

# A Moment Analysis for the $J/\psi$ Hadronic Decay Processes

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In this paper the helicity formalism of the angular distributions for processes  $e^+e^- \rightarrow J/\psi \rightarrow V + X$ ,  $V \rightarrow P_1P_2$  or  $V \rightarrow P_1P_2P_3$  and  $X \rightarrow PP$  are given simultaneously. On the basis of the formalism a moment analysis is made and some relations are obtained which can be used for determining the spin  $J_x$  of the boson resonance  $X$  and the helicity amplitude ratios of the process  $J/\psi \rightarrow V + X$ .

## 1. INTRODUCTION

In Ref. [1] the helicity formula of the angular distributions for the  $e^+e^- \rightarrow J/\psi \rightarrow V + X$ ,  $V \rightarrow P_1P_2$  or  $P_1P_2P_3$  and  $e^+e^- \rightarrow J/\psi \rightarrow V + X$ ,  $X \rightarrow PP$  is given. Their vector meson angular distributions of moments are, respectively,

$$\begin{aligned} H_{J_x}(\theta_v, lm) &= \int W_{J_x}(\theta_v, \theta_1, \phi_1) D_{m_0}^l(\phi_1, \theta_1, 0) \sin \theta_1 d\theta_1 d\phi_1, \\ H_{J_x}(\theta_v, LM) &= \int W_{J_x}(\theta_v, \theta, \phi) D_{M_0}^L(\phi, \theta, 0) \sin \theta d\theta d\phi. \end{aligned} \quad (1)$$

and their moments are

$$\begin{aligned} m_{J_x}(lm) &= \int H_{J_x}(\theta_v, lm) \sin \theta_v d\theta_v, \\ M_{J_x}(LM) &= \int H_{J_x}(\theta_v, LM) \sin \theta_v d\theta_v. \end{aligned} \quad (2)$$

where  $\theta_v$  is the angle between the incident positron direction and the emitting vector meson direction in the  $J/\psi$  rest frame;  $(\theta_1, \phi_1)$  describes the direction of the momentum of the pseudoscalar meson

$P_1$  in the rest frame of the vector meson  $V$  for the double pseudoscalar meson decay and  $\theta_1$  and  $\phi_1$  are the polar and azimuthal angles of the normal to the decay plane for the triple pseudoscalar meson decay;  $\theta$  and  $\phi$  are polar and azimuthal angles of the pseudoscalar  $P$  in the  $X$  rest frame. We choose the  $z$  axis parallel to the moving direction of the vector meson  $V$  in the  $J/\psi$  rest frame; the  $e^+$  and  $e^-$  beams are in the  $x$ - $z$  plane. It has been pointed out[2] that the moments are very similar to that of the  $J/\psi$  radiative decay process. It is too simple to give analogous relations to that in Ref.[3] as a criterion for determining the spin-parity of the resonance  $X$  since only two parameters  $l$  and  $m$  or  $L$  and  $M$  are contained. In order to surmount the difficulty, the photon angular distribution of the moment and the weighted moment by choosing appropriate weight functions are used to discuss the determination of the spin of the  $\xi$ ,  $\nu$ -E puzzle and the  $\theta$ -G problem in the  $J/\psi$  radiative decay, and some effective criteria are given[2,4,5,6]. Our procedure is different from that of Ref.[3].

For process  $e^+e^- \rightarrow J/\psi \rightarrow V + X$ , where  $V$  decays into two or three pseudoscalar mesons and  $X$  decays into a pair of pseudoscalar mesons simultaneously, the situation will be different. It is easy to find the angular distribution as follows

$$W_{J_x}(\theta_v, \theta_1, \phi_1, \theta, \phi) \propto \sum_{\substack{\lambda_v, \lambda'_v \\ \lambda_x, \lambda'_x}} I_{\lambda_j, \lambda'_j}(\theta_v) A_{\lambda_v, \lambda_x} A_{\lambda'_v, \lambda'_x} D_{\lambda_v, 0}^{1*}(\phi_1, \theta_1, 0) \cdot D_{\lambda_v, 0}^1(\phi_1, \theta_1, 0) D_{-\lambda_x, 0}^{J_x*}(\phi, \theta) D_{-\lambda'_x, 0}^{J_x}(\phi, \theta). \tag{3}$$

The vector meson angular distribution of the moment is

$$H_{J_x}(\theta_v, LMlm) = \int W_{J_x}(\theta_v, \theta_1, \phi_1, \theta, \phi) D_{m, 0}^L(\phi_1, \theta_1, 0) \cdot D_{m, 0}^L(\phi, \theta) \sin \theta_1 d\theta_1 d\phi_1 \sin \theta d\theta d\phi. \tag{4}$$

The moment is

$$M_{J_x}(LMlm) = \int H_{J_x}(\theta_v, LMlm) \sin \theta_v d\theta_v \tag{5}$$

By using Eq.(3) and the properties of the  $D$ -function we find

$$H_{J_x}(\theta_v, LMlm) \propto \sum_{\substack{\lambda_v, \lambda'_v \\ \lambda_x, \lambda'_x}} I_{\lambda_j, \lambda'_j}(\theta_v) g_{\lambda_v, \lambda_x, \lambda'_v, \lambda'_x}(J_x - \lambda'_x LM | J_x - \lambda_x) \cdot (1 \lambda'_v lm | 1 \lambda_v)(J_x 0 L 0 | J_x 0)(1 0 0 | 1 0). \tag{6}$$

where  $g_{\lambda_v, \lambda_x, \lambda'_v, \lambda'_x} = A_{\lambda_v, \lambda_x} A_{\lambda'_v, \lambda'_x}$ . From parity conservation we have

$$g_{\lambda_v, \lambda_x, \lambda'_v, \lambda'_x} = g_{-\lambda_v, -\lambda_x, -\lambda'_v, -\lambda'_x} = \varepsilon g_{-\lambda_v, -\lambda_x, \lambda'_v, \lambda'_x} = \varepsilon g_{\lambda_v, \lambda_x, -\lambda'_v, -\lambda'_x}. \tag{7}$$

where  $\varepsilon = \eta_x (-1)^{J_x}$  and  $\eta_x$  is the parity of the particle  $X$ . The definition and its final expression of  $I_{\lambda_j, \lambda'_j}(\theta_v)$  have been given in Ref. [2] (Eqs.(6), (7)). In the present case  $I_{\lambda_j, \lambda'_j}(\theta_v)$  have the same form except for substitution  $\theta_y$  by  $\theta_v$ . It is obvious that the following relation is satisfied

$$I_{\lambda_j, \lambda'_j}(\theta_v) = (-1)^{\lambda'_j - \lambda_j} I_{-\lambda_j, -\lambda'_j}(\theta_v) = I_{\lambda'_j, \lambda_j}(\theta_v). \tag{8}$$

Defining

$$K_{\lambda_j, \lambda'_j} = \int I_{\lambda_j, \lambda'_j}(\theta_v) \sin \theta_v d\theta_v. \tag{9}$$

we have

$$\begin{aligned} K_{1,1} &= K_{-1,-1} = \frac{8}{3} p^2, \\ K_{1,0} &= K_{0,1} = -K_{-1,0} = -K_{0,-1} = 0, \\ K_{1,-1} &= K_{-1,1} = \frac{4}{3} p^2, \end{aligned} \tag{10}$$

$$K_{0,0} = \frac{8}{3} p^2.$$

and

$$\begin{aligned} M_{J_x}(LMlm) &\propto \sum_{\substack{\lambda_v, \lambda'_v \\ \lambda_x, \lambda'_x}} K_{\lambda_j, \lambda'_j} g_{\lambda_v, \lambda_x, \lambda'_v, \lambda'_x}(J_x - \lambda'_x LM | J_x - \lambda_x) \\ &\cdot (1 \lambda'_v lm | 1 \lambda_v)(J_x 0 L 0 | J_x 0)(1 0 1 0 | 1 0). \end{aligned} \tag{11}$$

It is an important point that the vector meson angular distribution of the moment  $H_{J_x}(\theta_v, LMlm)$  and the moment itself  $H_{J_x}(LMlm)$  include four parameters  $L, M, l$  and  $m$ . Therefore, it is possible to obtain some relations between them which are helpful for determining the spin of the X and the helicity amplitude ratios of the process  $J/\psi \rightarrow V + X$  more easily by fitting the data.

## 2. MOMENTS AND RELATIONS

Eq.(6) can be rewritten as

$$H_{J_x}(\theta_v, LMlm) \propto i_{J_x}^{M, m}(\theta_v)(J_x 0 L 0 | J_x 0)(1 0 1 0 | 1 0). \tag{12}$$

where the multipole parameter reads

$$\begin{aligned} i_{J_x}^{M, m}(\theta_v) &= \sum_{\substack{\lambda_v, \lambda'_v \\ \lambda_x, \lambda'_x}} I_{\lambda_j, \lambda'_j}(\theta_v)(J_x - \lambda'_x LM | J_x - \lambda_x) \\ &\cdot (1 \lambda'_v lm | 1 \lambda_v) \cdot g_{\lambda_v, \lambda_x, \lambda'_v, \lambda'_x}, \end{aligned} \tag{13}$$

and

$$\begin{aligned} \lambda_j &= \lambda_v - \lambda_x, \quad \lambda'_j = \lambda'_v - \lambda'_x, \quad M = \lambda'_x - \lambda_x, \\ m &= \lambda_v - \lambda'_v, \quad L \leq 2J_x, \quad l \leq 2. \end{aligned} \tag{14}$$

From Eqs.(7), (8) and the properties of C-G coefficients we have

$$t_{J_x, L, l}^{M, m^*}(\theta_v) = (-1)^{L+l} t_{J_x, L, l}^{M, m}(\theta_v), \tag{15}$$

$$t_{J_x, L, l}^{M, m^*}(\theta_v) = (-1)^{L+l+M+m} t_{J_x, L, l}^{-M, -m^*}(\theta_v), \tag{16}$$

$$t_{J_x, L, l}^{M, m^*}(\theta_v) = (-1)^{M+m} t_{J_x, L, l}^{-M, -m}(\theta_v). \tag{17}$$

We take  $L + l$  to be even so that the multipole parameter is real. From Eq.(16) we have

$$t_{J_x, L, l}^{M, m^*}(\theta_v) = (-1)^{M+m} t_{J_x, L, l}^{-M, -m^*}(\theta_v), \quad (L + l) = \text{even} \tag{18}$$

Substituting Eqs.(15)-(17) into Eq.(12) one finds

$$H_{J_x}(\theta_v, LMlm) = (-1)^{L+l} H_{J_x}^*(\theta_v, LMlm), \tag{19}$$

$$H_{J_x}(\theta_v, LMlm) = (-1)^{L+l+M+m} H_{J_x}(\theta_v, L - Ml - m), \tag{20}$$

$$H_{J_x}(\theta_v, LMlm) = (-1)^{M+m} H_{J_x}^*(\theta_v, L - Ml - m). \tag{21}$$

From Eq.(19) and (20) one can see that both  $H_{J_x}(\theta_v, LMlm)$  and the corresponding moment  $M_{J_x}$  are real when  $(L + l)$  is even. From Eqs.(20) and (5) we have

$$\begin{aligned} H_{J_x}(\theta_v, LMlm) &= (-1)^{M+m} H_{J_x}(\theta_v, L - Ml - m), \quad (L + l \text{ to be even}); \\ M_{J_x}(LMlm) &= (-1)^{M+m} M_{J_x}(L - Ml - m), \quad (L + l \text{ to be even}). \end{aligned} \tag{22}$$

If  $l$  is odd, the C-G coefficient  $(10l0 | 10) = 0$ . Therefore, only when  $L$  and  $l$  are even,  $H_{J_x}(\theta_v, LMlm)$  and  $M_{J_x}(LMlm)$  can be real and non-zero. We define the helicity amplitude ratios as the following

$$x = \frac{A_{11}}{A_{10}}, \quad y = \frac{A_{12}}{A_{10}}, \quad z_1 = \frac{A_{00}}{A_{10}}, \quad z_2 = \frac{A_{01}}{A_{10}}. \tag{23}$$

For the process under discussion,  $J_x^{z^*} = (2n)^+, n = 0, 1, 2, \dots$ . In the following, we give the vector meson angular distribution of the moment  $H_{J_x}(\theta_v, LMlm)$ , moment  $M_{J_x}(LMlm)$  and some relations for three cases  $J_x^{z^*} = 0^+, 2^+$  and  $4^+$ .

(1),  $J_x^{z^*} = 0^+$

In this case  $L$  and  $M$  must be zero. Hence

$$\begin{aligned} H_0(\theta_v, 00lm) &\propto \sum_{\lambda_v, \lambda'_v} I_{\lambda_j, \lambda'_j}(\theta_v) g_{\lambda_v, \lambda'_v} (1\lambda'_v lm | 1\lambda_v) \cdot (10l0 | 10), \\ M_0(00lm) &\propto \sum_{\lambda_v, \lambda'_v} K_{\lambda_j, \lambda'_j} g_{\lambda_v, \lambda'_v} (1\lambda'_v lm | 1\lambda_v) \cdot (10l0 | 10). \end{aligned} \tag{24}$$

There are only four independent and non-zero vector meson angular distributions of moments. They are  $H_0(\theta_v, 0000)$ ,  $H_0(\theta_v, 0020)$ ,  $H_0(\theta_v, 0021)$  and  $H_0(\theta_v, 0022)$ . Because there are only two

independent helicity amplitudes  $A_{10}$  and  $A_{00}$  for the process  $J/\psi \rightarrow V + X(0^+)$ , only one helicity amplitude ratio  $z_1$  appears in the expression of  $H_0$ . For example

$$\begin{aligned}
 H_0(\theta_v, 0000) &\propto 2g_{1010}I_{1,1}(\theta_v) + g_{0000}I_{0,0}(\theta_v) \sim 2p^2[(1 + \cos^2\theta_v) + z_1^2\sin^2\theta_v], \\
 H_0(\theta_v, 0021) &\propto \frac{2}{5}\sqrt{3}g_{1000}I_{1,0}(\theta_v) \sim \frac{\sqrt{6}}{5}p^2 \cdot z_1 \sin 2\theta_v,
 \end{aligned}
 \tag{25}$$

The relation can be found,

$$\begin{aligned}
 2H_0(\theta_v, 0000) - 5H_0(\theta_v, 0020) - 5\sqrt{6}H_0(\theta_v, 0022) \\
 \propto 6g_{1010}[I_{1,1}(\theta_v) + I_{1,-1}(\theta_v)] \sim 12p^2.
 \end{aligned}
 \tag{26}$$

We notice that in the right hand side of Eq.(26) no  $z_1$  dependent term appears and all terms are independent of  $\theta_v$ .

$$(2), J_z^2 = 2^+$$

In this case  $L$  can be 0, 2 or 4,  $|M| \leq L, l \leq 2$  and  $|m| \leq l$ . Considering Eqs.(22) and (14) there are 33 independent and non-zero  $H_2(\theta_v, LMlm)$ . Because there are five independent helicity amplitudes:  $A_{10}, A_{11}, A_{12}, A_{00}$  and  $A_{01}$  for the process  $J/\psi \rightarrow V + X(2^+)$ , four helicity amplitude ratios  $x, y, z_1$  and  $z_2$  will appear in the expressions of  $H_2$  and the corresponding moment  $M_2$ . We can obtain many relations among them.

By analogy with Eq.(26) we have

$$\begin{aligned}
 2H_2(\theta_v, 0000) - 5H_2(\theta_v, 0020) - 5\sqrt{6}H_2(\theta_v, 0022) \\
 \propto 6\{g_{1112}I_{1,1}(\theta_v) + g_{1111}I_{0,0}(\theta_v) + g_{1010}[I_{1,1}(\theta_v) + I_{1,-1}(\theta_v)]\} \\
 \sim 6p^2[y^2(1 + \cos^2\theta_v) + 2z_1^2\sin^2\theta_v] + 12p^2,
 \end{aligned}
 \tag{27}$$

where the term containing  $x^2$  and  $y^2$  appears at the right hand side of the equation. Other relations can be found as well,

$$\begin{aligned}
 14\sqrt{5}H_2(\theta_v, 2200) - 35\sqrt{5}H_2(\theta_v, 2220) \\
 + 28\sqrt{3}H_2(\theta_v, 4200) - 70\sqrt{3}H_2(\theta_v, 4220) = 0,
 \end{aligned}
 \tag{28}$$

$$\begin{aligned}
 \sqrt{3}H_2(\theta_v, 4100) + 5\sqrt{3}H_2(\theta_v, 4120) \\
 - \sqrt{10}H_2(\theta_v, 2100) - 5\sqrt{10}H_2(\theta_v, 2120) = 0,
 \end{aligned}
 \tag{29}$$

$$\begin{aligned}
 21H_2(\theta_v, 4021) - 6H_2(\theta_v, 0021) \\
 - 5\sqrt{6}H_2(\theta_v, 222 - 1) - 6\sqrt{10}H_2(\theta_v, 422 - 1) = 0,
 \end{aligned}
 \tag{30}$$

$$\begin{aligned}
 54H_2(\theta_v, 2200) + 270H_2(\theta_v, 2220) - 26\sqrt{15}H_2(\theta_v, 4200) \\
 - 130\sqrt{15}H_2(\theta_v, 4220) \propto 14\sqrt{6}g_{0101}I_{1,-1}(\theta_v) \\
 \sim 34.3p^2z_1^2\sin^2\theta_v,
 \end{aligned}
 \tag{31}$$

$$\begin{aligned}
 & 270H_2(\theta_v, 0021) - 1485H_2(\theta_v, 2021) + 715H_2(\theta_v, 4021) \\
 & \propto \left( \frac{94}{\sqrt{3}} g_{1101} + 20\sqrt{3} g_{1000} \right) I_{1,0}(\theta_v) \\
 & \sim (38.4xz_2 + 24.5z_1)p^2 \sin 2\theta_v,
 \end{aligned} \tag{32}$$

$$\begin{aligned}
 & 26\sqrt{15}H_2(\theta_v, 4100) + 130\sqrt{15}H_2(\theta_v, 4120) \\
 & - 81\sqrt{2}H_2(\theta_v, 2100) - 405\sqrt{2}H_2(\theta_v, 2120) \\
 & \propto 42\sqrt{2} g_{0001} I_{1,0}(\theta_v) \sim 42p^2 z_1 z_2 \sin 2\theta_v,
 \end{aligned} \tag{33}$$

$$\frac{H_2(\theta_v, 212 - 2)}{\sqrt{2} H_2(\theta_v, 2022)} = x \operatorname{ctg} \theta_v, \tag{34}$$

$$- \frac{4H_2(\theta_v, 2021) + 3H_2(\theta_v, 4021)}{4H_2(\theta_v, 0022)} = z_1 \operatorname{ctg} \theta_v, \tag{35}$$

$$\frac{14\sqrt{6} M_2(2200) - 35\sqrt{6} M_2(2220)}{60 M_2(0022)} = y, \tag{36}$$

$$\frac{\sqrt{2} M_2(2121)}{M_2(2022)} = z_1. \tag{37}$$

(3)  $J_x^* = 4^+$

In this case  $L$  can be 0, 2, 4, 6 or 8,  $l \leq 2$  and  $|m| \leq 1$ . From Eq.(14) we know  $|M| \leq 4$ . By a further consideration Eq.(22) we find 65 independent and non-zero  $H_4(\theta_v, L M l m)$ . In view of the limit given by Eq.(14) there are only five independent helicity amplitudes, i.e.,  $A_{10}, A_{11}, A_{12}, A_{00}$  and  $A_{01}$  for the process  $J/\psi \rightarrow V + X(4^+)$ . Consequently, only four helicity amplitude ratios, i.e.,  $x, y, z_1$  and  $z_2$ , appear in the  $H_4(\theta_v, L M l m)$  and corresponding moments  $M_4(L M l m)$ . On the analogy of Eqs.(27)-(33), we can now find many relations. For example,

$$\begin{aligned}
 & 2H_4(\theta_v, 0000) - 5H_4(\theta_v, 0020) - 5\sqrt{6} H_4(\theta_v, 0022) \\
 & \propto 6\{g_{1212} I_{1,1}(\theta_v) + g_{1111} I_{0,0}(\theta_v) + g_{1010} [I_{1,1}(\theta_v) + I_{1,-1}(\theta_v)]\} \\
 & \sim 6p^2 [y^2(1 + \cos^2 \theta_v) + 2x^2 \sin^2 \theta_v] + 12p^2,
 \end{aligned} \tag{38}$$

Being similar to Eq.(27), Eq.(38) is independent of  $J_x$  being 2 or 4. But relation (38) is different from Eq.(26). Other relations are

$$\begin{aligned}
 & 14\sqrt{5} H_4(\theta_v, 2200) - 35\sqrt{5} H_4(\theta_v, 2220) \\
 & + 28\sqrt{3} H_4(\theta_v, 4200) - 70\sqrt{3} H_4(\theta_v, 4220) \\
 & \propto - \left( \frac{180}{11} + \frac{108}{13} \right) \sqrt{3} g_{1012} I_{1,-1}(\theta_v) \sim -42.7 p^2 y \sin^2 \theta_v,
 \end{aligned} \tag{39}$$

$$\begin{aligned} & \sqrt{3} H_4(\theta_v, 4100) + 5\sqrt{3} H_4(\theta_v, 4120) \\ & - \sqrt{10} H_4(\theta_v, 2100) - 5\sqrt{10} H_4(\theta_v, 2120) \\ & \propto -\frac{42\sqrt{3}}{143} g_{0001} I_{1,0}(\theta_v) \sim -0.36 p^2 z_1 z_2 \sin 2\theta_v, \end{aligned} \tag{40}$$

$$\begin{aligned} & 21H_4(\theta_v, 4021) - 6H_4(\theta_v, 0021) - 5\sqrt{6} H_4(\theta_v, 222 - 1) \\ & - 6\sqrt{10} H_4(\theta_v, 422 - 1) \propto \left( -\frac{162}{143} \sqrt{3} g_{1101} - \frac{744}{715} \sqrt{3} g_{1000} \right. \\ & \left. + \frac{1008}{143} \sqrt{\frac{3}{10}} g_{0012} \right) I_{1,0}(\theta_v) \sim (-1.39 x z_2 - 1.27 z_1 + 2.73 z_1 y) p^2 \sin 2\theta_v, \end{aligned} \tag{41}$$

$$\begin{aligned} & 54H_4(\theta_v, 2200) + 270H_4(\theta_v, 2220) - 26\sqrt{15} H_4(\theta_v, 4200) \\ & - 130\sqrt{15} H_4(\theta_v, 4220) = 0, \end{aligned} \tag{42}$$

$$\begin{aligned} & 270H_4(\theta_v, 0021) - 1485H_4(\theta_v, 2021) + 715H_4(\theta_v, 4021) = 0, \\ & 26\sqrt{15} H_4(\theta_v, 4100) + 130\sqrt{15} H_4(\theta_v, 4120) \end{aligned} \tag{43}$$

$$- 81\sqrt{2} H_4(\theta_v, 2100) - 405\sqrt{2} H_4(\theta_v, 2120) = 0. \tag{44}$$

and

$$\frac{\sqrt{15}}{3} \frac{H_4(\theta_v, 212 - 2)}{H_4(\theta_v, 2022)} = x \operatorname{ctg} \theta_v, \tag{45}$$

$$\frac{81H_4(\theta_v, 0021) - 1001H_4(\theta_v, 4021)}{162H_4(\theta_v, 0022)} = z_1 \operatorname{ctg} \theta_v, \tag{46}$$

$$\frac{-5\sqrt{10} M_4(2220) + 2\sqrt{10} M_4(2200)}{45 M_4(2022)} = y, \tag{47}$$

$$\frac{2}{3} \sqrt{15} \frac{M_4(2121)}{M_4(2022)} = z_2. \tag{48}$$

### 3. ANALYSIS AND DISCUSSIONS

In section 2 we have pointed out the difference between Eq.(26) and Eq.(27) (or Eq.(28)). If the vector meson angular distribution of moment given by the experiment,  $[2H_{J_x}(\theta_v, 0000) - 5H_{J_x}(\theta_v, 0020) - 5\sqrt{6} H_{J_x}(\theta_v, 0022)]$  is independent of  $\theta_v$ , we can conclude that the spin-parity of the particle X is  $J_{\frac{1}{2}}^{\pm} = 0^+$ . For the opposite case, we can only rule out the possibility of  $J_{\frac{1}{2}}^{\pm} = 0^+$ , but we can not determine  $J_{\frac{1}{2}}^{\pm}$  being 2 or 4.

In order to determine the spin-parity of the particle X, Eqs.(28)-(33) and Eqs.(39)-(44) are

very useful. Especially, Eqs.(28) and (30) as well as Eqs.(31) and (42) are more sensitive.

After the assignment of the spin-parity of the particle X, we can determine four helicity amplitude ratios  $x$ ,  $y$ ,  $z_1$  and  $z_2$ , respectively, through fitting experimental data by using the relations of the vector meson angular distribution of moment or moment itself, for example, Eqs.(34)-(37) for  $J_x = 2^+$  and Eqs.(45)-(48) for  $J_x = 4^+$ . This procedure is more simple and clear than the method of fitting in with four helicity amplitude ratios from one angular distribution. It is noteworthy that Eqs.(34)-(37) and Eqs.(45)-(48) are all proportions, that common factors appeared in the expressions of the vector meson angular distribution of moments, and that moments are all canceled out and the equations are rigorous. It is more convenient and straightforward for determining the values of  $x$ ,  $y$ ,  $z_1$  and  $z_2$ .

For the process  $e^+e^- \rightarrow J/\psi \rightarrow V + X$ , where V continues to decay into two or three pseudoscalar mesons and X decays into three pseudoscalar mesons simultaneously, the situation will be more complicated. However, it may provide some useful clue on the exotic state with  $J^{PC} = 1^{-+}$ . We will discuss this problem in the next work.

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