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$SO(6)\supset SO(5)$ 同位标量因子的计算

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摘　　要

本文用求约化矩阵元和逐步计算原理相互结合的方法计算了部分
$SO(6)\supset SO(5)$
同位标量因子.

一、引　言

我们知道相互作用玻色子模型（IBM）在描述核低能激发态方面获得了显著的成功.它最吸引人的特点是用群论来处理，并阐明了核动力学对称性的存在和精确解.目前由Iachello[1] 进一步提出利用超代数工具同时描述偶偶核和奇偶核，这两类核的能级属于同一个超多重态.这就是最近几年提出的，并获得一定的成功的 $U(6/4)$、$U(6/12)$，$U(6/20)$ 等超对称性的模型.把上述超对称模型的基本思想进一步推广，超对称自然而然地可以同时描述一个与偶偶核相邻的同位素和同中子异荷素.它们处于某一个超对称性的超多重态中.事实上，已有人用 $U(6/12\nu+4\pi)$ 来讨论相应核区[3]，而我们也做过用

$$U(6/12\nu+20\pi)$$

同时描述 $^{197}_{79}Au_{118}$、$^{198}_{79}Au_{119}$、$^{196}_{78}Pt_{118}$ 和 $^{197}_{28}Pt_{119}$[2]. 这样这个超对称性所含有的超多重态中除了三个核外还加上一个奇奇核 $^{198}_{79}Au_{119}$. 这一推广，当然把中子和质子看作不同的对象来处理.

用上述思想所作的各种讨论都局限于低能激发能谱的超多重态上.当然这是超多重态的主要指标之一.但对电磁跃迁（$B(E2)$）等未作任何讨论，我们认为不作讨论的原因主要是在这个核区的奇奇核电磁跃迁数据太少，使理论和实验不能作比较.在计算电磁跃迁 $B(E2)$ 时需要用 $SO(6)\supset SO(5)$ 的同位标量因子. 此文就想用计算约化矩阵元和逐步计算原理方法算出部分 $SO(6)\supset SO(5)$ 的同位标量因子.在第二节中提出计算原理

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及相关 6j 系数，第三节给出结果，第四节作一简短的讨论.

二、计 算 原 理

我们采用约化矩阵元方法，这种方法在文献[5]中已作详述. 逐步计算原理方法是:

设 $G\supset g$，群 G 的不可约表示由 (Λ) 表示，而子群 g 的不可约表示由 (λ_i) 表示. 它们满足

$$(\Lambda)=\sum_i(\alpha_i\lambda_i), \tag{2.1}$$

其中 α_i 是多重指标. 则有[4]

$$\sum_{\alpha_{1,23}}\langle\Lambda_1(\Lambda_2\Lambda_3)\alpha_{23}\Lambda_{23},\ \alpha_{1,23}\Lambda|(\Lambda_1\Lambda_2)\alpha_{12}\Lambda_{12}\Lambda_3\alpha_{12,3}\Lambda\rangle\langle\Lambda_1\alpha_1\lambda_1,\ \Lambda_{23}\alpha_{23}\lambda_{23}\|\Lambda\alpha\lambda\alpha_{1,23}\rangle$$
$$=\sum_{\substack{\alpha_2\lambda_2,\alpha_3\lambda_3,\\ \alpha_{12}\lambda_{12}}}\langle\Lambda_1\alpha_1\lambda_1,\ \Lambda_2\alpha_2\lambda_2\|\Lambda_{12}\alpha_{12}\lambda_{12}\rangle\langle\Lambda_2\alpha_2\lambda_2,\ \Lambda_3\alpha_3\lambda_3\|\Lambda_{23}\alpha_{23}\lambda_{23}\rangle$$
$$\cdot\langle\Lambda_{12}\alpha_{12}\lambda_{12},\ \Lambda_3\alpha_3\lambda_3\|\Lambda\alpha_{12,3}\lambda\rangle\langle\lambda_1\lambda_2(\lambda_{12})\lambda_3\lambda|\lambda_1\lambda_2\lambda_3(\lambda_{23})\lambda\rangle, \tag{2.2}$$

其中 $\langle\Lambda_1(\Lambda_2\Lambda_3)\alpha_{23}\Lambda_{23},\ \alpha_{1,23}\Lambda|(\Lambda_1\Lambda_2)\alpha_{12}\Lambda_{12}\Lambda_3\alpha_{12,3}\Lambda\rangle$ 和 $\langle\lambda_1\lambda_2(\lambda_{12})\lambda_3\lambda\|\lambda_1(\lambda_2\lambda_3)(\lambda_{23})\lambda\rangle$ 分别是群 G 和 g 的广义 6j 系数; α_{12}, α_{23}, $\alpha_{12,3}$ 等是区别多重态的指标;

$$\langle\Lambda_1\alpha_1\lambda_1\Lambda_{23}\alpha_{23}\lambda_{23}\|\Lambda\alpha\lambda\rangle\cdots\cdots$$

等等是群 $G\supset g$ 的同位标量因子.

相应 6j 系数应满足正交条件[4]

$$\sum_{\alpha_{23}\Lambda_{23}}\langle(\Lambda_1\Lambda_2)\alpha_{12}\Lambda_{12}\Lambda_3;\ \alpha\Lambda|\Lambda_1(\Lambda_2\Lambda_3)\alpha_{23}\Lambda_{23};\ \alpha'\Lambda\rangle^*\langle\Lambda_1(\Lambda_2\Lambda_3)\alpha_{23}\Lambda_{23};\ \alpha'\Lambda|(\Lambda_1\Lambda_2)\alpha'_{12}\Lambda'_{12}\Lambda_3;\ \alpha\Lambda\rangle$$
$$=\delta_{\alpha_{12},\alpha'_{12}}\delta_{\Lambda_{12},\Lambda'_{12}}\delta_{\alpha\alpha'}. \tag{2.3}$$

设 $SO(6)$ 群的不可约表示由 $(\sigma_1\sigma_2\sigma_3)$ 表示，$SO(5)$ 群的不可约表示由 $\langle\tau_1\tau_2\rangle$ 表示，利用 (2.2) 式和 (2.3) 式可算出 $SO(6)$ 和 $SO(5)$ 部分 6j 系数如下.

$$((\sigma00)((100)(1/2\ 1/2\ 1/2))(3/2\ 1/2\ 1/2)(\sigma+3/2\ 1/2\ 1/2)|$$
$$\cdot((\sigma00)(100))(\sigma+1,00)(1/2\ 1/2\ 1/2)(\sigma+3/2\ 1/2\ 1/2))=1,$$
$$((\sigma00)((100)(1/2\ 1/2\ 1/2))(3/2\ 1/2\ 1/2)(\sigma-3/2\ 1/2-1/2)|$$
$$\cdot((\sigma00)(100))(\sigma-10,0)(1/2\ 1/2\ 1/2)(\sigma-3/2\ 1/2-1/2))=1,$$
$$((\sigma00)((100)(1/2\ 1/2\ 1/2))(3/2\ 1/2\ 1/2)(\sigma+1/2\ 3/2\ 1/2)|$$
$$\cdot((\sigma00)(100))(\sigma10)(1/2\ 1/2\ 1/2)(\sigma+1/2\ 3/2\ 1/2))=1,$$
$$((\sigma00)((100)(1/2\ 1/2\ 1/2))(3/2\ 1/2\ 1/2)(\sigma-1/2\ 3/2-1/2)|$$
$$\cdot((\sigma00)(100))(\sigma10)(1/2\ 1/2\ 1/2)(\sigma-1/2\ 3/2-1/2))=1,$$
$$((\sigma00)((100)(1/2\ 1/2\ 1/2))(1/2\ 1/2-1/2)(\sigma-1/2\ 1/2\ 1/2)|$$
$$\cdot((\sigma00)(100))(\sigma-1,00)(1/2\ 1/2\ 1/2)(\sigma-1/2\ 1/2\ 1/2))$$
$$=-\left(\frac{\sigma+1}{3(\sigma+3)}\right)^{1/2},$$

$$((\sigma 00)((100)(1/2\ 1/2\ 1/2))(3/2\ 1/2\ 1/2)(\sigma-1/2\ 1/2\ 1/2)|$$
$$\cdot((\sigma 00)(100))(\sigma-1,00)(1/2\ 1/2\ 1/2)(\sigma-1/2\ 1/2\ 1/2))$$
$$=-\left(\frac{2(\sigma+4)}{3(\sigma+3)}\right)^{1/2},$$

$$((\sigma 00)((100)(1/2\ 1/2\ 1/2))(1/2\ 1/2-1/2)(\sigma-1/2\ 1/2\ 1/2)|$$
$$\cdot((\sigma 00)(100))(\sigma 10)(1/2\ 1/2\ 1/2)(\sigma-1/2\ 1/2\ 1/2))$$
$$=-\left(\frac{2(\sigma+4)}{3(\sigma+3)}\right)^{1/2},$$

$$((\sigma 00)((100)(1/2\ 1/2\ 1/2))(3/2\ 1/2\ 1/2)(\sigma-1/2\ 1/2\ 1/2)|$$
$$\cdot((\sigma 00)(100))(\sigma 10)(1/2\ 1/2\ 1/2)(\sigma-1/2\ 1/2\ 1/2))$$
$$=\left(\frac{\sigma+1}{3(\sigma+3)}\right)^{1/2},$$

$$((\sigma 00)((100)(1/2\ 1/2\ 1/2))(1/2\ 1/2-1/2)(\sigma+1/2\ 1/2-1/2)|$$
$$\cdot((\sigma 00)(100))(\sigma+1,0,0)(1/2\ 1/2\ 1/2)(\sigma+1/2\ 1/2-1/2))$$
$$=-\left(\frac{\sigma+3}{3(\sigma+1)}\right)^{1/2},$$

$$((\sigma 00)((100)(1/2\ 1/2\ 1/2))(3/2\ 1/2\ 1/2)(\sigma+1/2\ 1/2-1/2)|$$
$$\cdot((\sigma 00)(100))(\sigma+1,0,0)(1/2\ 1/2\ 1/2)(\sigma+1/2\ 1/2-1/2))$$
$$=-\left(\frac{2\sigma}{3(\sigma+1)}\right)^{1/2},$$

$$((\sigma 00)((100)(1/2\ 1/2\ 1/2))(1/2\ 1/2-1/2)(\sigma+1/2\ 1/2-1/2)|$$
$$\cdot((\sigma 00)(100))(\sigma 10)(1/2\ 1/2\ 1/2)(\sigma+1/2\ 1/2-1/2))$$
$$=-\left(\frac{2\sigma}{3(\sigma+1)}\right)^{1/2},$$

$$((\sigma 00)((100)(1/2\ 1/2\ 1/2))(3/2\ 1/2\ 1/2)(\sigma+1/2\ 1/2-1/2)|$$
$$\cdot((\sigma 00)(100))(\sigma 10)(1/2\ 1/2\ 1/2)(\sigma+1/2\ 1/2-1/2))$$
$$=\left(\frac{\sigma+3}{3(\sigma+1)}\right)^{1/2}.$$

$SO(5)$ 的 $6j$ 系数如下

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 3/2\ 1/2\rangle\langle\tau+3/2\ 1/2\rangle|$$
$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau+1,0\rangle\langle 1/2\ 1/2\rangle\langle\tau+3/2\ 1/2\rangle\rangle=1,$$

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 3/2\ 1/2\rangle\langle\tau+1/2,3/2\rangle|$$
$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau 1\rangle\langle 1/2\ 1/2\rangle\langle\tau+1/2\ 3/2\rangle\rangle=1,$$

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 3/2\ 1/2\rangle\langle\tau-1/2\ 3/2\rangle|$$
$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau 1\rangle\langle 1/2\ 1/2\rangle\langle\tau-1/2\ 3/2\rangle\rangle=1,$$

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 3/2\ 1/2\rangle\langle\tau-3/2\ 1/2\rangle|$$
$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau-1,0\rangle\langle 1/2\ 1/2\rangle\langle\tau-3/2\ 1/2\rangle\rangle=-1,$$

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 3/2\ 1/2\rangle\langle\tau+1/2\ 1/2\rangle|$$
$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau 1\rangle\langle 1/2\ 1/2\rangle\langle\tau+1/2\ 1/2\rangle\rangle=-\left\langle\frac{2\tau+5}{5\langle\tau+1\rangle}\right\rangle^{1/2},$$

表 1 $SO(6)\supset SO(5)$ 同位标量因子

$(\sigma_1\sigma_2\sigma_3)$ / $\langle\tau_1\tau_2\rangle$ $\langle v\ 1/2\rangle$	$(\sigma+3/2\ 1/2\ 1/2)$	$(\sigma+1/2\ 3/2\ 1/2)$	$(\sigma+1/2\ 1/2-1/2)$
$\langle\tau+3/2\ 1/2\rangle$ $\langle 3/2\ 1/2\rangle$	$[(\sigma+\tau+4)(\sigma+\tau+5)(\sigma+\tau+6)(\tau+1)/4(\sigma+1)(\sigma+2)\times(\sigma+3)(2\tau+5)]^{1/2}$	$(-)[3(\sigma+\tau+5)(\sigma+\tau+4)\times(\sigma-\tau)(\tau+5)/8(\sigma+1)\times(\sigma+2)(\sigma+4)(2\tau+5)]^{1/2}$	$(-)[3(\sigma+\tau+4)(\sigma+\tau+5)(\sigma-\tau)(\tau+1)/8(\sigma+2)(\sigma+3)\sigma(2\tau+5)]^{1/2}$
$\langle\tau+1/2\ 1/2\rangle$ $\langle 3/2\ 1/2\rangle$	$(-)[3(\sigma+\tau+4)(\sigma+\tau+5)(\sigma-\tau+1)\tau/20(\sigma+1)(\sigma+2)\times(\sigma+3)(2\tau+5)]^{1/2}$	$[(\sigma+\tau+4)(5\sigma-3\tau+5)^2(\tau+4)/40(\sigma+1)(\sigma+2)(\sigma+4)\times(2\tau+5)]^{1/2}$	$(-)[9(\sigma+\tau+4)(\sigma+\tau+5)^2\tau/40(\sigma+2)(\sigma+3)\sigma(2\tau+5)]^{1/2}$
$\langle\tau-1/2\ 1/2\rangle$ $\langle 3/2\ 1/2\rangle$	$[3(\sigma-\tau+1)(\sigma-\tau+2)(\sigma+\tau+4)(\tau+3)/20(\sigma+1)(\sigma+2)\times(\sigma+3)(2\tau+1)]^{1/2}$	$[(\sigma-\tau+1)(\tau-1)(5\sigma+3\tau+14)^2/40(\sigma+1)(\sigma+2)(\sigma+4)(2\tau+1)]^{1/2}$	$(-)[9(\sigma-\tau+1)(\sigma-\tau+2)^2(\tau+3)/40(\sigma+2)(\sigma+3)\sigma\times(2\tau+1)]^{1/2}$
$\langle\tau-3/2\ 1/2\rangle$ $\langle 3/2\ 1/2\rangle$	$[(\sigma-\tau+1)(\sigma-\tau+2)(\sigma-\tau+3)(\tau+2)/4(\sigma+1)(\sigma+2)\times(\sigma+3)(2\tau+1)]^{1/2}$	$[3(\sigma-\tau+1)(\sigma+\tau+3)(\sigma-\tau+2)(\tau-2)/8(\sigma+1)(\sigma+2)\times(\sigma+4)(2\tau+1)]^{1/2}$	$[3(\sigma-\tau+1)(\sigma-\tau+2)(\sigma+\tau+3)(\tau+2)/8(\sigma+2)(\sigma+3)\sigma\times(2\tau+1)]^{1/2}$
$\langle\tau+1/2\ 1/2\rangle$ $\langle 1/2\ 1/2\rangle$	$[3(\sigma-\tau+1)(\sigma+\tau+4)(\sigma+\tau+5)/10(\sigma+1)(\sigma+2)\times(\sigma+3)]^{1/2}$	$[9(\sigma+\tau+4)(\tau+4)\tau/20(\sigma+1)\times(\sigma+2)(\sigma+4)]^{1/2}$	$(-)[(\sigma+\tau+4)(2\sigma-3\tau)^2/20(\sigma+2)(\sigma+3)\sigma]^{1/2}$
$\langle\tau-1/2\ 1/2\rangle$ $\langle 1/2\ 1/2\rangle$	$[3(\sigma-\tau+1)(\sigma-\tau+2)(\sigma+\tau+4)/10(\sigma+1)(\sigma+2)\times(\sigma+3)]^{1/2}$	$[9(\sigma-\tau+1)(\tau+3)(\tau-1)/20\times(\sigma+1)(\sigma+2)(\sigma+4)]^{1/2}$	$[(\sigma-\tau+1)(2\sigma+3\tau+9)^2/20(\sigma+2)(\sigma+3)\sigma]^{1/2}$

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 1/2\ 1/2\rangle\langle\tau+1/2\ 1/2\rangle|$$

$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau 1\rangle\langle 1/2\ 1/2\rangle\langle\tau+1/2\ 1/2\rangle\rangle=-\left\langle\frac{3\tau}{5\langle\tau+1\rangle}\right\rangle^{1/2},$$

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 3/2\ 1/2\rangle\langle\tau-1/2\ 1/2\rangle|$$

$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau 1\rangle\langle 1/2\ 1/2\rangle\langle\tau-1/2\ 1/2\rangle\rangle=\left\langle\frac{2\tau+1}{5\langle\tau+2\rangle}\right\rangle^{1/2},$$

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 1/2\ 1/2\rangle\langle\tau-1/2\ 1/2\rangle|$$

$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau 1\rangle\langle 1/2\ 1/2\rangle\langle\tau-1/2\ 1/2\rangle\rangle=-\left\langle\frac{3\langle\tau+3\rangle}{5\langle\tau+2\rangle}\right\rangle^{1/2},$$

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 1/2\ 1/2\rangle\langle\tau+1/2\ 1/2\rangle|$$

$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau+1,0\rangle\langle 1/2\ 1/2\rangle\langle\tau+1/2\ 1/2\rangle\rangle=\left\langle\frac{2\tau+5}{5\langle\tau+1\rangle}\right\rangle^{1/2},$$

$$\langle\langle\tau_0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 3/2\ 1/2\rangle\langle\tau+1/2\ 1/2\rangle|$$

$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau+1,0\rangle\langle 1/2\ 1/2\rangle\langle\tau+1/2\ 1/2\rangle\rangle=-\left\langle\frac{3\tau}{5\langle\tau+1\rangle}\right\rangle^{1/2},$$

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 1/2\ 1/2\rangle\langle\tau-1/2\ 1/2\rangle|$$

$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\rangle\langle\tau-1,0\rangle\langle 1/2\ 1/2\rangle\langle\tau-1/2\ 1/2\rangle\rangle=-\left\langle\frac{2\tau+1}{5\langle\tau+2\rangle}\right\rangle^{1/2},$$

$$\langle\langle\tau 0\rangle\langle\langle 10\rangle\langle 1/2\ 1/2\rangle\rangle\langle 3/2\ 1/2\rangle\langle\tau-1/2\ 1/2\rangle|$$

$$\cdot\langle\langle\tau 0\rangle\langle 10\rangle\langle\tau-1,0\rangle\langle 1/2\ 1/2\rangle\langle\tau-1/2\ 1/2\rangle\rangle=-\left\langle\frac{3\langle\tau+3\rangle}{5\langle\tau+2\rangle}\right\rangle^{1/2}.$$

$$\begin{pmatrix} (\sigma 00) & (3/2\,1/2\,1/2) & \| & (\sigma_1\,\sigma_2\,\sigma_3) \\ \langle \tau\ 0\rangle & \langle v\ 1/2\rangle & \| & \langle \tau_1 \tau_2\rangle \end{pmatrix}$$

$(\sigma-1/2\ \ 3/2-1/2)$	$(\sigma-1/2\ \ 1/2\ \ 1/2)$	$(\sigma-3/2\ \ 1/2-1/2)$
$[3(\sigma-\tau)(\sigma+\tau+4)(\sigma-\tau-1)(\tau+5)/8\sigma(\sigma+2)(\sigma+3)(2\tau+5)]^{1/2}$	$[3(\sigma-\tau-1)(\sigma+\tau+4)(\sigma-\tau)(\tau+1)/8(\sigma+1)(\sigma+2)(\sigma+4)(2\tau+5)]^{1/2}$	$(-)[(\sigma-\tau-1)(\sigma-\tau-2)(\sigma-\tau)(\tau+1)/4(\sigma+1)(\sigma+2)(\sigma+3)\times(2\tau+5)]^{1/2}$
$[(\sigma-\tau)(5\sigma+3\tau+15)^2(\tau+4)/40(\sigma+2)\sigma(\sigma+3)(2\tau+5)]^{1/2}$	$(-)[9(\sigma-\tau-1)^2(\sigma-\tau)\tau/40(\sigma+1)(\sigma+2)(\sigma+4)(2\tau+5)]^{1/2}$	$(-)[3(\sigma-\tau-1)(\sigma-\tau)(\sigma+\tau+3)\tau/20(\sigma+1)(\sigma+2)(\sigma+3)(2\tau+5)]^{1/2}$
$(-)[(\sigma+\tau+3)(5\sigma-3\tau+6)^2(\tau-1)/40(\sigma+2)\sigma(\sigma+3)(2\tau+1)]^{1/2}$	$[9(\sigma+\tau+2)^2(\sigma+\tau+3)(\tau+3)/40(\sigma+1)(\sigma+2)(\sigma+4)(2\tau+1)]^{1/2}$	$(-)[3(\sigma+\tau+2)(\sigma+\tau+3)(\sigma-\tau)\times(\tau+3)/20(\sigma+1)(\sigma+2)(\sigma+3)\times(2\tau+1)]^{1/2}$
$[3(\sigma-\tau+1)(\sigma+\tau+3)(\sigma+\tau+2)\times(\tau-2)/8(\sigma+2)(\sigma+3)\sigma(2\tau+1)]^{1/2}$	$[3(\sigma+\tau+2)(\sigma+\tau+3)(\sigma-\tau+1)\times(\tau+2)/8(\sigma+1)(\sigma+2)(\sigma+4)\times(2\tau+1)]^{1/2}$	$[(\sigma+\tau+2)(\sigma+\tau+3)(\sigma+\tau+1)\times(\tau+2)/4(\sigma+1)(\sigma+2)(\sigma+3)\times(2\tau+1)]^{1/2}$
$(-)[9(\sigma-\tau)(\tau+4)\tau/20(\sigma+2)(\sigma+3)\sigma]^{1/2}$	$(-)[(\sigma-\tau)(2\sigma+3\tau+8)^2/20(\sigma+1)(\sigma+2)(\sigma+4)]^{1/2}$	$[3(\sigma-\tau)(\sigma+\tau+3)(\sigma-\tau-1)/10(\sigma+1)(\sigma+2)(\sigma+3)]^{1/2}$
$[9(\sigma+\tau+3)(\tau-1)(\tau+3)/20(\sigma+2)\times(\sigma+3)\sigma]^{1/2}$	$(-)[(\sigma+\tau+3)(2\sigma-3\tau-1)^2/20(\sigma+1)(\sigma+2)(\sigma+4)]^{1/2}$	$(-)[3(\sigma-\tau)(\sigma+\tau+3)(\sigma+\tau+2)/10(\sigma+1)(\sigma+2)(\sigma+3)]^{1/2}$

三、计 算 结 果

根据第二节中的计算原理和两个群的 $6j$ 系数的结果，可以算得部分 $SO(6)\supset SO(5)$ 的同位标量因子．其结果列于表 1．表 1 所示的是 $SO(6)\supset SO(5)$ 同位标量因子

$$\begin{pmatrix} (\sigma\,0\,0) & (3/2\,1/2\,1/2) & \| & (\sigma_1\,\sigma_2\,\sigma_3) \\ \langle \tau 0\rangle & \langle v\,1/2\rangle & \| & \langle \tau_1\,\tau_2\rangle \end{pmatrix} \tag{3.1}$$

的值．

其中相规约为，令 $\langle \tau^m\rangle$ 是 $SO(6)$ 的不可约表示 (σ) 约化到群 $SO(5)$ 的不可约表示 $\langle \tau_i\rangle$ 的最高权，则有

$$\begin{pmatrix} (\sigma_1) & (\sigma_2) & \| & (\sigma) \\ \langle \tau_1^m\rangle & \langle \tau_2\rangle & \| & \langle \tau^m\rangle \end{pmatrix} > 0, \tag{3.2}$$

同样对第二节中两个群的 $6j$ 系数也有类似的相规约．

经过冗长乏味的计算，我们所求的同位标量因子满足

$$\sum_{\alpha,\sigma} \begin{pmatrix} (\sigma') & (\sigma'') & \| & \alpha & (\sigma) \\ \alpha'\langle \tau'\rangle, & \alpha''\langle \tau''\rangle & \| & & \langle \tau\rangle \end{pmatrix}^* \begin{pmatrix} (\sigma) & \alpha & \| & (\sigma') & (\sigma'') \\ \langle \tau\rangle & & \| & \alpha'''\langle \tau'''\rangle, & \alpha''''\langle \tau''''\rangle \end{pmatrix}$$
$$= \delta_{\tau',\tau'''}\delta_{\tau''\tau''''}\delta_{\alpha',\alpha'''}\delta_{\alpha''\alpha''''}, \tag{3.3}$$

$$\sum_{\substack{\tau'\tau'' \\ \alpha'\alpha''}} \begin{pmatrix} \alpha & (\sigma) & \| & (\sigma') & (\sigma'') \\ & \langle \tau\rangle & \| & \alpha'\langle \tau'\rangle, & \alpha''\langle \tau''\rangle \end{pmatrix}^* \begin{pmatrix} (\sigma') & (\sigma'') & \| & (\sigma''') & \\ \alpha'\langle \tau'\rangle, & \alpha''\langle \tau''\rangle & \| & \langle \tau\rangle & \alpha''' \end{pmatrix} = \delta_{\alpha\alpha'''}\delta_{\sigma\sigma'''}, \tag{3.4}$$

表 2 $SO(6)\supset SO(5)$ 同位标量因子 $\left(\begin{matrix}(\sigma|0) & (1/2\ 1/2\ 1/2) \\ \langle\tau 0\rangle & \langle 1/2\ 1/2\rangle\end{matrix}\middle\|\begin{matrix}(\sigma_1\sigma_2\sigma_3) \\ \langle\tau_1\tau_2\rangle\end{matrix}\right)$, $\left(\begin{matrix}(\sigma|0) & (1/2\ 1/2\ 1/2) \\ \langle\tau 1\rangle & \langle 1/2\ 1/2\rangle\end{matrix}\middle\|\begin{matrix}(\sigma_1\sigma_2\sigma_3) \\ \langle\tau_1\tau_2\rangle\end{matrix}\right)$

$(\sigma_1\sigma_2\sigma_3)$ $\langle\tau_1\tau_2\rangle$	$(\sigma+1/2\ 3/2\ 1/2)$ $\langle\tau+1/2\ 1/2\rangle$	$(\sigma+1/2\ 3/2\ 1/2)$ $\langle\tau-1/2\ 1/2\rangle$	$(\sigma+1/2\ 1/2\ -1/2)$ $\langle\tau+1/2\ 1/2\rangle$	$(\sigma+1/2\ 1/2\ -1/2)$ $\langle\tau-1/2\ 1/2\rangle$
$\langle\tau 0\rangle$	$\left[\frac{3(\sigma+\tau+4)(\sigma+3)(\tau+4)}{8(\sigma+2)(\sigma+4)(\tau+3)}\right]^{1/2}$	$\left[\frac{3(\sigma-\tau+1)(\sigma+3)(\tau-1)}{8(\sigma+2)(\sigma+4)\tau}\right]^{1/2}$	$\left[\frac{(\sigma+\tau+4)(\sigma+3)\tau}{8(\sigma+2)\sigma(\tau+3)}\right]^{1/2}$	$\left[\frac{(\sigma-\tau+1)(\sigma+3)(\tau+3)}{8(\sigma+2)\sigma\tau}\right]^{1/2}$
$\langle\tau 1\rangle$	$(-)\left[\frac{(\sigma+\tau+4)(\sigma+1)(\tau+4)}{8(\sigma+2)(\sigma+4)(\tau+1)}\right]^{1/2}$	$\left[\frac{(\sigma-\tau+1)(\sigma+1)(\tau-1)}{8(\sigma+2)(\sigma+4)(\tau+2)}\right]^{1/2}$	$\left[\frac{3(\sigma+\tau+4)(\sigma+1)\tau}{8(\sigma+2)\sigma(\tau+1)}\right]^{1/2}$	$(-)\left[\frac{3(\sigma+1)(\sigma-\tau+1)(\tau+3)}{8(\sigma+2)\sigma(\tau+2)}\right]^{1/2}$
$(\sigma_1\sigma_2\sigma_3)$ $\langle\tau_1\tau_2\rangle$	$(\sigma-1/2\ 3/2\ -1/2)$ $\langle\tau+1/2\ 1/2\rangle$	$(\sigma-1/2\ 3/2\ -1/2)$ $\langle\tau-1/2\ 1/2\rangle$	$(\sigma-1/2\ 1/2\ 1/2)$ $\langle\tau+1/2\ 1/2\rangle$	$(\sigma-1/2\ 1/2\ 1/2)$ $\langle\tau-1/2\ 1/2\rangle$
$\langle\tau 0\rangle$	$(-)\left[\frac{3(\sigma+1)(\sigma-\tau)(\tau+4)}{8(\sigma+2)\sigma(\tau+3)}\right]^{1/2}$	$\left[\frac{3(\sigma+\tau+3)(\sigma+1)(\tau-1)}{8(\sigma+2)\sigma\tau}\right]^{1/2}$	$(-)\left[\frac{(\sigma+1)(\sigma-\tau)\tau}{8(\sigma+2)(\sigma+4)(\tau+3)}\right]^{1/2}$	$\left[\frac{(\sigma+\tau+3)(\sigma+1)(\tau+3)}{8(\sigma+2)(\sigma+4)\tau}\right]^{1/2}$
$\langle\tau 1\rangle$	$(-)\left[\frac{(\sigma-\tau)(\sigma+3)(\tau+4)}{8(\sigma+2)\sigma(\tau+1)}\right]^{1/2}$	$(-)\left[\frac{(\sigma+\tau+3)(\sigma+3)(\tau-1)}{8(\sigma+2)\sigma(\tau+2)}\right]^{1/2}$	$\left[\frac{3(\sigma-\tau)(\sigma+3)\tau}{8(\sigma+2)(\sigma+4)(\tau+1)}\right]^{1/2}$	$\left[\frac{3(\sigma+\tau+3)(\sigma+3)(\tau+3)}{8(\sigma+4)(\sigma+2)(\tau+2)}\right]^{1/2}$

(3.3) 和 (3.4) 式正交归一条件

四、结　论

因上述计算原理还可以计算另外一些 $SO(6)\supset SO(5)$ 的同位标量因子，正如表 2 所示，它们分别表明

$$\left(\begin{matrix}(\sigma 10) & (1/2\ 1/2\ 1/2) \\ \langle\tau 0\rangle & \langle 1/2\ 1/2\rangle\end{matrix}\right\Vert\left.\begin{matrix}(\sigma_1\sigma_2\sigma_2) \\ \langle\tau_1\tau_2\rangle\end{matrix}\right) \text{ 和 } \left(\begin{matrix}(\sigma 10) & (1/2\ 1/2\ 1/2) \\ \langle\tau 1\rangle & \langle 1/2\ 1/2\rangle\end{matrix}\right\Vert\left.\begin{matrix}(\sigma_1\sigma_2\sigma_3) \\ \langle\tau_1\tau_2\rangle\end{matrix}\right)$$

两个同位标量因子，其它还可以计算下去，但是对讨论奇奇核各种超对称性的超多重态是够用了．再者计算下去两个不可约表示的克罗内克的积出现多重性，这样就要引入附加指标来区别同一个不可约表示．这种情况我们以后再作讨论．

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SOME ISOSCALAR FACTORS FOR THE GROUP CHAIN $SO(6)\supset SO(5)$

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ABSTRACT

Some isoscalar factors for the group chain $SO(6)\supset SO(5)$ are calculated by using the method of reduced matrix elements and the building-up principle.