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Kaon Condensation at Finite Temperature

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The discussion on kaon condensation at zero temperature given by Brown et al., is generalized to that at finite temperature. It shows that the critical nuclear density is little influenced by temperature. A new phase may be formed in heavy ion reaction.

Based on $SU(3) \times SU(3)$ chiral Lagrangians, recent studies show that kaons might condense at a nuclear density about three times that of the normal nuclear matter [1]. Brown et al. [2] considered that this phenomenon can be understood as the dense nuclear matter "clears" $q\bar{q}$ condensations out the QCD vacuum. This also implies that a new phase will appear, which is different from either the quark plasma or the usual dense hadronic matter. The new matter phase might also appear in heavy ion collisions besides existing in neutron stars. As a high temperature may be produced in heavy ion collisions, the discussion on kaon condensation at zero temperature naturally should be generalized to that at finite temperature to see how the condensation changes with temperature.

In this paper, we will follow the approach proposed by Brown et al. [2]. which simplifies the problem to an $SU(2) \times SU(2)$ subgroup, a V-spin sigma model. The simplified $SU(2) \times SU(2)$ chiral Lagrangian may be written as

$$\begin{aligned} \mathscr{L} &= \mathscr{L}_0 + \mathscr{L}_{SB}, \\ \mathscr{L}_0 &= \bar{D}[i\gamma\partial - g(\Sigma + i\vec{V}\cdot\vec{K}\gamma_5)]D \\ &\quad + \frac{1}{2}[(\partial_\mu\Sigma)^2 + (\partial_\mu\vec{K})^2] - \frac{1}{4}\lambda(\Sigma^2 + \vec{K}^2 + \vec{V}^2)^2, \\ \mathscr{L}_{SB} &= c\Sigma, \end{aligned} \tag{1}$$

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where D is the V-spin doublet for baryons, and Σ is a V-spin scalar and a space scalar.

$$V_1 = \frac{1}{4}\lambda_4, \quad V_2 = \frac{1}{2}\lambda_5,$$
$$V_3 = \frac{1}{2}(\lambda_3/2 + \sqrt{3}\lambda_8/2), \tag{2}$$

Thus the form alism developed by Campbell, Dashen and Manassah for the pion condensation [2, 4] could be adopted to study the kaon condensation. The real time Green function method proposed in a series of previous papers [3] is used to study the influence of temperature on the condensation. The terms correlated to the nucleon are treated classically, K and Σ field and their conjugate momenta are canonically quantized.

$$K = \sum_k \frac{1}{\sqrt{2\omega'_k V}}(a_k + b^{\pm}_{-k})e^{ik\cdot x}, \quad K^* = \sum_k \frac{1}{\sqrt{2\omega'_k V}}(a_k + b_{-k})e^{-ik\cdot x}$$
$$P_K = i\sum_k \sqrt{\frac{\omega'_k}{2V}}(a^{+}_{k} - b_{-k})e^{-ik\cdot x}, \quad P_{K^*} = -i\sum_k \sqrt{\frac{\omega'_k}{2V}}(a_k - b^{\pm}_{-k})e^{ik\cdot x} \tag{3}$$
$$\Sigma = \sum_k \frac{1}{\sqrt{2\omega_k V}}(\tilde{\sigma}_k + \tilde{\sigma}^{+}_{k})e^{ik\cdot x}, \quad P_\Sigma = i\sum_k \sqrt{\frac{\omega_k}{2V}}(\tilde{\sigma}^{+}_{k} - \tilde{\sigma}_{-k})e^{-ik\cdot x}$$

Performing a Bogoliubov transformation

$$\tilde{\sigma}_k = \sqrt{N_\Sigma}\,\delta_k + \sigma_k, \quad \delta_k = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}, \tag{4}$$

we substitute it into the Hamiltonian of the system, and define the Green's functions

$$G_1 = \langle\langle \sigma_k | \sigma^{+}_{k} \rangle\rangle, \quad G_2 = \langle\langle \sigma^{\pm}_{-k} | \sigma^{+}_{k} \rangle\rangle,$$
$$G_3 = \langle\langle a_k | a^{+}_{k} \rangle\rangle, \quad G_4 = \langle\langle a^{\pm}_{-k} | a^{+}_{k} \rangle\rangle, \tag{5}$$

Under the first-order pairing cut-off approximation, we solved the equation of motion for the Green's functions by the approach given in Ref. [3], and obtained

$$\sum_p \frac{\langle \sigma^{+}_{p}\sigma^{+}_{-p}\rangle + \langle \sigma^{+}_{p}\sigma_p\rangle}{\omega_p} = \frac{T^2}{12}V. \tag{6}$$

N_Σ is determined by $\delta\langle :H: \rangle / \delta N_\Sigma = 0$.

Let $\langle \Sigma \rangle_T \equiv f_K(T)$, then we have

$$\frac{2f^2_K(T)}{f^2_K(0)} - \frac{f_K(0)}{f_K(T)} = 1 - \frac{5}{6}\frac{T^2}{f_K(0)}. \tag{7}$$

According to Refs.[2] and [4], the energy density of matter is

$$\begin{aligned} \varepsilon^{\text{eff}} &= \varepsilon^V + \varepsilon^H + \varepsilon^C, \\ \varepsilon^V &= f_K(0)f_K(T)\left[-\frac{1}{2}\mu^2\sin^2\theta + m_K^2(1-\cos\theta)\right], \\ \varepsilon^H &= \varepsilon_0^H + \rho(\cos\theta - 1)\Sigma^{KN}, \\ \varepsilon^C &= \text{the energy correlated only to the nucleon.} \end{aligned} \tag{8}$$

where μ is the chemical potential associated with the charge conservation and θ is the chiral angle. For small θ, we have

$$\begin{aligned} \varepsilon^{\text{eff}} &= \varepsilon(0) + (f_K(0)f_K(T)m_K^2 - f_K(0)f_K(T)\mu^2 - \rho\Sigma^{KN})\frac{1}{2}\theta^2 \\ &+ O(\mathscr{L}_{SB}^2) + O(\theta^4) + \cdots\cdots. \end{aligned} \tag{9}$$

Brown et al. considered that the KN interaction is weak, and then expected that ε^C and ε_0^H will not strongly depend on the chiral angle. As the chiral angle deviates from zero, the energy will be lowered if the coefficient of θ^2 is negative. Thus we obtain the critical nucleon density

$$\rho_c = f_K(0)f_K(T)(m_K^2 - \mu^2)/\Sigma^{KN}. \tag{10}$$

The time required to reach the high density in heavy ion collisions is very short, so the strangeness number is conserved. For $\mu = 0$, $f_K(0) \cong f_\pi = 93$ MeV, $m_K = 494$ MeV and $\Sigma^{KN} \cong \langle N|\hat{\Sigma}|N\rangle = \left\langle N\left|\frac{1}{2}(m_n + m_s)(\bar{u}u + \bar{s}s)\right|N\right\rangle \cong 570$ MeV [2], we have

$$\begin{aligned} T &= 0 & \rho_c &\cong 2.7\rho_0, \\ T &= 100\text{MeV} & \rho_c &\cong 2.2\rho_0, \\ T &= 200\text{MeV} & \rho_c &\cong 1.7\rho_0. \end{aligned} \tag{11}$$

It can be seen from the above results that

(1) It is easier to clear $q\bar{q}$ pairs out from the QCD vacuum after increasing the temperature, but the critical baryon density for kaon condensation is little influenced by the temperature. Thus kaon condensation state may exist in a wide range of temperature, and a new phase might be formed in heavy ion reactions.

(2) As the first-order pairing cut-off approximation is equivalent to the Hartree approximation, the change of quantities related to the nucleon is not large at finite temperatures($T < 200$ MeV), so the approximation conditions given by Brown are still satisfied.

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