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Deconfinement Transition of the Nontopological Soliton Model at Finite Temperature

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The temperature dependence of σ_{vac} describing the extent of gluon condensation and of color-dielectric constant K_{vac} describing the confinement of quarks are discussed in the framework of nontopological soliton model at finite temperature. The mechanism of deconfinement transition is analysed.

The nontopological soliton model [1,2] has been very successful in describing the static properties of isolated hadron. The model has two main features, one is the introduction of a phenomenological scalar field σ of the quantum number 0^{++} of vacuum, accounting for the gluon condensation arising from the nonlinear interaction of color fields [2]; the other is the introduction of the color-dielectric function $K(\sigma)$ which assures the confinement of quarks.

The covariant and renormalisable Lagrangian of this model has the following form [2]

$$L = \bar{\Psi}(i\gamma_\mu\partial^\mu - g\sigma)\Psi + 1/2\partial_\mu\sigma\partial^\mu\sigma - U(\sigma); \tag{1}$$

$$U(\sigma) = \frac{a}{2!}\sigma^2 + \frac{b}{3!}\sigma^3 + \frac{c}{4!}\sigma^4 + B, \tag{2}$$

where B is the bag constant and a, b, c are adjustable parameters which are used to fit the static properties of hadrons. The values of these parameters are not unique. $U(\sigma)$ has two minima, one at $\sigma = 0$ and the other lower one at

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$$\sigma_{vac} = \frac{3|b|}{2c}\left(1 + \sqrt{1 - \frac{8ac}{b^2}}\right); \tag{3}$$

where σ_{vac} describes the extent of gluon condensation in a QCD vacuum.

At zero temperature, the nonlinearity of $U(\sigma)$ provides a soliton bag for hadron. The external pressure supplied by gluon condensation in the QCD vacuum ensures the stability of the bag, and the confinement takes place. The "family" parameter $f = b^2/ac$ [2] characterizes the shape of $U(\sigma)$. The values of $f = 3$ and $f = \infty$ correspond to two limiting cases, in which the soliton solution still exists, as illustrated in Fig.1.

At finite temperature, the deconfinement transition of the model has been considered by some authors [3]. In this paper, we use the technique proposed by Linde [4], which takes the Gibbs average of the field equation, to investigate the influence of the thermal excitation on gluon condensation and the temperature dependence of color-dielectric constant close to the vacuum state. The physical mechanism of deconfinement is analysed. Some new results are obtained.

The field equation of σ can be obtained from the Langrangian Eq.(1). In order to discuss the quantum fluctuation close to the vacuum state, it is convenient to perform a shift of σ field with respect to this state

$$\sigma \to \sigma' + v, \tag{4}$$

where $v = \langle 0|\sigma|0\rangle = \sigma_{vac}$. At finite temperature, we consider the thermodynamic equilibrium system described by Lagrangian Eq.(1). Following the method proposed by Linde [4], we take the Gibbs average of the shifted field equation. The Gibbs average of σ field is $v(T) = \langle\sigma\rangle_\beta = \sigma_{vac}(T)$. The shifted field σ' describes the thermal excitation of glueball around the vacuum state and satisfies the relation $\langle\sigma'\rangle_\beta = 0$. $\sigma_{vac}(T)$ corresponds to the vacuum state at finite temperature, which describes the extent of gluon condensation in the QCD vacuum. $\sigma_{vac}(T)$ may be chosen as the "order

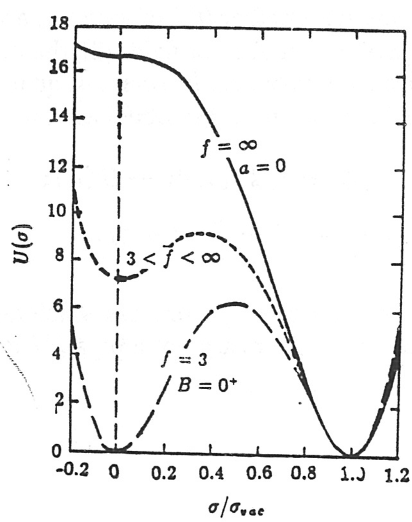

FIG. 1 Three forms for $U(\sigma)$. $f = 3$ and $f = \infty$ are the two limiting cases, in which the soliton bag still exists.

parameter" of deconfinement transition for the following reason. When the temperature is low, $\sigma_{vac}(T) \neq 0$, gluon condensation provides the condition for the existence of the soliton bag, and there is color confinement. At a certain temperature, $\sigma_{vac}(T)$ vanishes, the gluon condensation disappears and the soliton bag no longer exists, which means that the deconfinement transition takes place.

Taking into account that $v(T)$ is a constant field depending only on temperature, we obtain the equation

$$\left(a + \frac{c}{2}\langle\sigma'^2\rangle_\beta\right) v(T) + \frac{b}{2} v^2(T) + \frac{c}{6} v^3(T) + \frac{b}{2}\langle\sigma'^2\rangle_\beta + \frac{c}{6}\langle\sigma'^3\rangle_\beta + g\langle\bar{\Psi}\psi\rangle_\beta = 0. \quad (5)$$

Since our attention is concentrated on the effect of the excitation of the glueball on quark confinement, as in the case of Ref. [3], the contribution of fermions $g<\bar{\psi}\psi>_\beta$ is temporarily neglected. Consider the density of glueballs in the momentum space

$$\langle a_p^+ a_p\rangle = 1/[\exp(\sqrt{p^2 + m_\sigma^2}/T) - 1],$$

We have

$$\langle\sigma^{2'}\rangle_\beta = T^2 f(m_\sigma^2/T^2);$$

$$f(m_\sigma^2/T^2) = \frac{1}{2\pi^2}\int_0^\infty \frac{x^2 dx}{y[\exp(y) - 1]}, \quad (6)$$

where $y = \sqrt{x^2 + \frac{m_\sigma^2}{T^2}}$, $m_\sigma = [U''(\sigma_{vac})]^{1/2}$ is the mass of the glueball [2]. At high temperature $T >> m_\sigma$, $f(m_\sigma/T^2) = 1/12$.

There have been several linearized approximation schemes [5] for the term $<\sigma'^3>_\beta$, but we will not use such approximations. According to the dimensional analysis, this term is proportional to T^3. The proportional coefficient can approximately be taken as a constant α, which resembles the term $<\sigma'^2>_\beta = T^2/12$ at high temperature. Thus we have

$$\langle\sigma'^3\rangle_\beta = \alpha T^3; \quad (7)$$

Solving Equation (5) under the above approximation, we obtain the temperature dependence of $\sigma_{vac}(T)$ as illustrated in Fig. 2(a). At finite temperature, we can define the color dielectric constant as [2]

$$K_{vac}(T) = 1 + \theta(\sigma_{vac}(T))y^3(3y - 4); \quad (8)$$

where $y = \sigma_{vac}(T)/\sigma_{vac}(0)$, and $\sigma_{vac}(0) = \sigma_{vac}$. The temperature dependence of $K_{vac}(T)$ is shown in Fig. 2(b). Fig. 2 shows the results for the cases $\alpha > 0$, $\alpha = 0$, $\alpha < 0$ respectively. From the approximate treatment of $<\sigma'^3>_\beta$ in Ref. [5], we know that its contribution is positive definite, i.e. $\alpha > 0$. In this case, at a certain temperature $\sigma_{vac}(T) \to 0$, $K_{vac}(T) \to 1$ (c.f. Fig. 2), and the confinement of the nontopological soliton model is removed.

When the deconfinement transition occurs, the gluon condensation disappears, $\sigma_{vac}(T_c) = 0$. From Eq.(5) the relation satisfied by the critical temperature T_c is

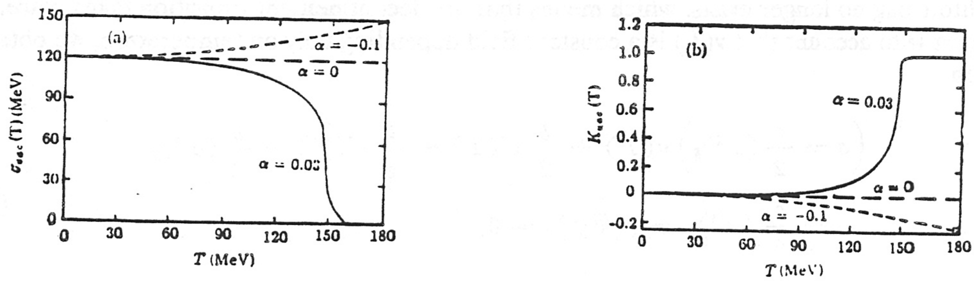

FIG. 2

(a) The temperature dependence of gluon condensation $\sigma_{vac}(T)$ in the QCD vacuum.

(b) The temperature dependence of color-dielectric constant $K_{vac}(T)$ in the QCD vacuum. The model parameters are taken from Ref.[2]: a = 1.908 fm^{-2}, b = 42.945 fm^{-1}, c = 177.34.

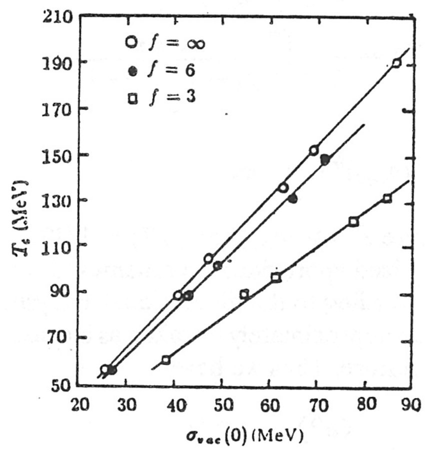

FIG. 3 The relations between critical temperature T_c and $\sigma_{vac}(0)$ for fixed values of f.

$$\left[\frac{b}{2}\langle\sigma'^2\rangle_\beta + \frac{c}{6}\langle\sigma'^3\rangle_\beta + g\langle\bar{\Psi}\Psi\rangle_\beta\right]_{T=T_c} = 0. \tag{9}$$

Starting from the above formalism, we have investigated the relation of T_c with both f and $\sigma_{vac}(0)$. Sixteen sets of values for the parameters a, b and c supplied by Ref. [2] have been used. It turns out that for fixed value of the shape parameter f of $U(\sigma)$, T_c increases linearly with the increase of the extent of gluon condensation $\sigma_{vac}(0)$. On the other hand, for fixed $\sigma_{vac}(0)$, the shape parameter f of $U(\sigma)$ affects the critical temperature T_c. The value of the critical temperature T_c is lowest for the case f = 3 and highest for the case $f = \infty$, as illustrated in Fig. 3. As has been pointed out previously (c. f. Fig.1), f = 3 and $f = \infty$ are associated with the two limiting cases of $B = 0^+$ and maximum B, respectively. We can see that T_c depends on B. The larger B is, the higher T_c will become.

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