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用路径积分法导出（1＋1）维费米场的非阿贝尔玻色化*

倪光炯　汪荣泰　陈苏卿
（复旦大学物理系，上海）

摘　要

本文利用随动表象下处理反常问题的路径积分方法[15]，给出(1 + 1)维空间费米场玻色化的统一推导方案．有外电场的无质量 Thirring 模型，以及有内部 $SU(N)$ 对称性的 Gross-Neveu 模型，分别作为阿贝尔和非阿贝尔玻色化的例子，作了具体的讨论．

一、引　言

在(1 + 1)维空间的费米场与玻色场，在量子场论的水平上存在如下的对应关系[1-4]：

$$i\bar{\psi}\partial\!\!\!/\psi \longleftrightarrow \frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi, \tag{1.1}$$

$$j_\mu = \bar{\psi}\gamma_\mu\psi \longleftrightarrow \frac{\beta}{2\pi}\,\varepsilon_{\mu\nu}\partial^\nu\phi, \tag{1.2}$$

$$j^5_\mu = \bar{\psi}\gamma_\mu\gamma^5\psi \longleftrightarrow \frac{\beta}{2\pi}\,\partial_\mu\phi, \tag{1.3}$$

$$\sigma_\pm = \bar{\psi}\,\frac{1}{2}\,(1 \pm \gamma_5)\psi \longleftrightarrow K^2 \!:\exp(\pm i\beta\phi):. \tag{1.4}$$

等等．其中 β 是与模型有关的常数(在下面 Thirring 模型中，$\beta^2 = 4\pi(1 + g^2/\pi)^{-1}$)．但是对有多个分量的费米场，例如 ψ 有内部 $SU(N)$ 或 $O(N)$ 对称性的情形，把上述"阿贝尔"玻色化法则简单地推广是有困难的． 于是 Witten 提出了新的"非阿贝尔"玻色化方案[5]近年来对有关玻色化的问题讨论很多[6-14]，本文企图统一地用路径积分方法导出玻色化，形式上与文献[15]处理反常的方案有些相似．为易于理解起见，在第二、三两节中分别以单分量费米场的无质量 Thirring 模型以及有 N 分量的 Gross-Neveu 模型作为阿贝尔和非阿贝尔玻色化的实例，加以具体的讨论．

二、无质量 Thirring 模型(有外电场)

在闵空间有外电场的无质量 Thirring 模型之拉氏密度为：

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$$\mathscr{L} = i\bar{\psi}\not{\partial}\psi + e j^{\mu} A_{\mu} - \frac{g^2}{2} j^{\mu} j_{\mu}, \tag{2.1}$$

这里取[16] $g_{\mu\nu} = \mathrm{diag}(g_{11}, g_{00})$，$g_{11} = -g_{00} = 1$，$j_{\mu} = \bar{\psi}\gamma_{\mu}\psi$,

$$\gamma^0 = \sigma_2, \quad \gamma^1 = i\sigma_1, \quad \gamma_5 = \gamma^0\gamma^1 = \sigma_3, \tag{2.2}$$

注意在(1 + 1)维空间的 γ 矩阵有一个重要关系，它是玻色化的依据：

$$\gamma^{\mu}\gamma_5 = \varepsilon^{\mu\nu}\gamma_{\nu}. \tag{2.3}$$

($\varepsilon^{01} = 1$)，由(2.1)式写出闵空间生成泛函的路径积分形式：

$$Z_M = \int \prod_x d\bar{\psi} d\psi \exp\left\{i\int d^2x \mathscr{L}\right\}. \tag{2.4}$$

为处理(2.1)中的流-流自耦合项，我们引入辅助场 $k^{\mu}(x)$ 和 $h^{\mu}(x)$[16]：

$$\begin{aligned} Z_M &= \int \prod_{x,\mu} d\bar{\psi} d\psi dk^{\mu}(x)\delta(k^{\mu}(x) - \bar{\psi}\gamma^{\mu}\psi) \\ &\quad \times \exp\left\{i\int d^2x \left[i\bar{\psi}\not{\partial}\psi + e k^{\mu}A_{\mu} - \frac{g^2}{2}k^{\mu}k_{\mu}\right]\right\} \\ &= \int \prod_{x,\mu} d\bar{\psi} d\psi dk^{\mu}\frac{dh^{\mu}(x)}{2\pi}\exp\left[i\int d^2x h^{\mu}(k_{\mu} - \bar{\psi}\gamma_{\mu}\psi)\right] \\ &\quad \times \exp\left\{i\int d^2x \left[i\bar{\psi}\not{\partial}\psi + e k^{\mu}A_{\mu} - \frac{g^2}{2}k^{\mu}k_{\mu}\right]\right\}, \end{aligned} \tag{2.5}$$

现在按下述规则从闵空间解析延拓到欧空间[16]：

$$\begin{aligned} & x_M^0 \to -ix_{2E}, \quad \bar{\psi}_M \to -i\bar{\psi}_E, \\ & A_{0M} \to iA_{2E}, \quad k_{0M} \to ik_{2E}, \quad h_{0M} \to ih_{2E}, \\ & x_M^1 = x_{1E}, \quad \psi_M = \psi_E, \quad \gamma_M^0 = \gamma_{2E}, \quad \gamma_M^1 = i\gamma_{1E}, \\ & A_{1M} = A_{1E}, \quad k_{1M} = k_{1E}, \quad h_{1M} = h_{1E}, \end{aligned} \tag{2.6}$$

则

$$\begin{aligned} Z_M \to Z_E = \int \prod_{\mu,x} d\bar{\psi} d\psi dk_{\mu}\frac{dh_{\mu}}{2\pi}\exp\Big\{-\int dx^2 \Big[&-i\bar{\psi}\not{\partial}\psi \\ &+ \bar{\psi}\gamma_{\mu}\psi h_{\mu} + \frac{g^2}{2}k_{\mu}k_{\mu} - ek_{\mu}A_{\mu} - h_{\mu}k_{\mu}\Big]\Big\}. \end{aligned} \tag{2.7}$$

(已略去脚标 E)。配方后可以完成对 k_{μ} 的泛函高斯积分(给出 N_g)而得

$$Z_E = N_g \int \prod_{x,\mu} d\bar{\psi} d\psi \frac{dh_{\mu}}{2\pi}\exp\left\{-\int d^2x \left[\bar{\psi}(-i\not{D})\psi - \frac{1}{2g^2}(eA_{\mu} + h_{\mu})^2\right]\right\} \tag{2.8}$$

其中

$$-i\not{D} = -i\not{\partial} + \gamma_{\mu}h_{\mu} \tag{2.9}$$

是一个厄米算符。下面作手征变换[15]：

$$\begin{aligned} \psi \to \psi^t &= e^{i\alpha(x)\gamma_5 t}\psi, \\ \bar{\psi} \to \bar{\psi}^t &= \bar{\psi}e^{i\alpha(x)\gamma_5 t}, \end{aligned} \tag{2.10}$$

$$-i\not{D} \to -i\not{D}^t = e^{-i\alpha\gamma_5 t}(-i\not{D})e^{-i\alpha\gamma_5 t} = -i\not{D} + \gamma_5 t\not{\partial}\alpha(x). \tag{2.11}$$

在这种随动表象中，无量纲参数 t 在 (0, 1) 区间变化，因为(2.3)式 (在欧空间为 $i\gamma_5\gamma_{\mu} = -\varepsilon_{\mu\nu}\gamma_{\nu}$，$\varepsilon_{12} = 1$)，只要 $\alpha(x) = -i\beta(x)$，$\beta(x)$ 为实，并取

$$h_\mu = \varepsilon_{\mu\nu}\partial_\nu\beta, \tag{2.12}$$

在 $t=1$ 时便可使 h_μ 与 ψ 退耦而有

$$-i\not{D}^{t=1} = -i\not{\partial}, \tag{2.13}$$

与此相应，在任意“时刻” t 我们有厄米算符：

$$-i\not{D}^t = -i\not{\partial} + \gamma_\mu(1-t)h_\mu, \tag{2.14}$$

于是

$$\int\prod_x d\bar{\psi}d\psi\exp\left\{-\int d^2x\bar{\psi}(-i\not{D})\psi\right\} = e^{\Delta\Gamma}\int\prod_x d\bar{\psi}d\psi\exp\left\{-\int d^2x\bar{\psi}(-i\not{\partial})\psi\right\}, \tag{2.15}$$

其中

$$e^{\Delta\Gamma} = \prod_i e^{\delta\Gamma_i}, \quad \Delta\Gamma = \sum_{i=1}^{M-1}\delta\Gamma_i = \int_{t=0}^{t=1}\delta\Gamma(t), \tag{2.16}$$

而

$$e^{\delta\Gamma_i} = e^{\delta\Gamma(t)} = \frac{\int\prod_x d\bar{\psi}d\psi\exp\left\{-\int d^2x\bar{\psi}(-i\not{D}^{t_i})\psi\right\}}{\int\prod_x d\bar{\psi}d\psi\exp\left\{-\int d^2x\bar{\psi}(-i\not{D}^{t_{i+1}})\psi\right\}}. \tag{2.17}$$

文[15]中详细地讨论了如何计算这一连乘积中任一比值的方法，最后给出

$$\delta\Gamma(t) = -\frac{dt}{2\pi}\int d^2x\beta(x)(1-t)\varepsilon_{\mu\nu}(\partial_\mu h_\nu - \partial_\nu h_\mu). \tag{2.18}$$

分部积分后利用(2.12)，即得

$$\Delta\Gamma = -\frac{1}{\pi}\int_0^1 dt(1-t)\int d^2x h_\mu h_\mu, \tag{2.19}$$

代回(2.15)及(2.8)后，为完成对 h_μ 的泛函积分，我们在 h_μ 的复平面作解析延拓，记在虚轴上的 $h_\mu = iq_\mu$ （q_μ 为实），则可在 $g^2<\pi$ 的条件下完成对 q_μ 的泛函高斯积分而得

$$Z_E = N'_g\int\prod_x d\bar{\psi}d\psi\exp\left[-\int d^2x\bar{\psi}(-i\not{\partial})\psi\right]\exp\left\{-\int d^2x\frac{e^2}{2(\pi-g^2)}A_\mu A_\mu\right\}. \tag{2.20}$$

我们看到：经玻色化处理后，费米场的自耦合不见了，成为自由的仍无质量的费米场，同时代替原来无质量的电场，出现了质量为 $m_B = \frac{e}{\sqrt{\pi-g^2}}$ 的自由玻色子。当 $g^2=0$，此 Thirring 模型退化为 Schwinger 模型，$m_B = \frac{e}{\sqrt{\pi}}$。由 m_B 为实的要求定出 g^2 的临界值为 $(g^2)_{\text{crit}} = \pi$，这与用其它方法得到的结果一致[17,18]。

三、非阿贝尔的玻色化

让我们考察二维欧空间中有 N 个分量的 Gross-Neveu 模型[10,11]：

$$\mathscr{L}_E = -\bar{\psi}i\not{\partial}\psi + \mathscr{L}_{\text{int}} \tag{3.1}$$

其中

$$\mathscr{L}_{\text{int}} = -\frac{\lambda^2}{4N}[(\bar{\psi}_k\psi_k)^2 - (\bar{\psi}_k\gamma_5\psi_k)^2], \quad (k = 1, 2, \cdots N) \tag{3.2}$$

既有整体的 $U(1) \times U(1)$ 对称性,也有整体的 $SU(N)$ 不变性。

现在作一个 Fierz 型的变换,可以证明(从略):

$$\mathscr{L}_{\text{int}} = -\frac{\lambda^2}{2N}(\bar{\psi}\gamma_\mu G^a\psi)^2, \quad (a = 0, 1, 2, \cdots N^2 - 1) \tag{3.3}$$

其中

$$G^0 = \frac{1}{\sqrt{2N}} I, \quad G^b = T^b, \quad (b = 1, 2, \cdots N^2 - 1) \tag{3.3$'$}$$

T^b 是 $SU(N)$ 群的生成元,取为 $N \times N$ 的厄米矩阵,满足

$$\operatorname{tr}(T^aT^b) = \frac{1}{2}\delta^{ab}. \quad (a, b = 1, 2, \cdots N^2 - 1) \tag{3.4}$$

现在引入一个有 N^2 个分量的矢量辅助场 A_μ^a, $(a = 0, 1, \cdots N^2 - 1)$, 注意在泛函积分意义下成立下述恒等式:

$$\begin{aligned} \exp\left\{\frac{\lambda^2}{2N}\int(\bar{\psi}\gamma_\mu G^a\psi)^2 d^2x\right\} &= \int DA_\mu^0 D\boldsymbol{A}_\mu \exp\left\{-\int d^2x \right. \\ &\times \left[\frac{\lambda}{\sqrt{N}}\bar{\psi}\gamma_\mu\left(\boldsymbol{A}_\mu\cdot\boldsymbol{T} + A_\mu^0\frac{1}{\sqrt{2N}}\right)\psi \right. \\ &\left.\left. + \frac{1}{2}\boldsymbol{A}_\mu\cdot\boldsymbol{A}_\mu + \frac{1}{2}A_\mu^0A_\mu^0\right]\right\}. \end{aligned} \tag{3.5}$$

其中 $\boldsymbol{A}_\mu$ 表示分量为 $a = 1, 2, \cdots N^2 - 1$ 的 A_μ^a 的集合, $\boldsymbol{T}$ 的意义亦同。于是 G-N 模型(3.1)的生成泛函变为:

$$\begin{aligned} Z_E &= \int D\bar{\psi}D\psi\exp\left(-\int\mathscr{L}_E d^2x\right) \\ &= \int D\bar{\psi}D\psi DA_\mu^0 D\boldsymbol{A}_\mu\exp\left\{-\int d^2x\left[-\bar{\psi}i\partial\!\!\!/\psi\right.\right. \\ &\left.\left. + \frac{\lambda}{\sqrt{N}}\bar{\psi}\gamma_\mu\left(\boldsymbol{A}_\mu\cdot\boldsymbol{T} + \frac{1}{\sqrt{2N}}A_\mu^0\right)\psi + \frac{1}{2}\boldsymbol{A}_\mu\cdot\boldsymbol{A}_\mu + \frac{1}{2}A_\mu^0A_\mu^0\right]\right\}. \end{aligned} \tag{3.6}$$

接下来先使 ψ 与 A_μ^0 退耦,考虑到(3.6)中质量项 $\frac{1}{2}A_\mu^0A_\mu^0$ 的存在,我们作变换:

$$\begin{aligned} \psi &\rightarrow e^{i\eta(x)t}e^{\phi(x)\gamma_5 t}\psi, \\ \bar{\psi} &\rightarrow \bar{\psi}e^{-i\eta(x)t}e^{\phi(x)\gamma_5 t}, \end{aligned} \tag{3.7}$$

$$\begin{aligned} -i\partial\!\!\!/ \rightarrow -i\partial\!\!\!/^t &= e^{-i\eta t}e^{-\phi\gamma_5 t}(-i\partial\!\!\!/)e^{-i\eta t}e^{-\phi\gamma_5 t} \\ &= -i\partial\!\!\!/ - t\partial\!\!\!/\eta - t\varepsilon_{\mu\nu}\gamma_\mu\partial_\nu\phi, \end{aligned} \tag{3.8}$$

并要求当 $t = 1$ 时退耦,即取

$$\lambda' A_\mu^0 \equiv \frac{1}{\sqrt{2N}}\lambda A_\mu^0 = \varepsilon_{\mu\nu}\partial_\nu\phi + \partial_\mu\eta, \tag{3.9}$$

则类似于上节(2.18)式,可得

$$\Delta\Gamma = \int_{t=0}^{t=1}\delta\Gamma(t) = -\frac{1}{2\pi}\int d^2x\{(\partial_\mu\phi)(\partial_\mu\phi) + \partial_\mu\phi\varepsilon_{\mu\nu}\partial_\nu\phi\}。 \tag{3.10}$$

改(3.6)中的积分测度 $DA_\mu^0 = D\phi D\eta$，再以(3.9)式代入 $\frac{1}{2}A_\mu^0 A_\mu^0$ 项，我们有

$$Z_E = \int D\bar{\psi} D\psi D\boldsymbol{A}_\mu D\phi D\eta \exp\left\{-\int d^2x\left[-\bar{\psi} i\partial\!\!\!/\psi + \frac{\lambda}{\sqrt{N}}\bar{\psi}\gamma_\mu \boldsymbol{A}_\mu\cdot\boldsymbol{T}\psi + \frac{1}{2}\boldsymbol{A}_\mu\cdot\boldsymbol{A}_\mu + \frac{1}{2\lambda'^2}\left(1+\frac{\lambda'^2}{\pi}\right)(\partial_\mu\phi)^2 + \frac{1}{2\lambda'^2}(\partial_\mu\eta)^2 + \left(\frac{1}{\lambda'^2}+\frac{1}{2\pi}\right)\varepsilon_{\mu\nu}\partial_\mu\eta\partial_\nu\phi\right]\right\}. \tag{3.11}$$

上面的 $\phi(x)$ 和 $\eta(x)$ 都是 c 数函数，相当于我们在处理一个阿贝尔反常问题[15]．下面想使 ψ 与 $\boldsymbol{A}_\mu$ 退耦，由于 T^a 矩阵的不可对易性，就相当于处理非阿贝尔反常问题．

在随动表象下，引入分立参数 t_i，$t_0=1$，t_1，$\cdots t_M=1$，则

$$\begin{aligned}\psi^{t_i} &\to \psi^{t_{i+1}} = e^{\alpha_{i+1}(x)+\beta_{i+1}(x)\gamma_5}\psi^{t_i},\\ \bar{\psi}^{t_i} &\to \bar{\psi}^{t_{i+1}} = \bar{\psi}^{t_i}e^{-\alpha_{i+1}(x)+\beta_{i+1}(x)\gamma_5},\end{aligned} \tag{3.12}$$

这里 $\alpha_i(x)$，$\beta_i(x)$ 均为无限小矩阵（α_i，$\beta_i\in SU(N)$），同时诱导出算符的相应变换为

$$\begin{aligned}-iD\!\!\!\!/^{\,t_0} &= -iD\!\!\!\!/ = -i\partial\!\!\!/ + \frac{\lambda}{\sqrt{N}}\gamma_\mu \boldsymbol{A}_\mu\cdot\boldsymbol{T}\\ &\to -iD\!\!\!\!/^{\,t_i} = e^{\alpha_i(x)-\beta_i(x)\gamma_5}(-iD\!\!\!\!/)^{t_{i-1}}e^{-\alpha_i(x)-\beta_i(x)\gamma_5}\\ &= \left[\prod_{j=1}^{i} e^{-\alpha_j(x)+\beta_j(x)\gamma_5}\right]^{-1}(-iD\!\!\!\!/)\left[\prod_{j=1}^{i} e^{-\alpha_j(x)-\beta_j(x)\gamma_5}\right],\end{aligned} \tag{3.13}$$

因为连乘积中的 i 个指数因子总可以用 Baker-Hausdorff 公式

$$e^A e^B = e^C, \tag{3.14}$$

$$C = A + B + \frac{1}{2}[A, B] + \frac{1}{12}[A-B, [A, B]] - \frac{1}{24}[A, [B, [A, B]]] + \cdots,$$

化为一个指数因子，相应地引入连续的 t 变量：

$$\prod_{j=1}^{i} e^{-\alpha_j(x)-\beta_j(x)\gamma_5} = e^{-\bar{\alpha}(x,t)-\bar{\beta}(x,t)\gamma_5}, \tag{3.15}$$

这里 $\bar{\alpha}(x, t)$ 与 $\bar{\beta}(x, t)$ 是有限的矩阵，由 $t=1$ 时之退耦条件

$$iD\!\!\!\!/^{\,t=1} = -i\partial\!\!\!/, \tag{3.16}$$

可得

$$\frac{\lambda}{\sqrt{N}}\gamma_\mu \boldsymbol{A}_\mu\cdot\boldsymbol{T} = -i\gamma_\mu(\partial_\mu\alpha + \gamma_5\partial_\mu\beta). \tag{3.17}$$

（已记 $\bar{\alpha}(x, 1)=\alpha(x)$，$\bar{\beta}(x, 1)=\beta(x)$），任意时刻 t 之厄米算符为

$$-iD\!\!\!\!/^{\,t} = -i\gamma_\mu(\partial_\mu + V_\mu + a_\mu\gamma_5), \tag{3.18}$$

其中

$$V_\mu = \partial_\mu\alpha - \partial_\mu\bar{\alpha},\quad V_\mu^+ = -V_\mu, \tag{3.19}$$

$$a_\mu = \partial_\mu\beta - \partial_\mu\bar{\beta},\quad a_\mu^+ = a_\mu, \tag{3.20}$$

和上节一样，在随动表象中完成从 $t=0$ 到 $t=1$ 的变换后：

$$\int D\bar{\psi}D\psi\exp\left\{-\int d^2x\left[-\bar{\psi}i\partial\!\!\!/\psi + \frac{g}{\sqrt{N}}\bar{\psi}\gamma_\mu \boldsymbol{A}_\mu\cdot\boldsymbol{\Gamma}\psi\right]\right\}$$

$$= e^{\Delta\Gamma}\int D\bar{\Phi}D\phi\exp\left\{-\int d^2x[-\bar{\Phi}i\partial\!\!\!/\phi]\right\}. \tag{3.21}$$

引用文[15]中(5.15)式来计算

$$d\Gamma(t) = 2dt\frac{1}{4\pi}\sum_{m=0}^{1}B(m+1,2)\mathrm{Tr}\left\{(\partial_t\bar{\beta})\gamma_5\frac{1}{(2m)!}\left[\frac{d^{2m}}{dz^{2m}}\sum_{l_1+l_2=m+1}Q_+^{l_1}Q_-^{l_2}\right]_{z=0}\right\} \tag{3.22}$$

其中

$$\begin{aligned} Q_\pm &= F_{\mu\nu}^{(V)}\sigma_{\mu\nu} \pm F_{\mu\nu}^{(a)}\sigma_{\mu\nu}\gamma_5 + 2z\gamma_\mu a_\mu\gamma_5 \\ F_{\mu\nu}^{(V)} &= \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu] + [a_\mu, a_\nu] \\ F_{\mu\nu}^{(a)} &= \partial_\mu a_\nu - \partial_\nu a_\mu + [a_\mu, V_\nu] + [V_\mu, a_\nu] \\ \sigma_{\mu\nu} &= \frac{1}{4}[\gamma_\mu, \gamma_\nu] = \frac{i}{2}\gamma_5\varepsilon_{\mu\nu} \end{aligned} \tag{3.23}$$

以(3.19),(3.20)代入,并引入 $SU(N)$ 群的两个元素 $\bar{g}(x,t)$ 和 $\bar{U}(x,t)$,使得

$$\bar{g}(x,t) = e^{2i\bar{\beta}(x,t)}, \quad \bar{g}(x,1) = g(x) = e^{2i\beta(x)}, \tag{3.24}$$

$$\bar{U}(x,t) = e^{2\bar{\alpha}(x,t)}, \quad \bar{U}(x,1) = U(x) = e^{2\alpha(x)}. \tag{3.25}$$

于是(3.22)式可以化为

$$\begin{aligned} d\Gamma(t) = \frac{dt}{8\pi}\mathrm{tr}^c\int d^2x\varepsilon_{\mu\nu}\bar{g}^{-1}\partial_t\bar{g}\{&U^{-1}\partial_\mu UU^{-1}\partial_\nu U - U^{-1}\partial_\mu U\bar{U}^{-1}\partial_\nu\bar{U} \\ &- \bar{U}^{-1}\partial_\mu\bar{U}U^{-1}\partial_\nu U + \bar{U}^{-1}\partial_\mu\bar{U}\bar{U}^{-1}\partial_\nu U + g^{-1}\partial_\mu gg^{-1}\partial_\nu g \\ &- g^{-1}\partial_\mu g\bar{g}^{-1}\partial_\nu\bar{g} - \bar{g}^{-1}\partial_\mu\bar{g}g^{-1}\partial_\nu g + \bar{g}^{-1}\partial_\mu\bar{g}\bar{g}^{-1}\partial_\nu\bar{g}\} \end{aligned} \tag{3.26}$$

其中已完成对自旋矩阵的求迹,tr^c 只表示对色空间矩阵求迹。故(3.21)中

$$\Delta\Gamma = \int_{t=0}^{t=1}d\Gamma(t) = \sum_{s=1}^{8}\Gamma_s \tag{3.27}$$

其中 Γ_s 依次代表(3.26)中各项对 t 积分后的贡献,例如最后一项可写为:

$$\begin{aligned} \Gamma_8 &= \frac{1}{8\pi}\int_0^1 dt\int d^2x\ \mathrm{tr}^c\{\varepsilon_{\mu\nu}\bar{g}^{-1}\partial_t\bar{g}\bar{g}^{-1}\partial_\mu\bar{g}\bar{g}^{-1}\partial_\nu\bar{g}\} \\ &= \frac{1}{24\pi}\int_B d^3y\ \mathrm{tr}^c\{\varepsilon_{ijk}\bar{g}^{-1}\partial_i\bar{g}\bar{g}^{-1}\partial_j\bar{g}\bar{g}^{-1}\partial_k\bar{g}\} \end{aligned} \tag{3.28}$$

最后一步已将积分取在整个三维球 B 内进行,在 B 的边界 S^2(紧致的二维欧空间)上,$\bar{g}(x,t=1) = g(x)$,如(3.24)所示。(3.28)正是 Witten 首先推出的 Wess-Zumino 项[5]。这里我们看到,沿着球径向第三维无量纲坐标 $t(0\to 1)$ 是通过随动表象自然地引入的。

(3.11)中的质量项,用(3.17)后,可以化为:

$$\frac{1}{2}\boldsymbol{A}_\mu\cdot\boldsymbol{A}_\mu = \frac{1}{2}A_\mu A_\mu = -\frac{N}{4\lambda^2}\mathrm{tr}^c(\partial_\mu g^{-1}\partial_\mu g - \partial_\mu U^{-1}\partial_\mu U) \tag{3.29}$$

代回(3.1)—(3.11)诸式,我们得到

$$\begin{aligned} Z_E &= \int D\bar{\Phi}D\phi\exp\left\{-\int d^2x\left[-\bar{\Phi}i\partial\!\!\!/\phi - \frac{\lambda^2}{4N}[(\bar{\Phi}\phi)^2 - (\bar{\Phi}\gamma_5\phi)^2]\right]\right\} \\ &= \int D\bar{\Phi}D\phi\exp\left[-\int dx^2(-\bar{\Phi}i\partial\!\!\!/\phi)\right]\int DgDUD\phi D\eta\exp\left\{-\int d^2x\right. \end{aligned}$$

$$\times \left[-\frac{N}{4\lambda^2} \mathrm{tr}^c (\partial_\mu g^{-1} \partial_\mu g - \partial_\mu U^{-1} \partial_\mu U) + \frac{1}{2\lambda'^2} \left(1 + \frac{\lambda'^2}{\pi}\right) (\partial_\mu \phi)^2 \right.$$

$$\left. + \frac{1}{2\lambda'^2} (\partial_\mu \eta)^2 + \left(\frac{1}{\lambda'^2} + \frac{1}{2\pi}\right) \varepsilon_{\mu\nu} \partial_\mu \eta \partial_\nu \phi \right] + \sum_{s=1}^{8} \Gamma_s \Bigg\} \qquad (3.30)$$

$\left(\lambda' = \dfrac{\lambda}{\sqrt{2N}}\right)$. 正好 Witten 所指出[5]，各 $W-Z$ 项 $\Gamma_s(s=2, 3, 4, 6, 7, 8)$的前面可以分别乘上一个正整数 n_s，这是根据三阶同伦群 $\pi_3(SU(N))=Z$ 的性质。当 g 和 U 为取值在 $SU(N)$ 基本表示上的矩阵时，作为拓扑不变量的 Γ_s，其数值准确到 $\Gamma_s \to \Gamma_s + 2\pi$。这一不定性源于在三维球 B 内定义 $\bar{g}(x, t)$ (或 $\bar{U}(x, t)$) 时，它虽受到边界 $\partial B = S^2$ 上条件 $\bar{g}(x, 1) = g(x)$ 及 $\bar{g}(x, 0) = 1$ 的约束，但由它给出的从 B 到 $SU(N)$ 群流形的映照，其覆盖次数 n_s 可以是任意的。

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THE NONABELIAN BOSONIZATION OF FERMION FIELDS IN (1+1) DIMENSIONS BY PATH INTEGRAL METHOD

NI GUANGJIONG　WANG RONGTAI　CHEN SUQING

(*Fudan University, Shanghai*)

ABSTRACT

Using the path integral method for handling the anomaly problem under the comoving representation, we present a unified scheme for deriving the bononization of fermion fields in (1+1) dimensions. The massless Thirring model with an external electric field, and the Gross-Neveu model with internal $SU(N)$ symmetry, as two examples for abelian and nonabelian bosonization, are discussed respectively in some detail.