Y_K Y_\psi Y_{\bar{\psi}},$$
$$Z_1^\psi = Z_{e1} Z_2 Z_3^{1/2} = Y_K Y_\psi Y_{\bar{\psi}} Y_A, \to Z_{e1} = \frac{Z_1^\psi}{Z_2 Z_3^{1/2}} = \frac{Y_A}{Z_3^{1/2}} = Y_e^{-\frac{1}{2}},$$
$$Z_1^\theta = Z_2 Z_\theta^{1/2} = Y_K Y_\psi Y_{\bar{\psi}} Y_\theta, \to Z_\theta = Y_\theta^2,$$
$$Z_m = Y_m Y_\psi Y_{\bar{\psi}} Z_2^{-1} = Y_m Y_K^{-1}, \to Z_m = Y_m Y_K^{-1},$$
$$Z_3^\theta = Z_{e2}^2 Z_3 Z_\theta^{1/2} = Y_A^2 Y_\theta, \to Z_{e2} = Y_A / Z_3^{1/2} = Y_e^{-\frac{1}{2}};$$

由第4、5、6行可得:

$$\tilde{Z}_3 = (Y_A Y_V)^{-1}, \text{并已取}\ Z_U = \tilde{Z}_3, \tilde{Z}_1 = Z_{e3}(Z_K \tilde{Z}_3 Z_2)^{\frac{1}{2}} = Z_{e4}(Z_K \tilde{Z}_3 Z_2)^{\frac{1}{2}} = Y_V^{-1},$$
$$Z_1^\omega = Z_{e5}(Z_\omega \tilde{Z}_3)^{\frac{1}{2}} = (Y_\theta Y_V)^{-1},$$

由以上诸式可得:

$$(Z_1^\psi / Z_2) = (\tilde{Z}_1/\tilde{Z}_3) = Y_A.$$

而

$$Z_{e3} = Z_{e4} = \frac{\tilde{Z}_1}{(Z_K \tilde{Z}_3 Z_2)^{\frac{1}{2}}} = \frac{\tilde{Z}_1}{\tilde{Z}_3 Z_3^{1/2}}(Z_K Z_2)^{-\frac{1}{2}}(\tilde{Z}_3 Z_3)^{\frac{1}{2}},$$

$$Z_{e5}=\frac{\widetilde{Z}_1}{(Z_\theta Z_\omega \widetilde{Z}_3)^{\frac{1}{2}}}=\frac{\widetilde{Z}_1}{\widetilde{Z}_3 Z_3^{1/2}}(Z_\omega Z_\theta)^{-\frac{1}{2}}(\widetilde{Z}_3 Z_3)^{\frac{1}{2}}.$$

取 $Z_K=\widetilde{Z}_3 Z_3 Z_2^{-1}, Z_\omega=\widetilde{Z}_3 Z_3 Z_\theta^{-1}$，则有

$$Z_{e1}=Z_{e2}=Z_{e3}=Z_{e4}=Z_{e5}=Z_e=Y_e^{-\frac{1}{2}}.$$

这样我们便完成了阿贝尔手征群陪集纯规范场理论的重整化。

对殷鹏程、胡瑶光教授的帮助表示感谢。

附　　录

我们来验证(35)式是(33)和(34)式的解。

G_τ 是规范不变的泛函 $\delta G_\tau=0$，不包含鬼场，不是 C_a^+、U_i 的函数，所以

$$\frac{\delta G_\tau}{\delta C_a^+}=\frac{\delta G_\tau}{\delta U_i}=0,$$

满足(34)式；又

$$\hat{S}_0 * G_\tau=\sum_j\left(\frac{\delta \hat{S}_0}{\delta \phi_j}\frac{\delta G_\tau}{\delta U_i}+\frac{\delta \hat{S}_0}{\delta U_j}\frac{\delta G_\tau}{\delta \phi_j}\right)=\sum_j\frac{\delta \hat{S}_0}{\delta U_j}\frac{\delta G_\tau}{\delta \phi_j}=\sum_j\delta\phi_j\frac{\delta G_\tau}{\delta \phi_j}=\delta G_\tau=0.$$

式中 U_i 是与 $\delta\phi_i$ 相应的外源，$\hat{S}_0=\Sigma U_i\delta\phi_i+\cdots$，所以 $\frac{\delta \hat{S}_0}{\delta U_j}=\delta\phi_j$，因而 $\hat{S}_0 * G_\tau$ 是(33)和(34)式的解.

再证 $\hat{S}_0 * k$ 满足 $\hat{S}_0 * \hat{S}_0 * k=0$。必须注意，在 $\delta\phi_i$ 中除去 ξ 因子后，其对易性质和 ϕ_i 相反，因而 U_i 的对易性质和 ϕ_i 相反。为了方便，设 U_i 是反对易的 C 数而 ϕ_i 是对易的 C 数，于是

$$\begin{aligned}
\hat{S}_0 *(\hat{S}_0 * k)&=\hat{S}_0 * \sum_j\left(\frac{\delta \hat{S}_0}{\delta \phi_j}\frac{\delta k}{\delta U_j}+\frac{\delta \hat{S}_0}{\delta U_j}\frac{\delta k}{\delta \phi_j}\right)\\
&=\sum_{j,K}\left[\frac{\delta \hat{S}_0}{\delta \phi_K}\frac{\delta}{\delta U_K}\left(\frac{\delta \hat{S}_0}{\delta \phi_j}\frac{\delta k}{\delta U_j}+\frac{\delta \hat{S}_0}{\delta U_j}\frac{\delta k}{\delta \phi_j}\right)+\frac{\delta \hat{S}_0}{\delta U_K}\frac{\delta}{\delta \phi_K}\left(\frac{\delta \hat{S}_0}{\delta \phi_j}\frac{\delta k}{\delta U_j}\right.\right.\\
&\quad\left.\left.+\frac{\delta \hat{S}_0}{\delta U_j}\frac{\delta k}{\delta \phi_j}\right)\right]=\sum_{j,K}\left[\frac{\delta \hat{S}_0}{\delta \phi_K}\left(\frac{\delta^2 \hat{S}_0}{\delta U_K\delta\phi_j}\frac{\delta k}{\delta U_j}+\frac{\delta \hat{S}_0}{\delta \phi_j}\frac{\delta^2 k}{\delta U_K\delta U_j}\right.\right.\\
&\quad\left.+\frac{\delta^2 S_0}{\delta U_K\delta U_j}\frac{\delta k}{\delta \phi_j}-\frac{\delta \hat{S}_0}{\delta U_j}\frac{\delta^2 k}{\delta U_K\delta\phi_j}\right)\\
&\quad+\frac{\delta \hat{S}_0}{\delta U_K}\left(\frac{\delta^2 \hat{S}_0}{\delta \phi_K\delta\phi_j}\frac{\delta k}{\delta U_j}+\frac{\delta \hat{S}_0}{\delta \phi_j}\frac{\delta^2 k}{\delta \phi_K\delta U_j}+\frac{\delta^2 \hat{S}_0}{\delta \phi_K\delta U_j}\frac{\delta k}{\delta \phi_j}\right.\\
&\quad\left.\left.+\frac{\delta \hat{S}_0}{\delta U_j}\frac{\delta^2 k}{\delta \phi_K\delta\phi_j}\right)\right]=\sum_{j,K}\left[\frac{\delta}{\delta \phi_j}\left(\frac{\delta \hat{S}_0}{\delta \phi_K}\frac{\delta \hat{S}_0}{\delta U_K}\right)\frac{\delta k}{\delta U_j}\right.\\
&\quad\left.-\frac{\delta}{\delta U_j}\left(\frac{\delta \hat{S}_0}{\delta \phi_K}\frac{\delta \hat{S}_0}{\delta U_K}\right)\frac{\delta k}{\delta \phi_j}\right]=0.
\end{aligned}$$

运算过程运用了 $\frac{\delta^2 k}{\delta U_K\delta U_j}=\frac{\delta^2 k}{\delta \phi_K\delta\phi_j}=0$，根据(36)式，$k$ 为 U_i 和 ϕ_i 的线性函数. 又运用了

$$\sum_K\frac{\delta \hat{S}_0}{\delta \phi_K}\frac{\delta \hat{S}_0}{\delta U_K}=\hat{S}_0 * \hat{S}_0=0.$$

显然 $\hat{S}_0 * k$ 也满足鬼方程：

$$\frac{\delta(\hat{S}_0 * k)}{\delta C_a^+}+f_i'^a\frac{\delta(\hat{S}_0 * k)}{\delta U_i}=0.$$

$$\left(\frac{\delta}{\delta C_a^+}+f_i'^a\frac{\delta}{\delta U_i}\right)(\hat{S}_0 * k)=\left(\frac{\delta}{\delta C_a^+}+f_i'^a\frac{\delta}{\delta U_i}\right)\sum_j\left(\frac{\delta \hat{S}_0}{\delta \phi_j}\frac{\delta k}{\delta U_j}+\frac{\delta \hat{S}_0}{\delta U_j}\frac{\delta k}{\delta \phi_j}\right)$$

$$= \sum_i \frac{\delta}{\delta\phi_i}\left(\frac{\delta\hat{S}_0}{\delta C_a^+} + f_i'^a \frac{\delta\hat{S}_0}{\delta U_i}\right)\frac{\delta k}{\delta U_i} - \sum_i \frac{\delta\hat{S}_0}{\delta\phi_i} \frac{\delta}{\delta U_i}\left(\frac{\delta k}{\delta C_a^+} + f_i'^a \frac{\delta k}{\delta U_i}\right)$$

$$- \sum_i \frac{\delta}{\delta U_i}\left(\frac{\delta\hat{S}_0}{\delta C_a^+} + f_i'^a \frac{\delta\hat{S}_0}{\delta U_i}\right) \frac{\delta k}{\delta\phi_i} - \sum_i \frac{\delta\hat{S}_0}{\delta U_i} \frac{\delta}{\delta\phi_i}\left(\frac{\delta k}{\delta C_a^+} + f_i'^a \frac{\delta k}{\delta U_i}\right)$$

$$= 0.$$

已用了(32)式，又由(36)式得

$$\frac{\delta k}{\delta C_a^+} + f_i'^a \frac{\delta k}{\delta U_i} = \int d^4x \sum_i \{b_i f_i'^a \phi_i - f_i'^a b_i \phi_i\} = 0.$$

参 考 文 献

[1] 胡瑶光，量子场论，(1984)，189、118、184. 杨炳麟，量子场论导引，(1988)，533、537.
[2] Kazuo Fujikawa, *Phys. Rev. Lett.*, **42**,**18**(1979), 1195.
[3] 井思聪、阮图南，高能物理与核物理，**6**(1988)，773.
[4] 邱荣，高能物理与核物理，**2**(1991)，111.
[5] 汪容，量子规范理论，(1986)，129.

The Identity and Renormalization of Pure Gauge Fields in Coset Space of Abelian Chiral Group

Qiu Rong

(*Physics Department, Fuzhou University,* 350002)

Abstract

We derived the identity which combines the measure of the path-integral of the generating functional and the effective action of the pure gauge field in the coset space of the Abelian chiral group, and is invariant under chiral transformation. With this identity the renormalization equations are established. The renormalization of the pure gauge field theory in coset space of the Abelian chiral group is carried out by solving these equations.