High Energy Physics and Nuclear Physics

Volume 12, No. 3

Second Order Pion-Nucleus Optical Potential and the Double Charge Exchange Reaction*

Zou Bingsong and Jiang Huanqing

(Institute of High Energy Physics, Chinese Academy of Sciences, Beijing)

In the framework of the fixed-scattering field theory, we construct microscopically a second-order pion-nucleus Iso-elastic optical potential by including two-nucleon correlations. The double charge exchange reactions to the double isobaric analog states are calculated by using our theoretical optical potential. For the incident pion energies ranging from 0 to 300 MeV, the S and P partial waves are included in the πN amplitude.

I. INTRODUCTION

It has been a long-standing goal of nuclear structure physics to understand the two-nucleon correlations in nuclei. Unfortunately, in most nuclear reactions the effects of these correlations occur only as higher-order corrections. Since there are at least two like nucleons involved in the pion double-charge-exchange (DCX) reaction, it might be very sensitive to these correlations. In recent years a lot of DCX experimental data have been published. These data provide a good opportunity for us to investigate these correlations.

In the study of the double-charge-exchange reaction to the double isobaric analog state (DIAS DCX), the conventional first-order optical potential method encountered two difficulties [1]. 1) Around 50 MeV pion incident energies, the theoretical forward cross sections are much smaller than the experimental data; 2) In the (3.3) resonance region near 160 MeV, the theoretical disposition of the angular distribution is larger then the experimental value.

* Supported by National Natural Science Foundation of China.
Received on July 23, 1987

How to explain these phenomena? Now there basically exists two different directions for the theoretical development. One is to introduce new degrees of freedom in nuclei, such as Δ_{33} components [2], virtual meson exchange currents [3] and six-quark cluster [4], etc. The other one is to consider the higher order effects of nuclear correlations at the pure nucleonic level [5,6].

We think that even if we need to introduce the new degrees of freedom in nuclei, we must first ascertain the contributions of the pure nucleonic mechanisms. In this work we construct a second order pion-nucleus Iso-elastic optical potential microscopically in the framework of the fixed scattering field theory, and compare it with the first-order optical potential. Then, using this theoretical potential, the DIAS DCX differential cross sections are calculated and compared with the experimental data.

2. THEORETICAL FRAMEWORK

We refer the pion-nucleus elastic, single-charge-exchange (SCX) to the isobaric analog state and double-charge-exchange (DCX) to the double-isobaric analog state as Iso-elastic scattering. They are related by applying the isospin symmetry for the strong interaction. When isospin breaking effects can be ignored, the Iso-elastic scattering can be treated theoretically on the basis of an Iso-elastic optical potential of the following form:

$$\hat{U} = U_0 + U_1(\boldsymbol{\phi} \cdot \boldsymbol{T}) + U_2(\boldsymbol{\phi} \cdot \boldsymbol{T})^2, \tag{2.1}$$

where $\boldsymbol{\phi}$ is the pion and $\boldsymbol{T}$ the nuclear isospin operator, U_0, U_1 and U_2 are the so-called isoscalar, isovector and isotensor terms respectively.

In this framework, the pion wave function satisfies the following Klein-Gordon equation

$$(\nabla^2 + k^2)\phi(\boldsymbol{r}) = \hat{U}\phi(\boldsymbol{r}). \tag{2.2}$$

If $\hat{U}$ is given, we can obtain the Iso-elastic scattering cross sections by using the coupled channel method to solve this K-G equation. Therefore the problem now is how to calculate $\hat{U}$.

The relationship between $U_i(i = 0,1,2)$ and the physical elastic optical potential $U^{(m)}$ $(m = +,o,-)$ for π^+, π^0, and π^- can be expressed formally as[5]

$$\begin{aligned} U_2 &= (U^{(+)} + U^{(-)} - 2U^{(0)})/[T_0(2T_0 - 1)], \\ U_1 &= (U^{(0)} - U^{(+)})/T_0 + T_0U_2, \\ U_0 &= U^{(0)} - T_0U_2, \end{aligned} \tag{2.3}$$

where T_0 is the isospin quantum number of the nucleus.

In principle U_0, U_1 and U_2 may be calculated microscopically by an expansion in terms of the number of active nucleons. This is usually referred to as the density expansion:

$$U_i = U_i^{(1)} + U_i^{(2)} + \cdots \tag{2.4}$$

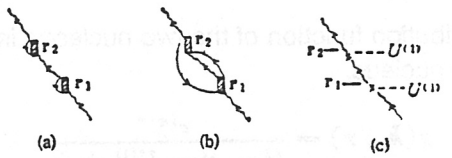

FIG. 1 Two-nucleon processes contributing to the pion-nucleus optical potential.

where the superscript (i) indicates the number of active nucleons.

In this paper we calculate the second-order optical potential $U_i^{(2)}$ in the framework of the fixed scattering center field theory [7].

The main terms contributing to the second-order optical potential are shown in Fig. 1. Figs. 1(a) and (b) are the direct and Pauli exchange terms respectively. Fig. 1(c) is a contribution from the iteration of the first-order optical potential. This term must be subtracted from the sum of Figs. 1(a) and (b) in order to avoid double counting.

The basic ingredient in our calculation is the πN scattering amplitude. For pion incident energies Tπ 300 MeV, *S* and *P* partial waves dominate the πN amplitude we may write

$$\hat{F}(\boldsymbol{k}', \boldsymbol{k}) = \hat{F}^{(s)}(\boldsymbol{k}', \boldsymbol{k}) + \hat{F}^{(p)}(\boldsymbol{k}', \boldsymbol{k}), \tag{2.5}$$

$$\hat{F}^{(s)}(\boldsymbol{k}', \boldsymbol{k}) = \frac{k_0^2}{4\pi} \cdot \frac{v(k')v(k)}{v^2(k_0)} \cdot \hat{\lambda}^{(s)}, \tag{2.6}$$

$$\hat{F}^{(p)}(\boldsymbol{k}', \boldsymbol{k}) = \frac{1}{4\pi} \cdot \frac{v(k')v(k)}{v^2(k_0)} [\hat{\lambda}^p \boldsymbol{k}' \cdot \boldsymbol{k} + i\hat{\lambda}^\sigma \cdot \boldsymbol{\sigma} \cdot (\boldsymbol{k}' \times \boldsymbol{k})], \tag{2.7}$$

where the form factor is taken to be $v(k) = (1 + k^2/\beta^2)^{-1}$ with $\beta = 4.82$ fm^{-1}, and k_0 is the corresponding on-shell momentum of the pion.

$$\begin{aligned} \hat{\lambda}_i^{(s)} &= \lambda_{00}^{(s)} + \frac{1}{2}\lambda_{01}^{(s)}(\boldsymbol{\phi} \cdot \boldsymbol{T}), \\ \hat{\lambda}_i^{p} &= \lambda_{00}^{(p)} + \frac{1}{2}\lambda_{01}^{(p)}(\boldsymbol{\phi} \cdot \boldsymbol{T}), \\ \hat{\lambda}_i^{\sigma} &= \lambda_{10}^{(p)} + \frac{1}{2}\lambda_{11}^{(p)}(\boldsymbol{\phi} \cdot \boldsymbol{T}), \end{aligned} \tag{2.8}$$

where $\lambda_{00}^{(s)}$, $\lambda_{01}^{(s)}$, $\lambda_{00}^{(p)}$, $\lambda_{01}^{(p)}$, $\lambda_{10}^{(p)}$, and $\lambda_{11}^{(p)}$ are obtained from the phase shift analysis of Ref. [8].

Within the framework of the fixed scattering center field theory, the second-order scattering operator can be expressed as

$$\hat{T}^{(2)}(\boldsymbol{k}', \boldsymbol{k}; \boldsymbol{r}_2 - \boldsymbol{r}_1) = \int \frac{d^3k_1}{(2\pi)^3} [-4\pi \hat{F}_2(\boldsymbol{k}', \boldsymbol{k}_1)] \cdot g(\boldsymbol{k}_1, \boldsymbol{r}_2 - \boldsymbol{r}_1) \cdot \Gamma(\boldsymbol{r}_2 - \boldsymbol{r}_1) \cdot [-4\pi \hat{F}_1(\boldsymbol{k}_1, \boldsymbol{k})], \tag{2.9}$$

where $\Gamma(r)$ is the pair distribution function of the two nucleons in the nucleus, $g(k_1, r)$ is the pion propagator in the nucleus,

$$g(k_1, r) = \frac{e^{ik_1 \cdot r}}{k_0^2 - k_1^2 - U^{(1)} + i\eta} \tag{2.10}$$

The pion-nucleus optical potential can be obtained by averaging the scattering operator over the nuclear wave function.

$$U^{(m)}(k', k) = \langle \phi_0; k', m | \hat{T} | \phi_0; k, m \rangle \tag{2.11}$$

where ϕ_0 is the nuclear ground state. k', k are the incoming and outgoing momenta of the pion respectively. $m = +,0,-$ indicates the charge state of the pion.

By substituting $U^{(m)}$ into Eq.(2.3), we obtain the second-order correction to the Iso-elastic optical potential. The details of the calculation of $U^{(m)}$ are very similar to that in Ref.[9]. For the nuclear density matrix we use the revised local Fermi gas model which has been shown to be a good approximation [10].

The final second-order optical potential in momentum space can be expressed as

$$U_i^{(2)}(k', k) = \int dR e^{-iq \cdot R} U_i^{(2)}(k', k, R) \tag{2.12}$$

where $q = k' - k$, $R = 1/2\ (r_1 + r_2)$, and

$$U_i^{(2)}(k', k, R) = k_0^2 \xi_i^{(ss)}(k_0, R) + k_0^2 e_{k'} \cdot e_k \xi_i^{(pp)}(k_0, R) + \frac{1}{2} k_0^2 (1 + e_{k'} \cdot e_k) \xi_i^{(sp)}(k_0, R) \tag{2.13}$$

here $\xi_i^{(ss)}(\xi_i^{(pp)})$ corresponds to the contribution from the pure $S(P)$ wave, $\xi_i^{(sp)}$ is the contribution from the interference term of the S and P wave amplitudes.

3. RESULTS AND DISCUSSIONS FOR THE SECOND-ORDER OPTICAL POTENTIAL

Since the P wave part dominates the πN amplitude in the (3.3) resonance region it is generally believed that only the P wave component should remain in calculating the second-order optical potential. But we find that this is not the case.

TABLE 1.

Parameters of the πN Amplitudes at 50 MeV and 164 MeV (unit: fm^{-3}).

T_π(MeV)	λ_{00}^s	λ_{01}^s	λ_{00}^p	λ_{01}^p
50	$-1.1 + 0.7i$	$-9.5 - 0.4i$	$8.0 + 1.0i$	$9.6 + 0.9i$
164	$-0.7 + 0.4i$	$-2.3 + 0.1i$	$3.2 + 9.5i$	$3.7 + 9.5i$

In Table 1 the parameters of the πN amplitude are given for T_π = 50 MeV and 164 MeV. At T_π = 50 MeV the S wave is obviously not negligible. At T_π = 164 MeV, since U_0 and U_1 are proportional to λ_{00}, and $\lambda_{00}^p \gg \lambda_{00}^s$, the effects of the S wave component to U0 and U1 are very small. In other word, the P wave is dominant. However, U_2 is proportional to λ_{01}, namely, the pure P wave contribution term $U_2^{(pp)} \propto (\lambda_{01}^p)^2$ and S-P interference term $U_2^{(ps)} \propto 2\lambda_{01}^s \cdot \lambda_{01}^p$. The real parts of the S and P wave amplitudes are comparable at 164 MeV (Re λ_{01} = 2.3 fm^{-3} and Re λ_{01} = 3.7 fm^{-3}). Therefore the contribution from the S and P wave interference term is not negligible for U_2. In this paper we calculate the Iso-elastic optical potential for ^{18}O by including both the S and P wave πN amplitudes.

In order to compare the contributions from S and P wave amplitudes easily, we consider the case of $\boldsymbol{k}' = \boldsymbol{k}$. Then the second-order optical potential in coordinate space is

$$U_i^{(2)}(k_0, k) = k_0^2[\xi_i^{(ss)}(k_0, R) + \xi_i^{(sp)}(k_0, R) + \xi_i^{(pp)}(k_0, R)] \tag{3.1}$$

For practical applications it is useful to parametrize the optical potential as a function of ρ and $\Delta\rho$. One of such parametrizations is suggested in Ref. [5]. For $\boldsymbol{k}' = \boldsymbol{k} = \boldsymbol{k}_0$ and T = 1, the following form of the optical potential is assumed,

$$\begin{aligned} U_0^{(2)}(k_0, R) &= -k_0^2\lambda_0\rho^2(R)/\rho_0, \\ U_1^{(2)}(k_0, R) &= -\frac{1}{2}k_0^2\lambda_1\rho(R)\Delta\rho(R)/\rho_0, \\ U_2^{(2)}(k_0, R) &= -k_0^2\lambda_2\frac{\Delta\rho(R)}{\rho_0}, \end{aligned} \tag{3.2}$$

where ρ is the density of the nucleus, $\Delta\rho$ the density distribution of the valence nucleons, ρ_0 the central density of the nucleus, which is taken to be 0.16 fm^{-3}, and

$$\lambda_i = \lambda_i^{(ss)} + \lambda_i^{(sp)} + \lambda_i^{(pp)}. \tag{3.3}$$

Since the Iso-elastic scattering near the (3,3) resonance is dominated by interactions near the surface of the nucleus, only the optical potential near the surface is important. It

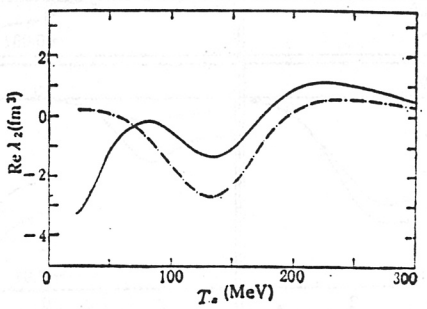

FIG. 2 Real part of λ_2 as a function of the pion incident energy Tπ. The dot dashed line is the result by considering only the P wave, and the solid line is that by considering both the S and P wave amplitudes.

is found that the λ_i has a weak dependence on R near the surface, thus, as a good approximation, it can be taken as a constant. We evaluate them at $R = 3.5$ fm in this paper.

In Fig. 2 we show the real part of λ_2 vs. the pion incident energies T_π ranging from 25 to 300 MeV. The dot-dashed line is the result by considering only the P wave, and the solid line is that by considering both the S and P wave amplitudes. We find that the contribution of the S wave part is quite important, even at the peak of the (3,3) resonance around 164 MeV. The effect of the S wave to U_2 is not negligible.

For the isoscalar and isovector parts (λ_0 and λ_1), the contribution of the S wave πN amplitude is not very important in the (3,3) resonance region . Only about 20 ~ 30% correction to the first-order optical potential attributes to the S wave component. Thus the P wave is dominant. The calculation coincides with our previous analysis.

In the following we will examine the importance of the second-order correction to the total optical potential. Since there is no U_2 term in the first-order optical potential, the second-order contribution is the leading term for U_2. We compare the total (first-plus second-order) optical potential with the first-order one at energies $T_\pi = 50$ MeV and 164 MeV for U_0 and U_1 in Fig. 3. It is found that the first-order contribution is the dominant one and the second-order contribution is only a small correction at 164 MeV. But at 50 MeV, the second-order contribution becomes very important, especially for U_1. The second-order contribution to U_1 is larger than the first-order one, and thus becomes the dominant one. This is due to the fact that $U_1^{(1)} \propto (\lambda_{01}^P + \lambda_{01}^I)$, while λ_{01}^P and λ_{01}^I are nearly equal and have an opposite sign at 50 MeV. This cancellation of S and P wave parts makes $U_1^{(1)}$ nearly zero. Therefore $U_1^{(2)}$ becomes a dominant term.

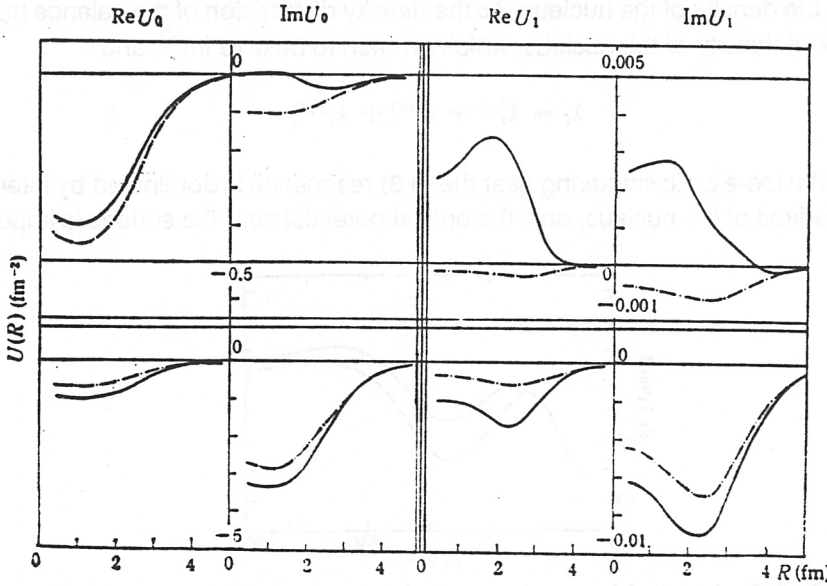

FIG. 3 Comparison of the second-order optical potential (solid line) to the first-order one (dot-dashed line) at $T = 50$ MeV (upper part of this figure) and 164 MeV (lower part).

4. APPLICATION OF THE ISO-ELASTIC OPTICAL POTENTIAL TO THE DIAS DCX REACTION

When only the S and P wave πN amplitudes are considered, the first-order optical potential has the following form:

$$U^{(1)}(\boldsymbol{k}', \boldsymbol{k}, R) = k_0^2(\bar{\xi} + \xi \boldsymbol{e}_{k'} \cdot \boldsymbol{e}_k) \tag{4.1}$$

where $\bar{\xi}$ and ξ correspond to the S and P wave term respectively.

The second-order optical potential obtained in Eq.(2.13) has a similar form:

$$U^{(2)}(\boldsymbol{k}', \boldsymbol{k}, R) = k_0^2(\Delta\bar{\xi} + \Delta\xi \boldsymbol{e}_{k'} \cdot \boldsymbol{e}_k) \tag{4.2}$$

where

$$\Delta\bar{\xi} = \xi^{(ss)} + \frac{1}{2}\xi^{(sp)}$$

$$\Delta\xi = \xi^{(pp)} + \frac{1}{2}\xi^{(sp)} \tag{4.3}$$

The specific form of the optical potential adopted in this paper is the one used in Refs.[5,6], namely,

$$U(R) = \nabla \cdot [\xi(R) + \Delta\xi(R)]\nabla - k^2[\bar{\xi}(R) + \Delta\bar{\xi}(R)] - \frac{1}{2}(p_1 - 1)\nabla^2\xi(R) - \frac{1}{2}(p_2 - 1)\nabla^2\xi(R) \tag{4.4}$$

Considering the true absorption of the pion an absorption term needs to be added to the optical potential, i.e.,

$$U(R) = U^{(1)}(R) + U^{(2)}(R) + U_{abs}(R) \tag{4.5}$$

where U_{abs} can be parametrized as the following form

$$U_{abs}(R) = 4\pi[B_0\rho^2(R) - \nabla C_0\rho^2(R)\nabla] \tag{4.6}$$

where the values of B_0, C_0 in the energy region between 0 to 300 MeV are obtained from Ref.[11].

By using the coordinate-space computer code PIESDEX [5] and the obtained optical potential, we solved the isospin-invariant coupled-channel Klein-Gordon equation. The DIAS DCX reaction $^{18}O(\pi^+, \pi^-)Ne^{18}(g.s.)$ is evaluated numerically.

In Fig. 4 we show the calculation for the angular distributions at $T_\pi = 50$ MeV. Curves 1 and 2 correspond to the results either without or with the inclusion of the true absorption term in the optical potential respectively. The result of the first-order potential is not given because it is too small ($d\sigma/d\Omega$ (0°) = 0.005 μb/sr). We can see that curve 1 is nearly one order of magnitude higher than the experimental data while curve 2 is more than one order

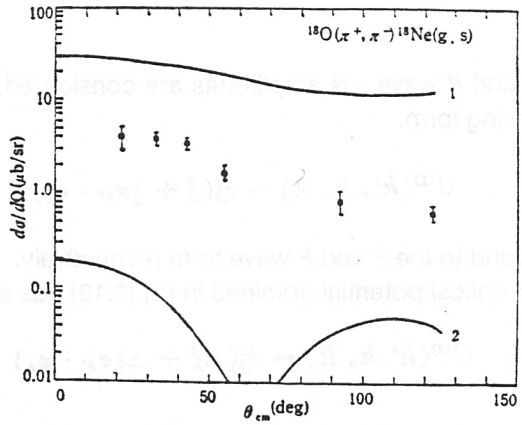

FIG. 4 Angular distribution at $T\pi$ = 50 MeV. Curves 1 and 2 correspond to the results without and with the inclusion of the true absorption term the optical potential respectively.

of magnitude lower than the experimental data. This is because our theoretical microscopical optical potential has the so-called Kisslinger singularity [12] for energies between 40 ~ 100 MeV. The Kisslinger singularity is the intrinsic defect of the Kisslinger - form optical potential. In the singularity region the results are very sensitive to the parameters

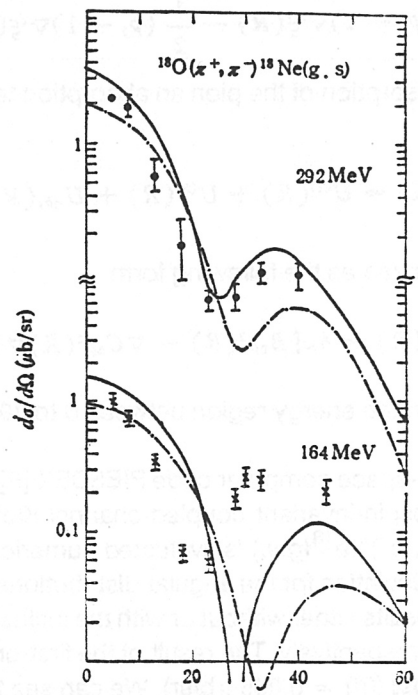

FIG. 5 Angular distributions at $T\pi$ = 164 MeV and 292 MeV. The dot dashed line and the solid line correspond to the results by using the first- and second-order optical potentials respectively.

of the optical potential and, therefore, are less meaningful. Nevertheless, from the comparison of the second-order optical potential with the first-order one, the nucleon-nucleon correlations play a very important role in the DIAS DCX reactions at low energies. We should use the DWBA method to study this reaction.

In Fig. 5 the angular distributions for $T\pi$ = 164 MeV and 292 MeV are shown. We found that the second-order optical potential gives some improvements on the results. The disposition of the angular distribution shifts toward smaller angles and the cross sections at large angles increase. But it is still far from satisfactory for $T\pi$ = 164 MeV.

In all our calculations the true absorption term is included in the second-order optical potential but curve 1 in Fig. 4. No free parameters are introduced.

5. SUMMARY AND CONCLUSIONS

From this investigation we may conclude as follows:

(1) At low energies around 50 MeV the strong cancellation between the *S* and *P* wave πN amplitudes leads to a forward-angle minimum for the DIAS DCX reactions in the conventional first-order optical method. Therefore the second-order optical potential becomes the dominant term and it can enhance the DIAS DCX forward-angle cross sections greatly at low energies. Even in the (3.3) resonance region it increases the cross section significantly.

(2) In the (3.3) resonance region it was believed that the (3.3) resonance part dominated the πN amplitude, therefore the contribution of the *S* wave was neglected. But our results show that though the contribution to the isoscalar and isovector parts of the optical potential from the *S* wave part is negligible, the effect to the isotensor part, especially to its real part, is very important. Even in the (3.3) resonance region the contribution of the *S* wave πN amplitude to the DIAS DCX reaction cannot be neglected. By considering both *S* and *P* wave πN amplitudes the angular distribution functions are improved. The dispositions shift to the smaller angles, the large-angle cross sections are enhanced. But still it cannot reproduce the experiment data satisfactorily.

To be short we conclude that within the pure-nucleonic approach, the two-nucleon correlations and the *S* and *P* wave πN amplitudes are all very important and should be considered together for the DIAS DCX reactions. However it is still not enough to explain the experimental data by considering only these factors. We need to improve the theory and study some new mechanisms.

ACKNOWLEDGMENTS

We are indebted to Professors T. E. O. Ericson, T. S. Ho, C. R. Ching, and Y. G. Li for their meaningful discussions with us.

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